Experimental Lagrangian statistics (1-point)

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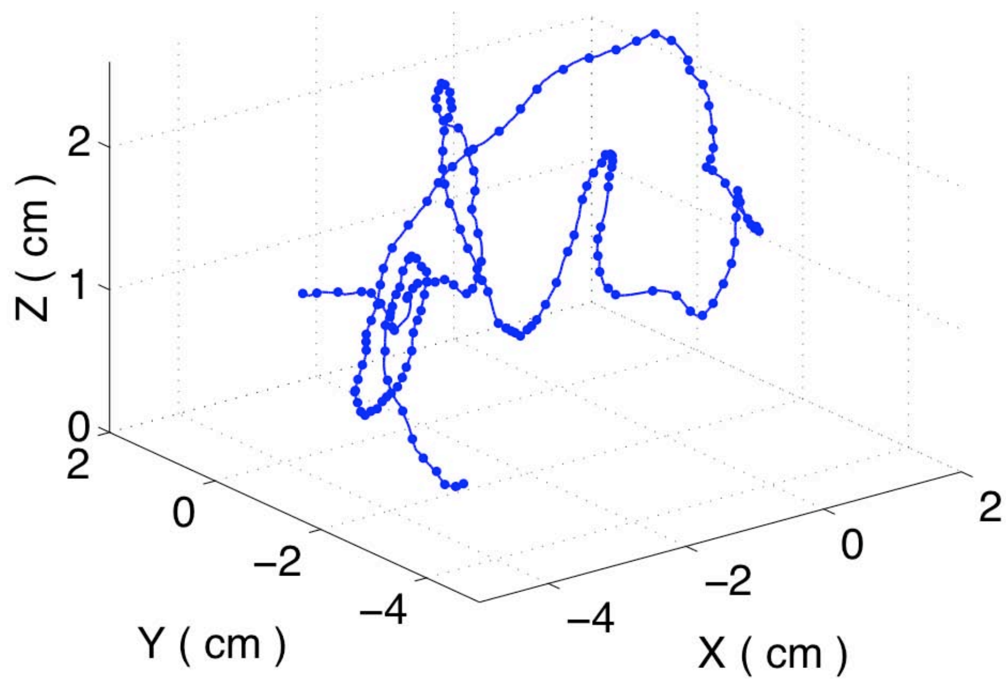
International Collaboration on Turbulence Research
http://ictr/cineca.it/ictr

Euler & Lagrange

- Euler : $\mathbf{u}(\mathbf{x}, t)$, $\mathbf{x}$ in {flow domain}

- Lagrange : $\mathbf{v}(\mathbf{x}_0, t)$, $\mathbf{x}_0$ in {initial positions}



1) **Motivations**
2) Measurement methods
3) “K41” measurements (2nd order qty)
4) Acceleration
5) Intermittency (higher orders)
6) [2-points: dispersion, Multipoints: gradients]
7) Note 1 : Inertial particles
8) Note 2 : Convection

Turbulent dispersion

B. Sawford, *Ann. Rev. Fluid Mech.*, **33** (2001)

$$\partial_t C + \vec{u}.\vec{\nabla} C = \kappa \Delta C$$

- $\kappa = 0$

$$C(\vec{x}, t) = \int_{s \leq t} \int_V d^3y dt \, p_1(\vec{x}, t; \vec{y}, s) \, S(\vec{y}, s)$$

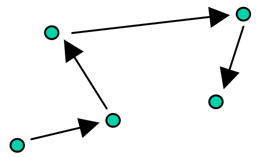
- $\kappa \neq 0$

Same equations at high Reynolds and Peclet numbers ,
Except very close to sources or boundaries
[Saffman, *JFM*, **8** (1960)]

The Kolmogorov picture (1 particle)

- Random walks

$$d\vec{V}(\vec{X},t) = -\gamma(\vec{V})dt + dG(t)$$



- White acceleration,

spectrum : $E_L^A(\omega) \propto \omega^0$

- Velocity spectrum,

$$E_L^V(\omega) = C_0 \epsilon\, \omega^{-2}$$

dimensionally : $\langle v(t)v(t+\tau)\rangle_t = C_0(\epsilon\tau)$

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Experiments Gifford - Hanna

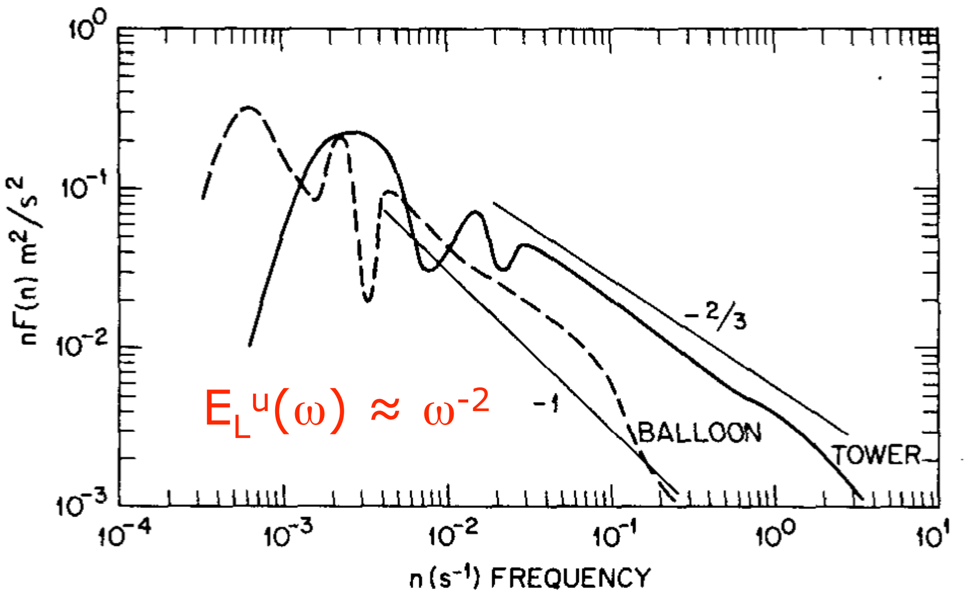
Gifford, *Month. Weath. Rev.*, **83**, 293, (1955)
Hanna, *J. Appl. Meteo.*, **20**, 242, (1981)

Neutral balloons + Doppler radar
- ratio $\beta = T_L / T_E$
- Re ≈ 25,000
- size 1m^3
- sampling 1Hz, 1h runs

