A New Approach to the Flux Rope Dynamo

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Introduction

- The dynamo effect, namely the amplification of a weak magnetic seed field by the motion of an electrically conducting fluid, is the most likely explanation for the omnipresence of magnetic fields in the universe.
- A dynamo can take two forms, the large scale dynamo, which generates magnetic fields with scales larger than that of the fluid motion.
- There also exists a small scale or *fluctuation* dynamo, where the largest scale of the magnetic field is comparable to the largest scale of motion.
- Here we present preliminary results for a new technique to model the fluctuation dynamo, using a 'synthetic' turbulent flow.

Motivation

The normal procedure for modelling the small scale dynamo is to numerically solve both the induction equation

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B}) + \eta \nabla^2 \mathbf{B}, \tag{1}$$

and the Navier-Stokes equation.

$$\rho \left(\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla)\mathbf{u} \right) = -\nabla p + \mathbf{j} \times \mathbf{B} + \nu \nabla^2 \mathbf{u} + \mathbf{F}. \tag{2}$$

Imposing both $\nabla \cdot \mathbf{B} = 0$, magnetic field is divergence free, and $\nabla \cdot \mathbf{u} = 0$, for incompressability. For the purpose of this study we shall assume the strength of the magnetic field is weak enough to ignore the effect of the Lorentz force, $\mathbf{j} \times \mathbf{B}$. Much progress has been, using this approach, in furthering our understanding of the mechanisms present in the fluctuation dynamo. In particular in understanding the kinematic phase in the high $\mathrm{Pr_m}$ limit. However, in rarefied plasma's, such as the solar corona and galactic halos, the magnetic field is believed to be concentrated in thin flux ropes. Here diffusion only acts over short length-scales, through magnetic reconnections, heuristically this can be thought of as seeking a solution to

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B}) + \eta \nabla^n \mathbf{B}, \tag{3}$$

where $n \to \infty$. By representing thin flux tubes as discretised lines, frozen into the flow, we introduce a new way to model the fluctuation dynamo numerically. Before describing the algorithms of the model we introduce the velocity field

Velocity field

Due to the limitations of DNS we have used a model of a turbulent flow, the Kinematic Simulation (KS) model. The KS model prescribes the flow velocity at a position $\mathbf{x}$ and time t through the summation of Fourier modes with randomly chosen parameters. These modes are mutually independent, therefore the advection of small eddies by large eddies is not included in the model. More precisely, the velocity field is prescribed to be

$$\mathbf{u}(\mathbf{x}, t) = \sum_{n=1}^{N} \left(\mathbf{A}_n \times \mathbf{k}_n \cos \psi_n + \mathbf{B}_n \times \mathbf{k}_n \sin \psi_n \right), \tag{4}$$

where $\psi_n = \mathbf{k}_n \cdot \mathbf{x} + \omega_n t$ and N is the number of modes. The unit vectors $\hat{\mathbf{k}}_n$ are chosen randomly, and $\mathbf{k}_n = k_n \hat{\mathbf{k}}_n$ where k_n is the wavenumber of the nth mode. We choose $\mathbf{A}_n$, and $\mathbf{B}_n$ randomly, imposing orthogonality with $\hat{\mathbf{k}}_n$, which gives the required spectrum as

$$|\mathbf{A}_n \times \hat{\mathbf{k}}_n| = A_n, \tag{5}$$

we proceed in the same fashion for $\mathbf{B}_n$. We then choose

$$A_n = B_n = \sqrt{\frac{2E(k_n)\Delta k_n}{3}}. \tag{6}$$

This ensures

$$\frac{1}{V} \int_V \frac{1}{2} |\mathbf{u}|^2 dV = \int_0^\infty E(k)\, dk \sim \sum_{n=1}^{N_k} E(k_n) \Delta k_n, \tag{7}$$

As we can see one of the main advantages of using the KS model is that we have complete control of the energy spectrum, $E(k_n)$, and in particular the slope of the spectrum. We adopt a normalised energy spectrum of the KS flow $E(k)$, which is a modification of the von Kármán energy spectrum,

$$E(k) = k^4 (1 + k^2)^{-(2+p/2)} e^{-1/2(k/k_N)^2}, \tag{8}$$

which reduces to $E(k) \propto k^{-p}$ in the inertial range $1 \ll k \ll k_N$, with $k = 1$ at the integral scale; $p = 5/3$ produces the Kolmogorov spectrum.

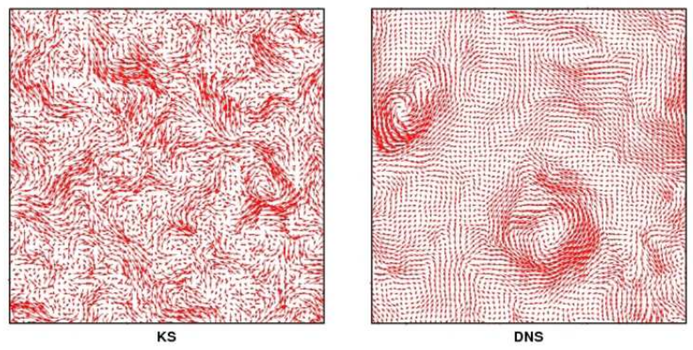

Figure 1: Comparison of velocity fields from KS and DNS with comparable Reynolds number. Despite the apparent differences, KS has been shown to be in good agreement with DNS for Lagrangian statistics.