NATIONAL CENTER FOR ATMOSPHERIC RESEARCH (NCAR)
BOULDER, CO 80307-3000

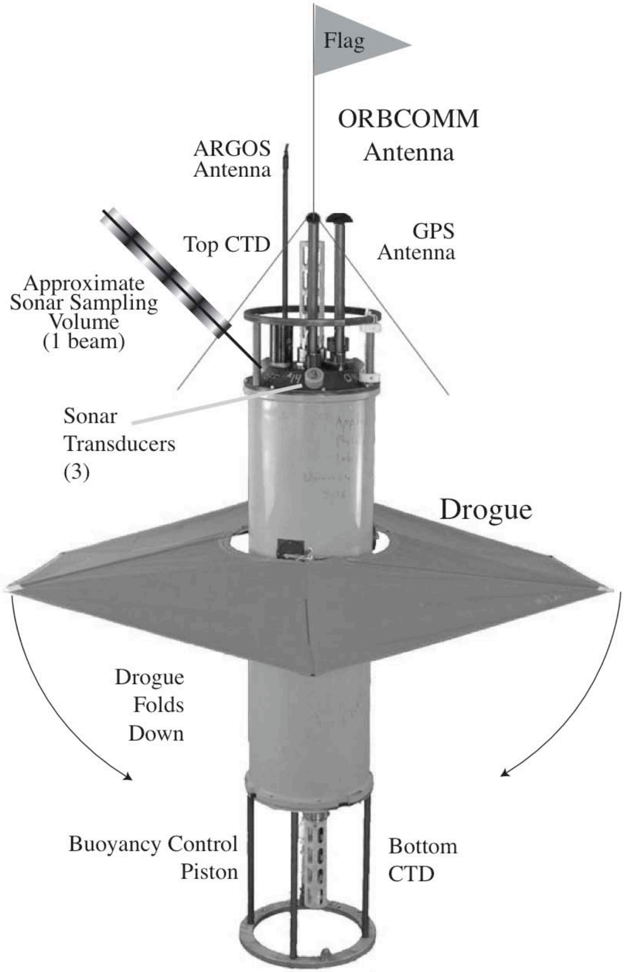
Revision A

May 1999

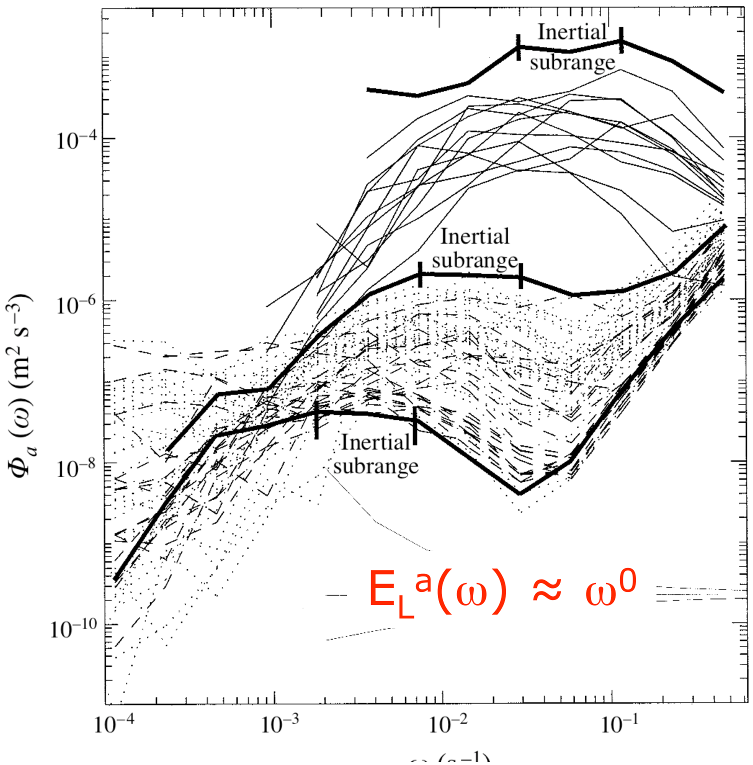


Experiments Lien D'Asaro

Lien, d'Asaro, Daikiri, *J. Fluid Mech.*, **362**, 177, (1998)



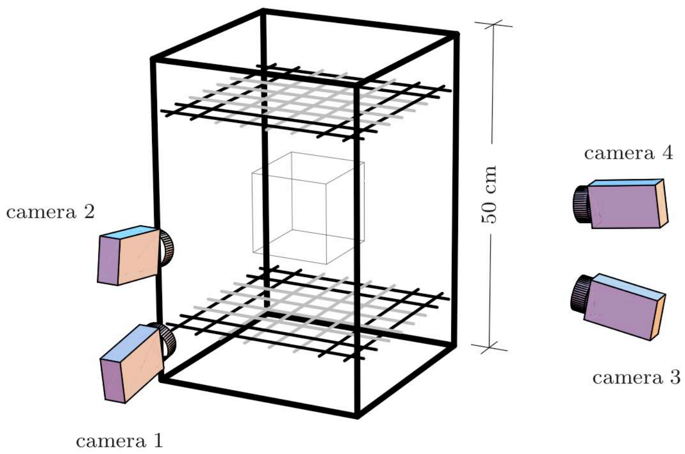
Floaters in ocean
- size 1.5m x 1.2m
- 1 Hz sampling
- 71 units

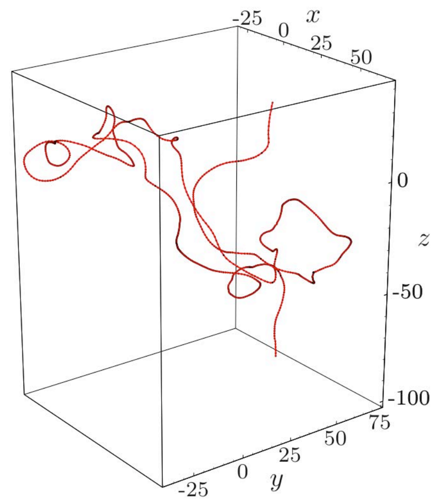


Experiment - Risø

Ott, Mann, *J. Fluid Mech.*, **422**, 207, (2000)

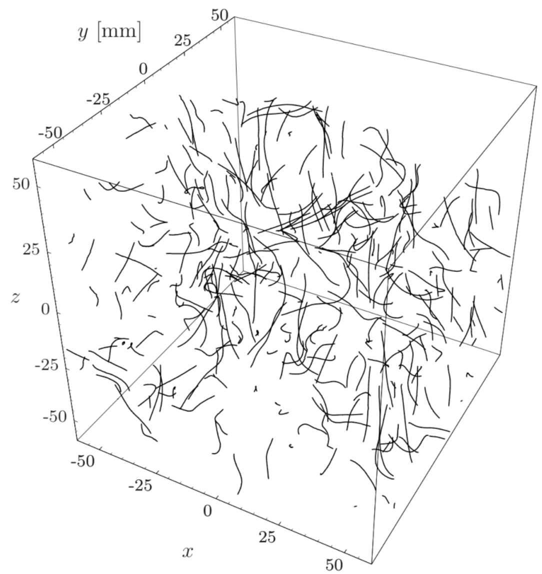
Snyder & Lumley, 1971
Sato & Yamamoto, 1987
Virant & Dracos, 1997





Particles : 600 μm
Pictures : 576 x 720 pixels
Rate : 25 images / sec
Record : 40 sec

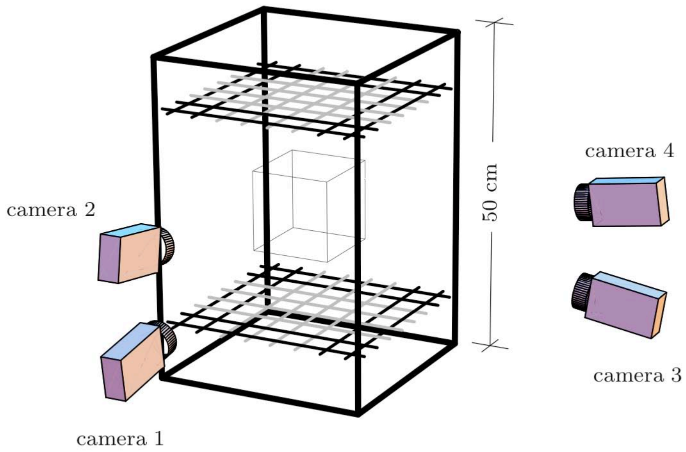
$R_\lambda \approx 90$

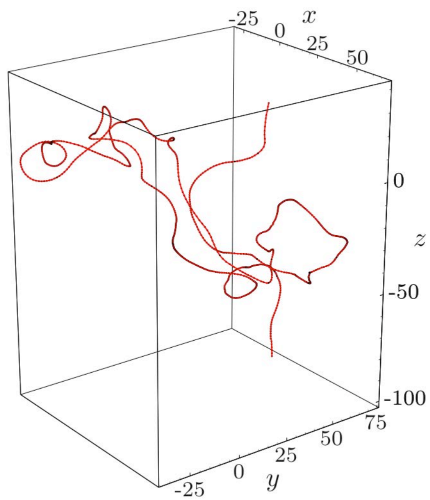


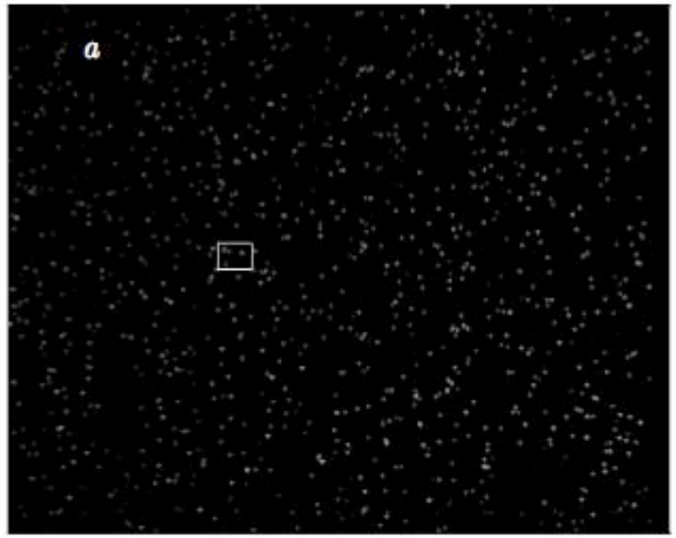
Particle Tracking Velocimetry

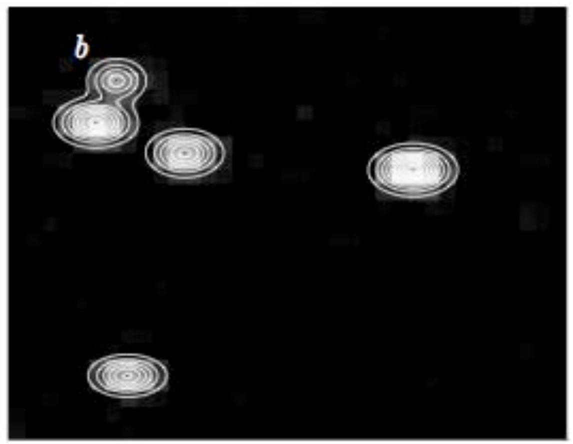
Ott, Mann, *J. Fluid Mech.*, **422**, 207, (2000)

>>Wiki software <<









Resolved PTV

Fully developped turbulence, $R_\lambda \approx 1000$

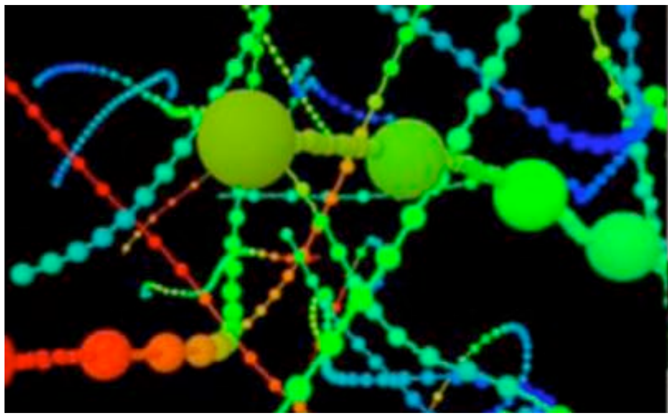
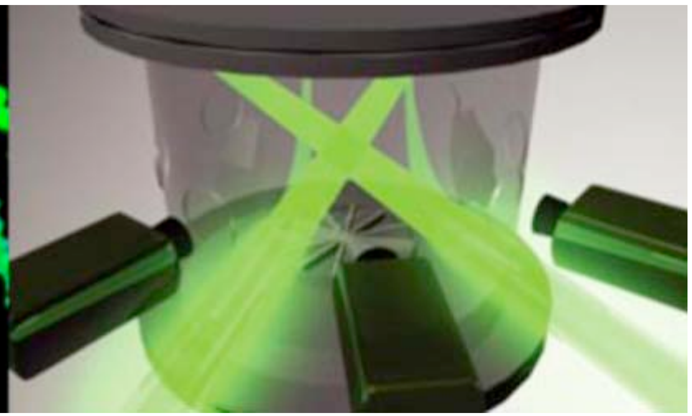
- Scale resolution : $L/\eta \approx 10{,}000$
- Time resolution : $T/\tau_\eta \approx 1{,}000$
- Lab experiment : $L \approx 10$ cm - 100 cm

  : $T \approx 0.1$ s - 1 s

  pixel size ≈ 10 μm (NB: max 1024^2)
  sample rate > 10 kHz
- Data size

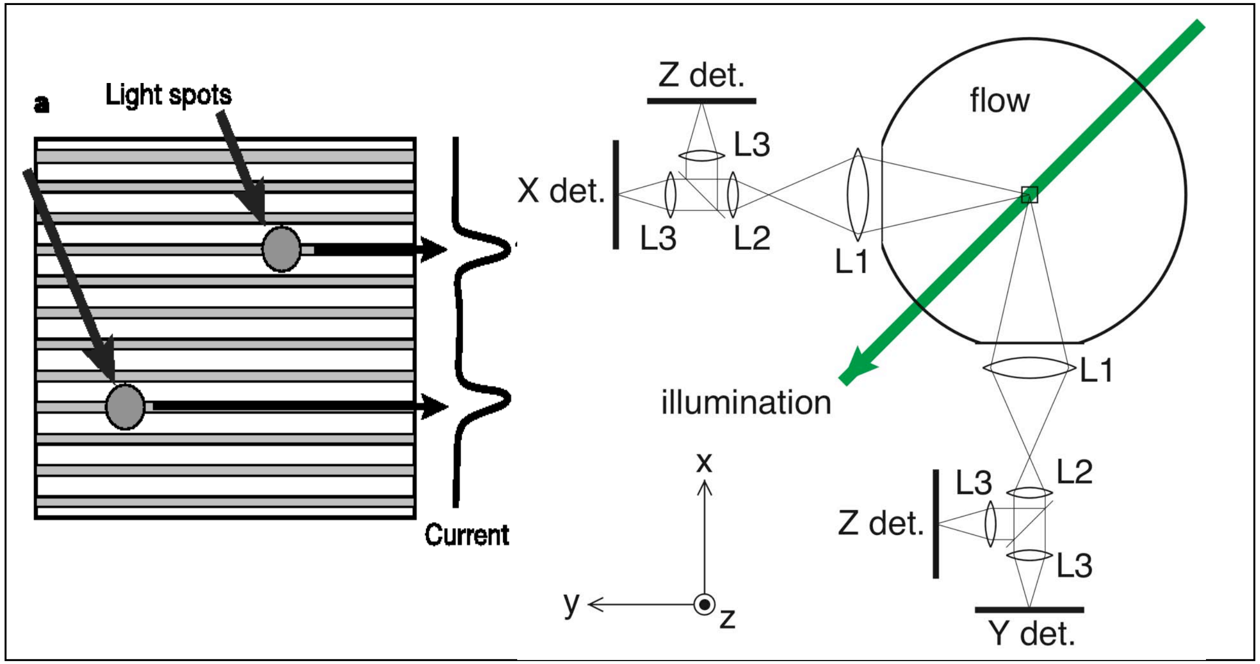
  $$N = (L/\eta)^2 \, (T/\tau_\eta) \, (10) \approx 10^{12} = 1\text{Tb} / \text{s}$$

  per video channel

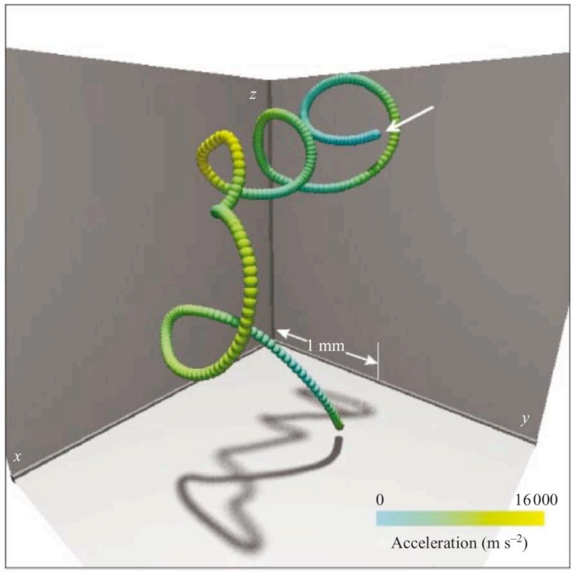
 

Silicon-strip detectors

Voth, Satyanarayan, Bodenschatz, *Phys. Fluids*, **10**, 2268, (2000)