The Flux Rope Model

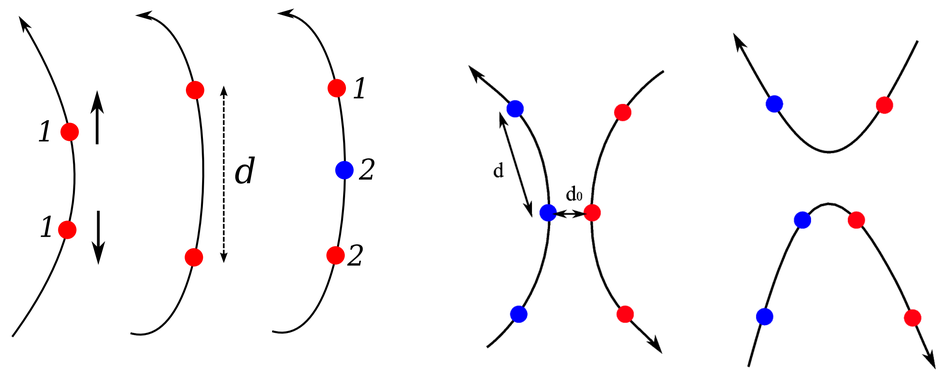

Figure 2: **(a)** Algorithm for introducing new points, and accounting for stretching of flux tube. **(b)** Reconnection algorithm commonly used in simulations of quantised vortices in superfluid Helium.

- Magnetic flux tubes are represented by discretised loops ($\nabla \cdot \mathbf{B}$) of fluid particles, whose only information is their position, and flags for particles behind and in front. We introduce a length scale d which acts as our resolution and introduce a new particle if the distance between any two particles on the loop becomes larger than d.
- When introducing new particles we must also account for the stretching of the flux tube, and effect on the strength of the magnetic field caused by reducing the width of a flux tube. $\psi = \int_S \mathbf{B} \cdot d\mathbf{S} \sim \mathcal{BA} = \text{constant}$, and $\mathcal{LA} = \text{constant}$. Hence, if we double the length, the strength of the magnetic field doubles.
- If the separation between two particles, which are not neighbours, becomes very close, they can reconnect. The distance which a reconnection can occur over is denoted by d_0 and is the diffusive length scale. We must also ensure that the orientations of the flux tube are sufficient to allow a reconnection, and ensure that parallel flux tubes with the same direction will not reconnect.

Testing the Model

As an initial test we consider a two dimensional shear flow with a Gaussian profile, $u_x = u_0 e^{-y^2/2}$. For this simple flow we can solve the induction equation (with $\eta = 0$) analytically, finding

$$|\mathbf{B}| = B_0 \sqrt{1 + u_0 y^2 e^{-y^2} t^2}. \tag{9}$$

We then subject our model to the same flow, and test that the total line length grows like $\int_{-\infty}^{\infty} B_0 \sqrt{1 + u_0 y^2 e^{-y^2} t^2} dy$, and also that at each point on the flux tube $|\mathbf{B}|$ correctly assigned.

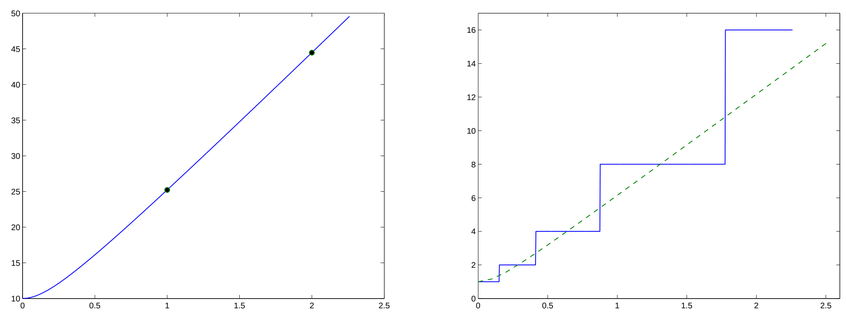

Figure 3: **(a)** t against $\mathcal{L}$ from flux rope model, two values of integral show good agreement. **(b)** t against $|\mathbf{B}|$ at $y = 1$ from model, dashed line gives value predicted from Eqn. 9

It is clear our model is doing a good job of approximating the true solution. The next stages of the project will involve testing the flux rope dynamo in the KS model. In particular we are interested in comparing this model with the induction equation and comparing rate of energy release, field morphology and curvature of the flux tubes and magnetic field strength.

Finally we note,

$$\mathbf{j} \times \mathbf{B} = (\nabla \times \mathbf{B}) \times \mathbf{B} = (\mathbf{B} \cdot \nabla)\mathbf{B} - \frac{1}{2} \nabla \mathbf{B}^2 = |\mathbf{B}| \frac{\partial \mathbf{B}}{\partial \ell}. \tag{10}$$

Leading on nicely to a non-linear flux tube model ...