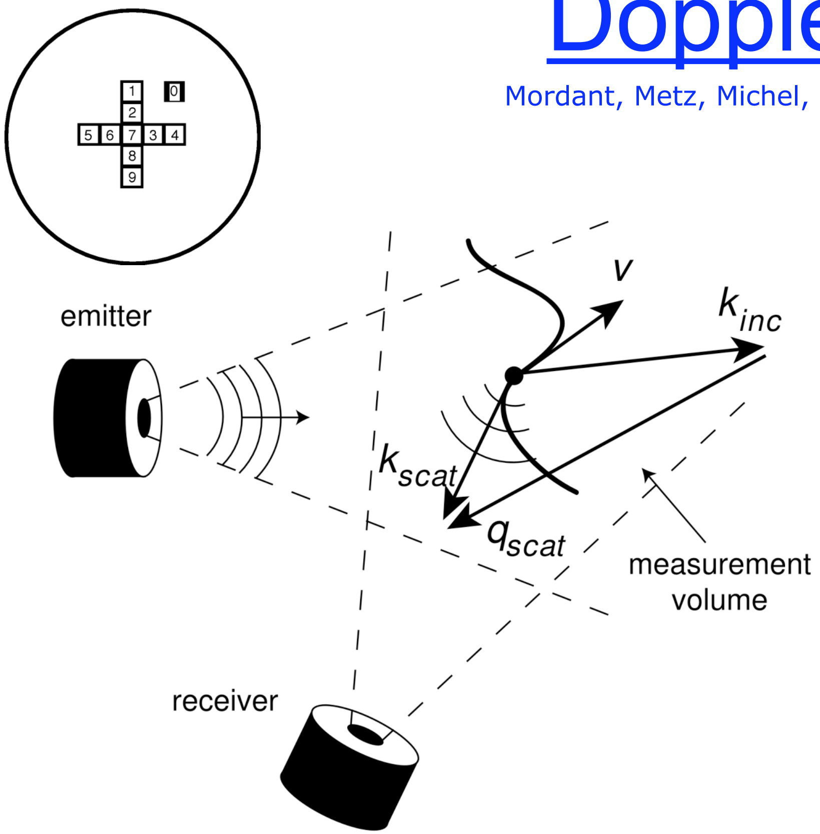


Particles : 10 μm
Pictures : 512 x 512 pixels
Rate : 70,000 images / sec
Record : 4000 images
$R_\lambda \approx 800$



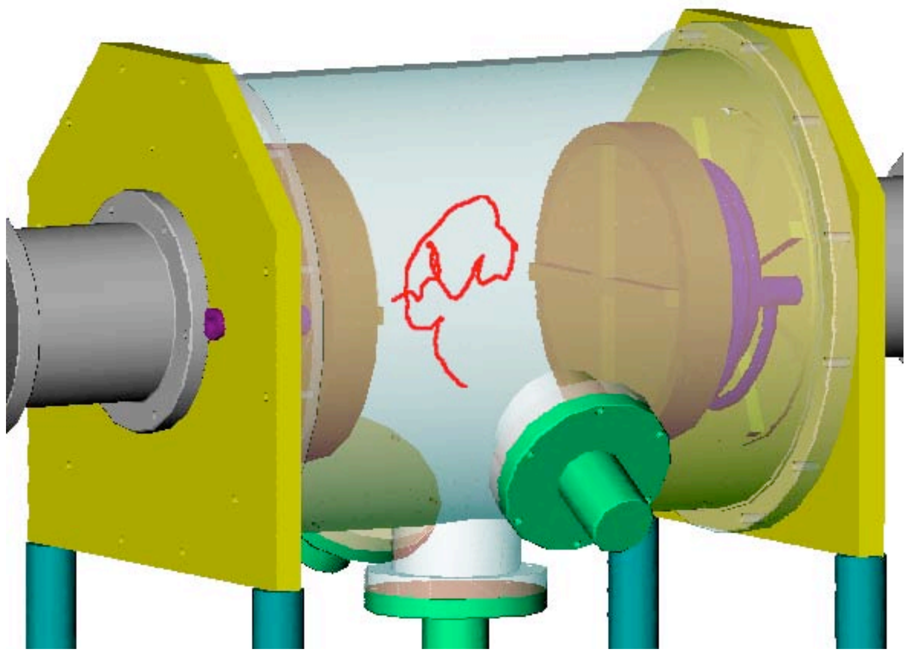
Doppler Acoustics

Mordant, Metz, Michel, Pinton, *Phys. Rev. Lett.*, **87**, 214501, (2001)



Array : radar tracking

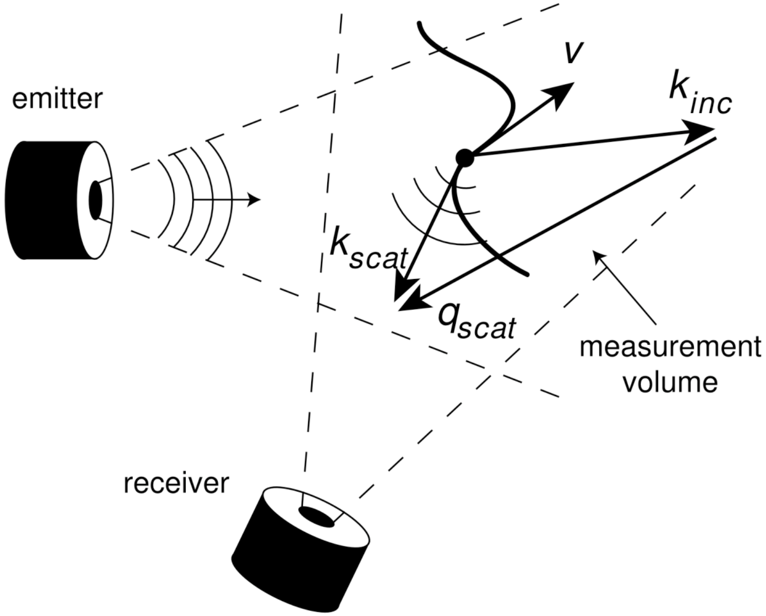
POSITION

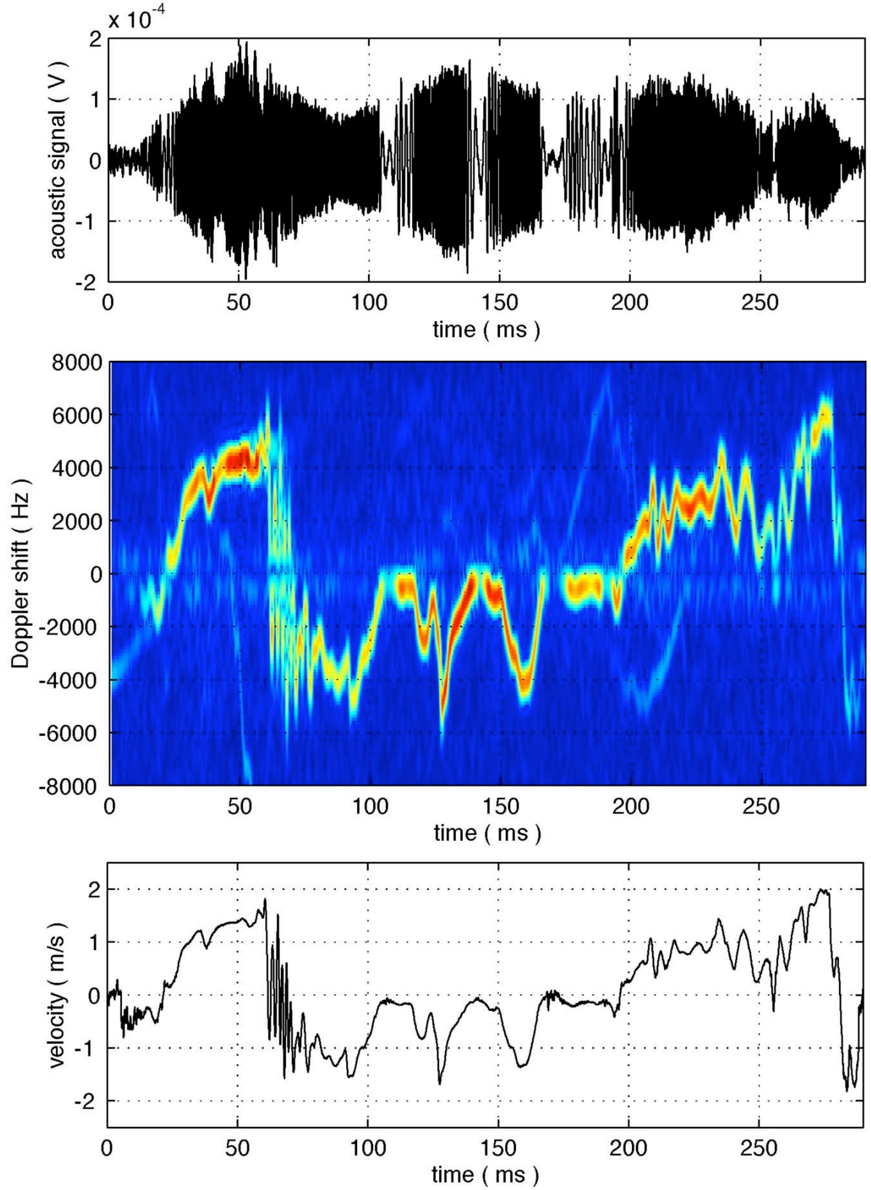


One 250 μm particle
Rate : 3,000 rec / sec
Record : 1 sec
$R_\lambda \approx 800$

Doppler Acoustics

Mordant, Pinton, Michel, *J. Acoust. Soc. Am.*, **112**, 108, (2002)





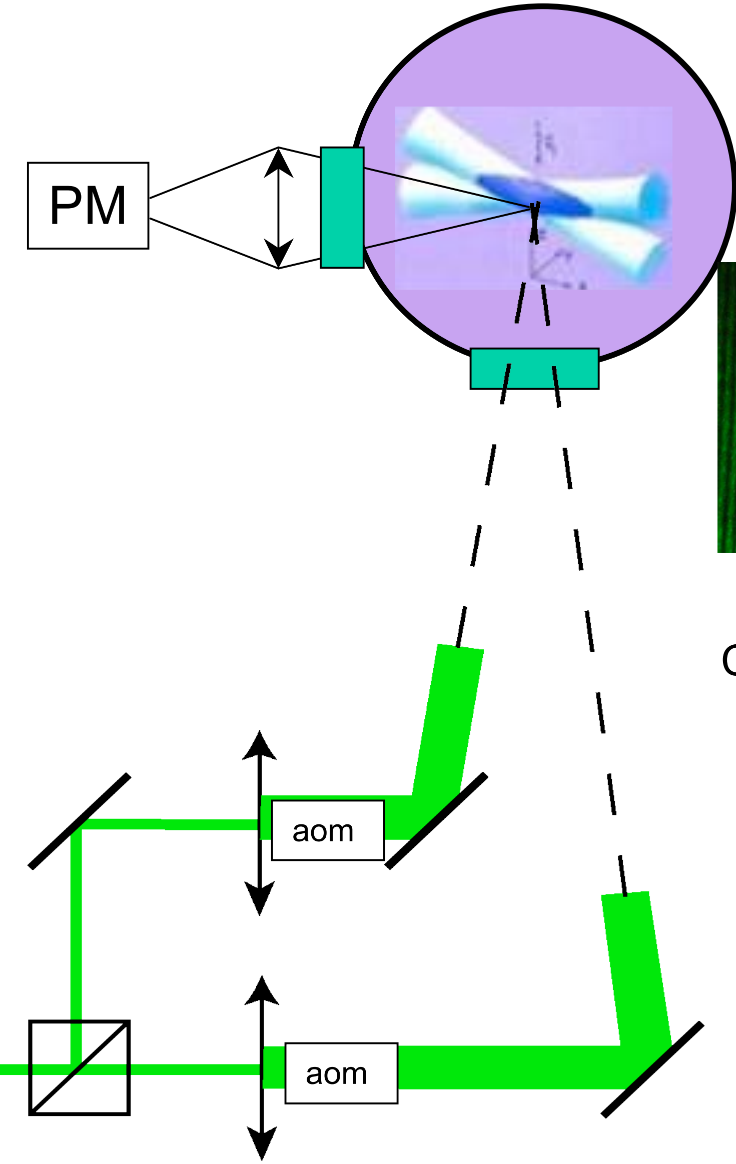
Doppler shift

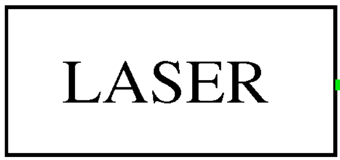
VELOCITY

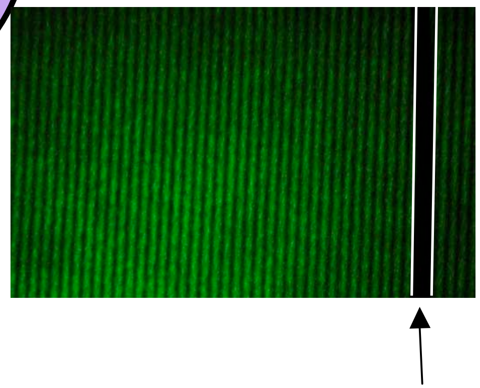
$$2\pi\ f(t) = \mathbf{q}.\mathbf{v}(t)$$

Extended Laser Doppler Velocimetry

Volk, Mordant, Verhille, Pinton, preprint *Europhys. Lett*, (2007)



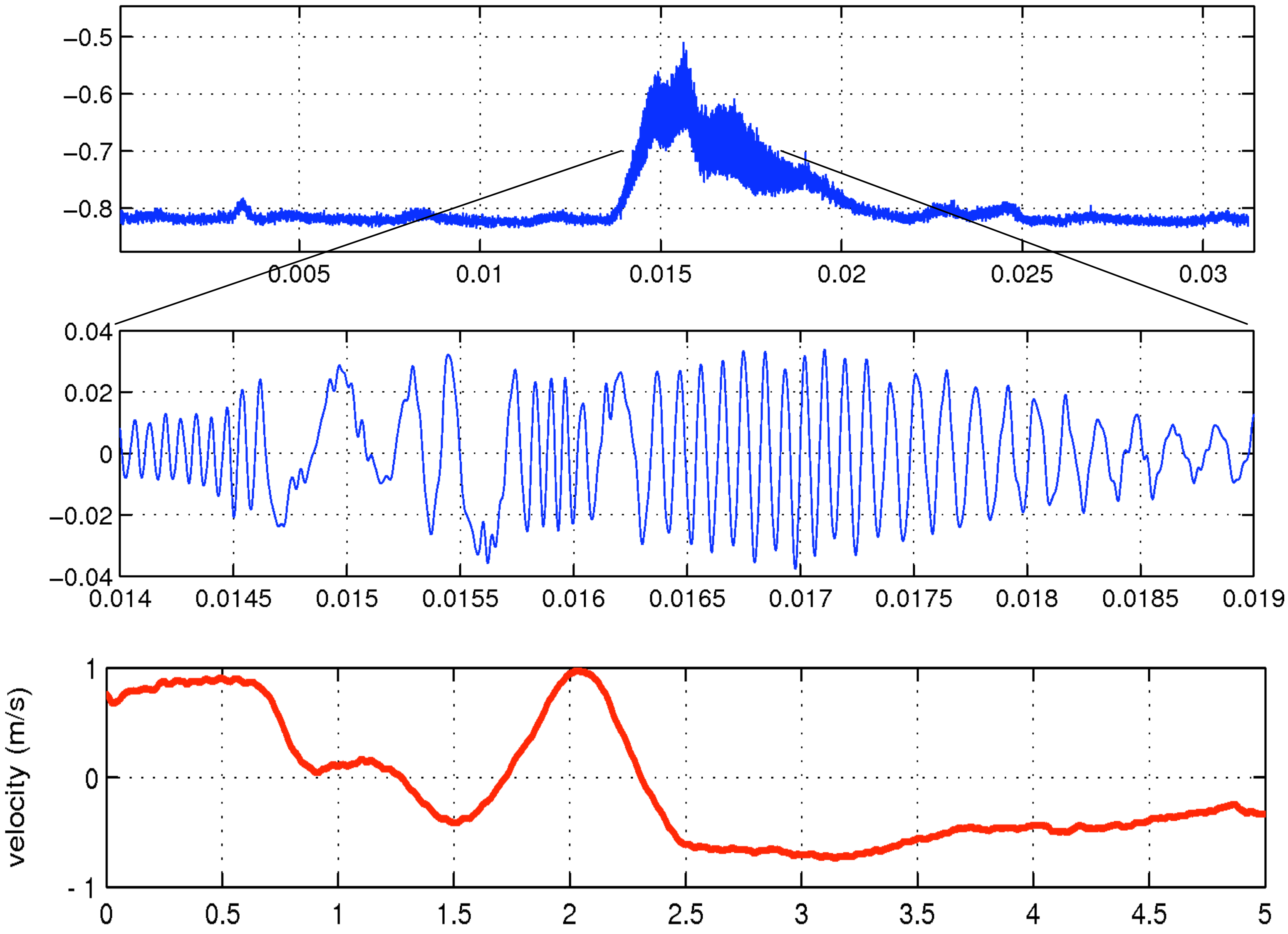




Calibrated wire, 100μm

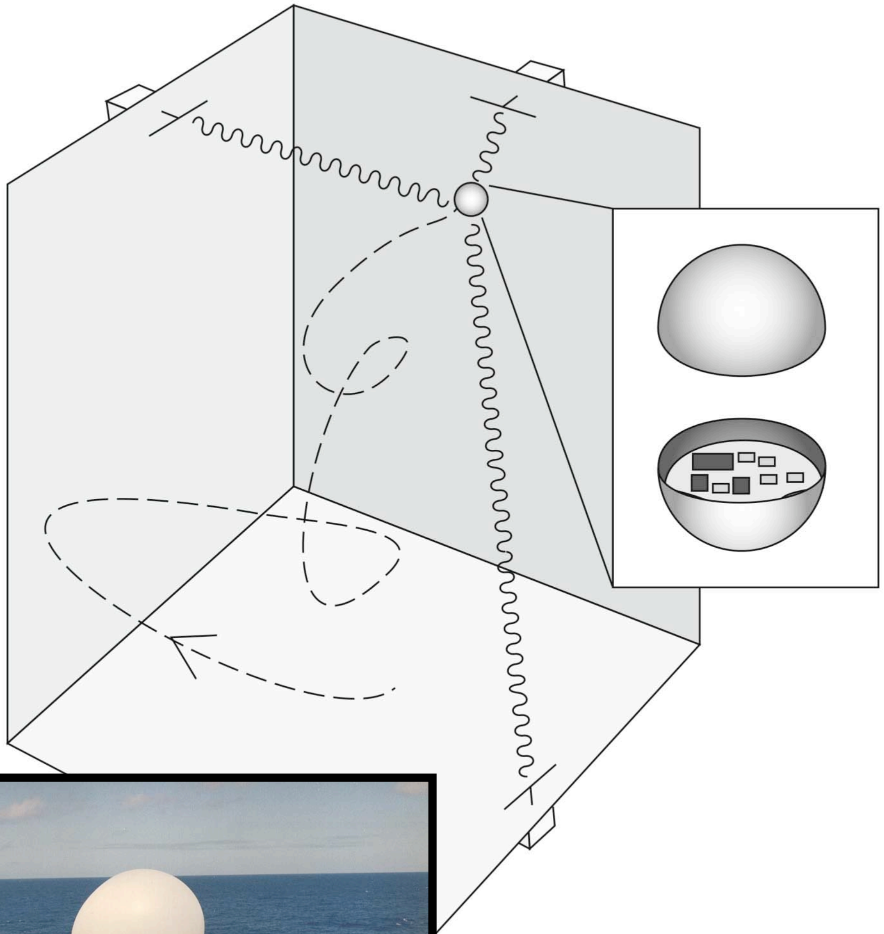
AML algorithm

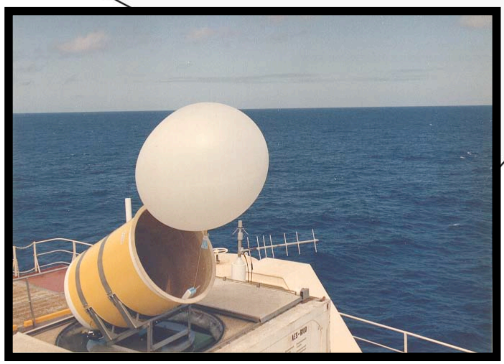
Mordant, Pinton, Michel, *J. Acoust. Soc. Am.*, **112**, 108, (2002)



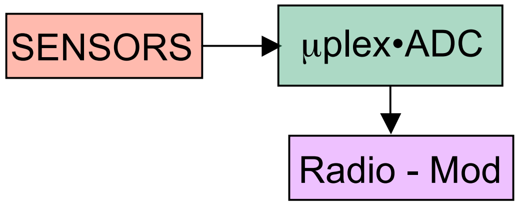
Instrumented particles

Shew, Gasteuil, Metz, Pinton, *Rev. Sci. Instr.*, **78**, 065105, (2007)

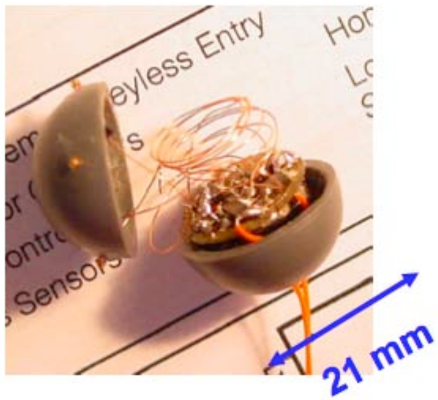




- measurement in moving frame
- principle :

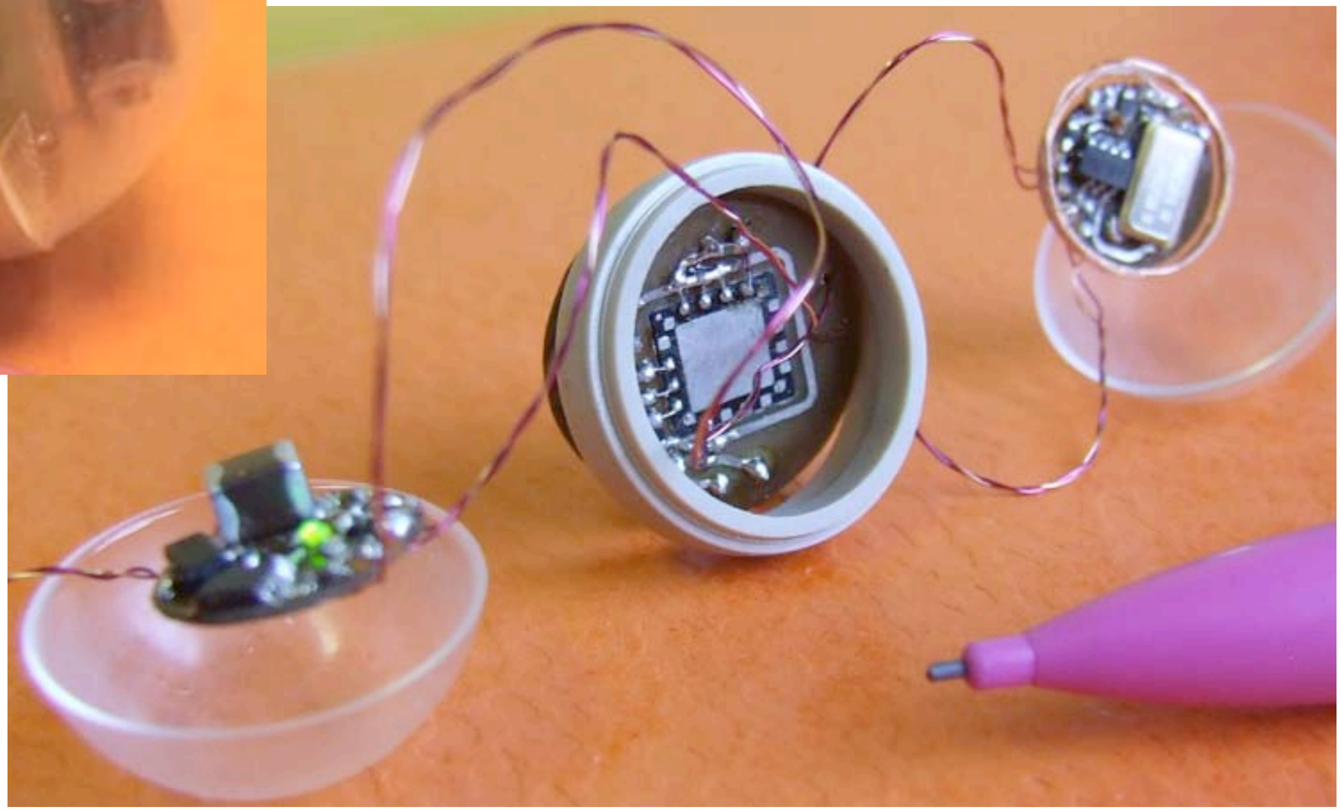


- probes currently avalailable
  temperature
  acceleration



Instrumented particles

3D- accelaration measurement
Digital (1 kHz)



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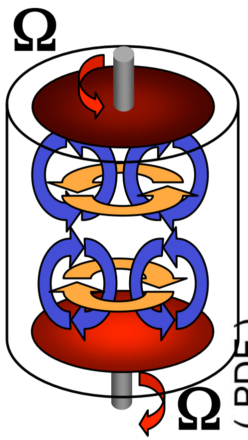
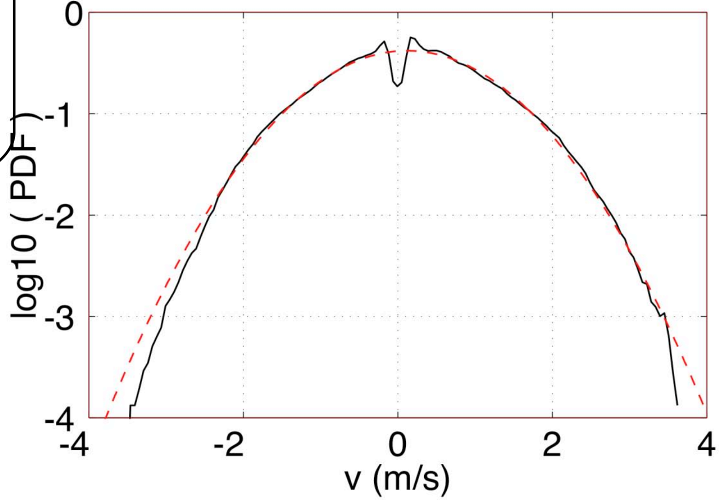
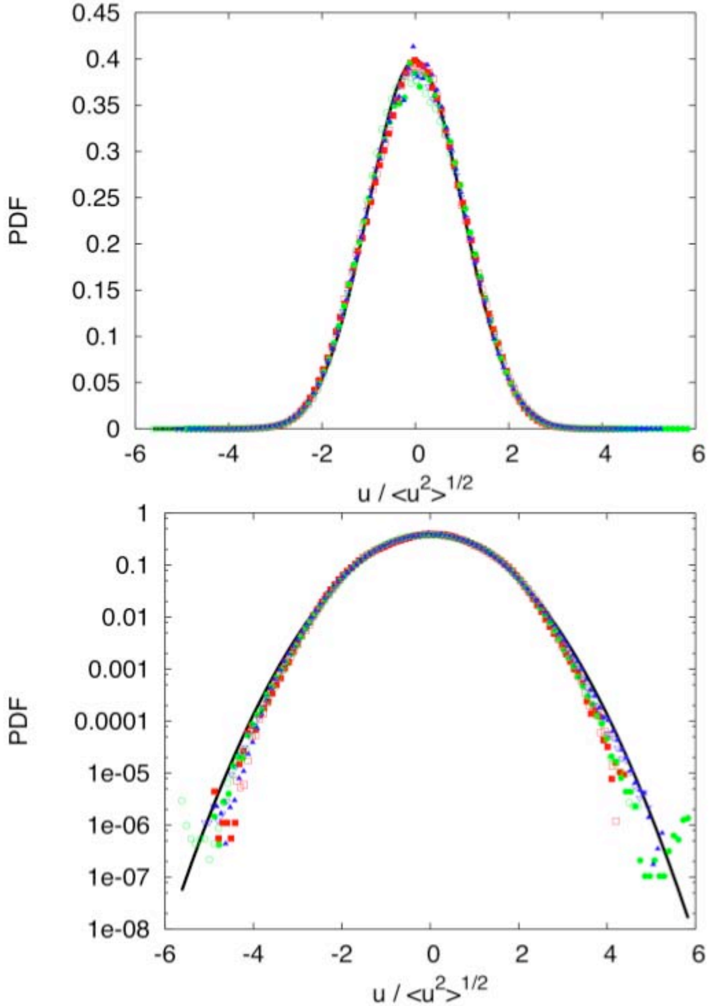
velocity statistics



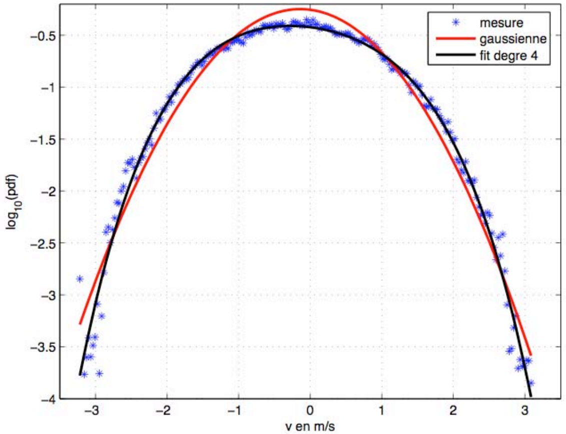
Acoustics, Mordant thesis



PTV, Ouellette thesis

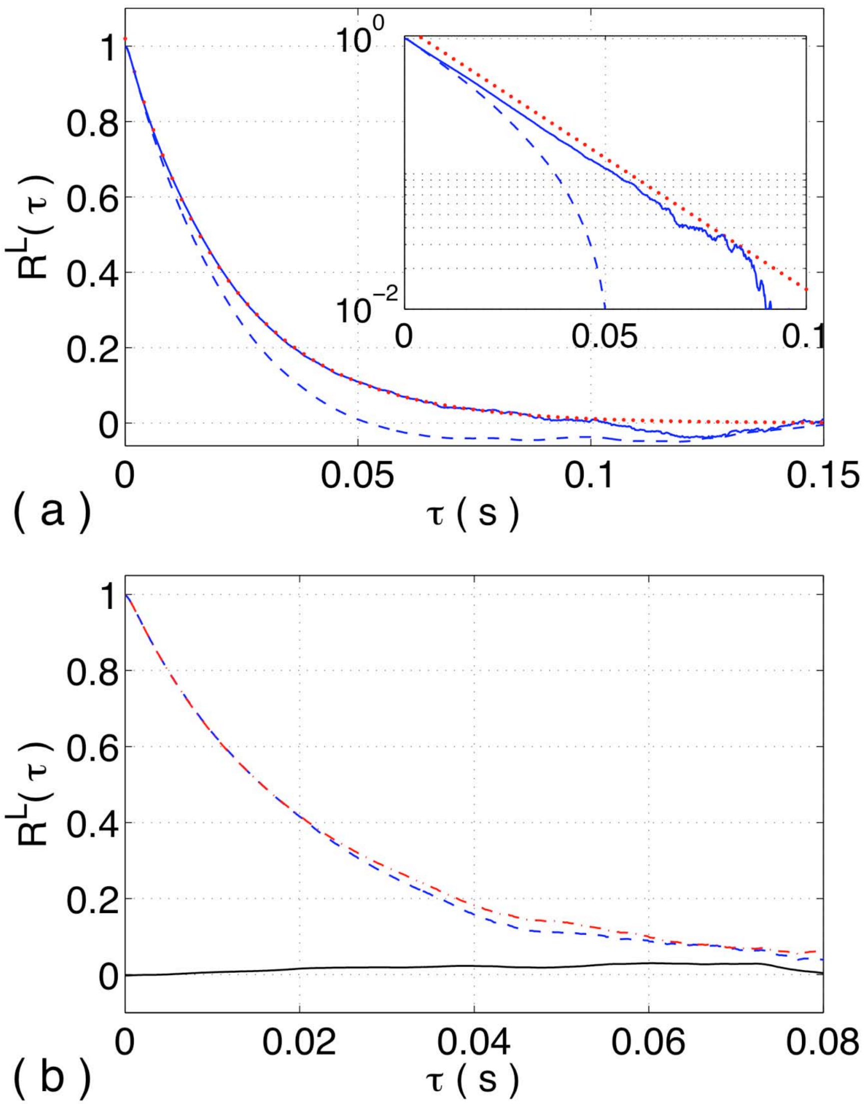


eLDV, Volk (2007)



velocity auto-correlation

Mordant, Metz, Michel, Pinton, *Phys. Rev. Lett.*, **87**, 214501, (2001)

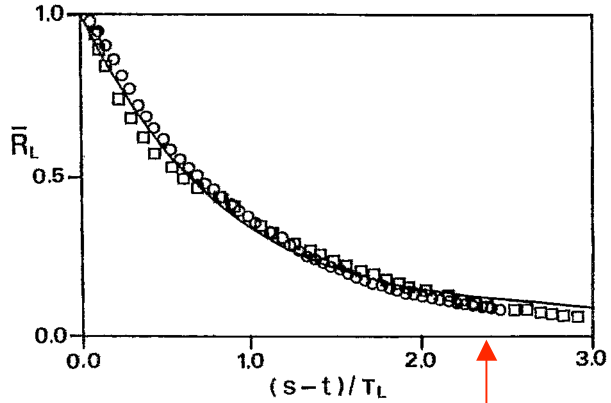


$R_L(\tau) = \langle v(t)v(t+\tau)\rangle / v^2$

$R_L(\tau) \approx \exp(-\tau / T_L)$

$T_L \approx 22$ ms (driving discs)

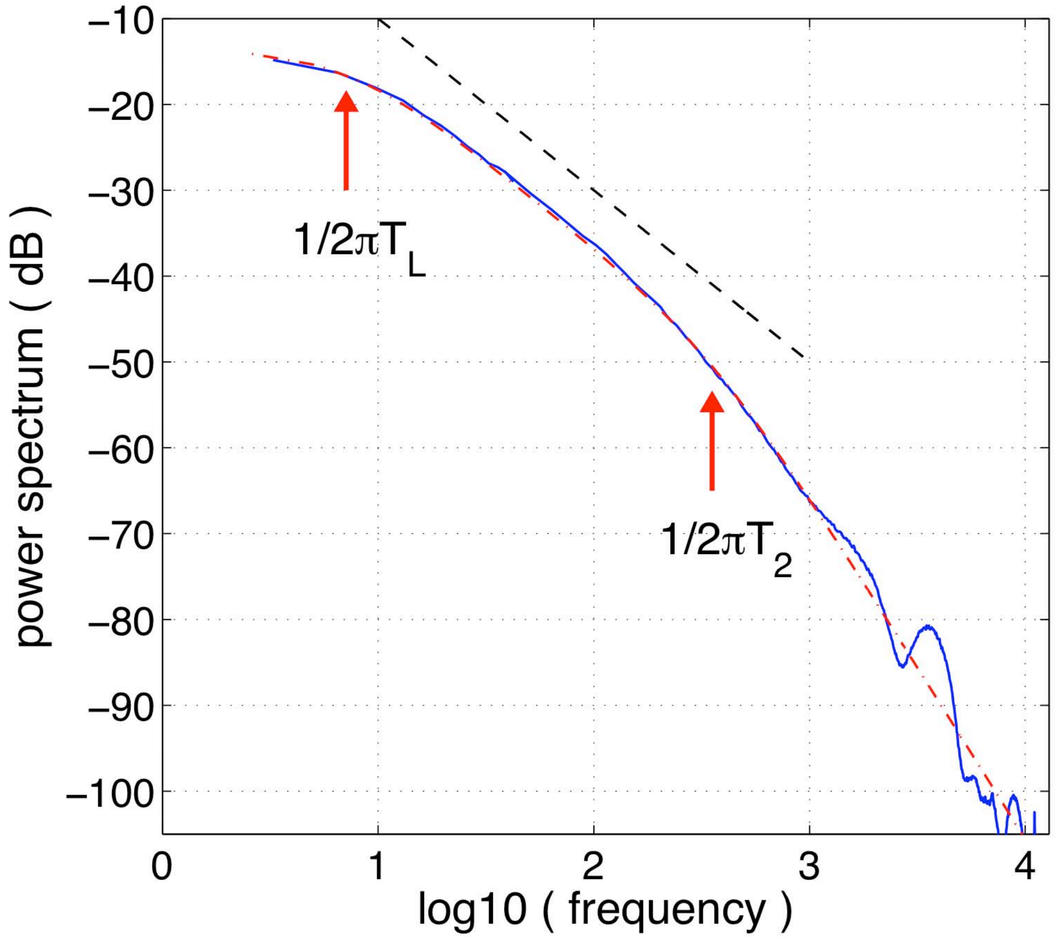
Valid $\tau \in [\,10\,\tau_\eta\,,\,4\,T_L\,]$



Yeung & Pope, 1989

velocity spectrum

Mordant, Metz, Michel, Pinton, *Phys. Rev. Lett.*, **87**, 214501, (2001)

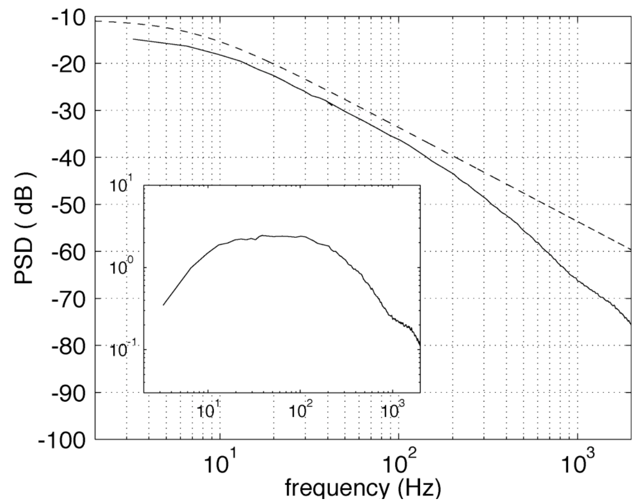


$$R_L(\tau) \approx \exp(-\tau / T_L)$$

$$E_L(\omega) = v^2\, T_L / (1 + T_L{}^2\, \omega^2)$$

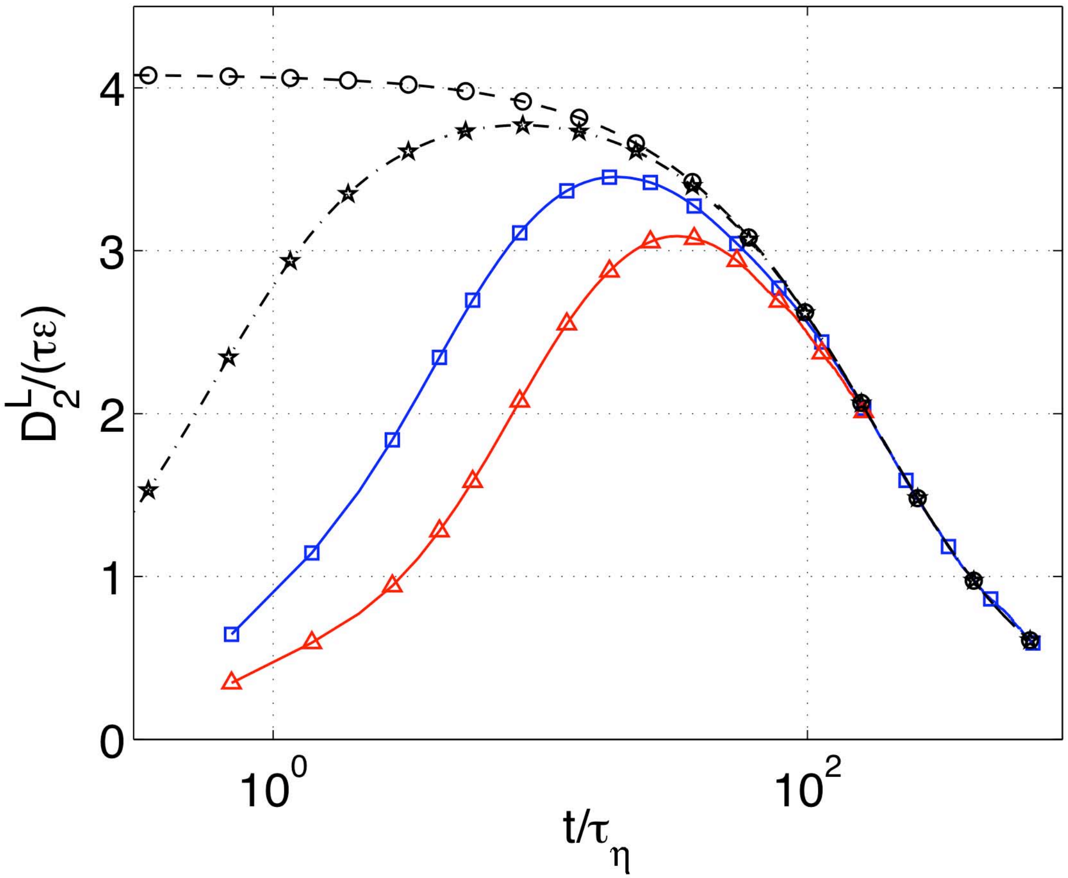
Kolmogorov

$$E_L(\omega) = C_0 \varepsilon\, \omega^{-2}$$



Second order structure function

Mordant, Metz, Michel, Pinton, *Phys. Rev. Lett.*, **87**, 214501, (2001)



1mm particle
(*)Sawford 2-exp. fit
0.25mm particle
(o) Exponential fit

$D^2_L(\tau) = < (v(t+\tau)-v(t))^2 >$

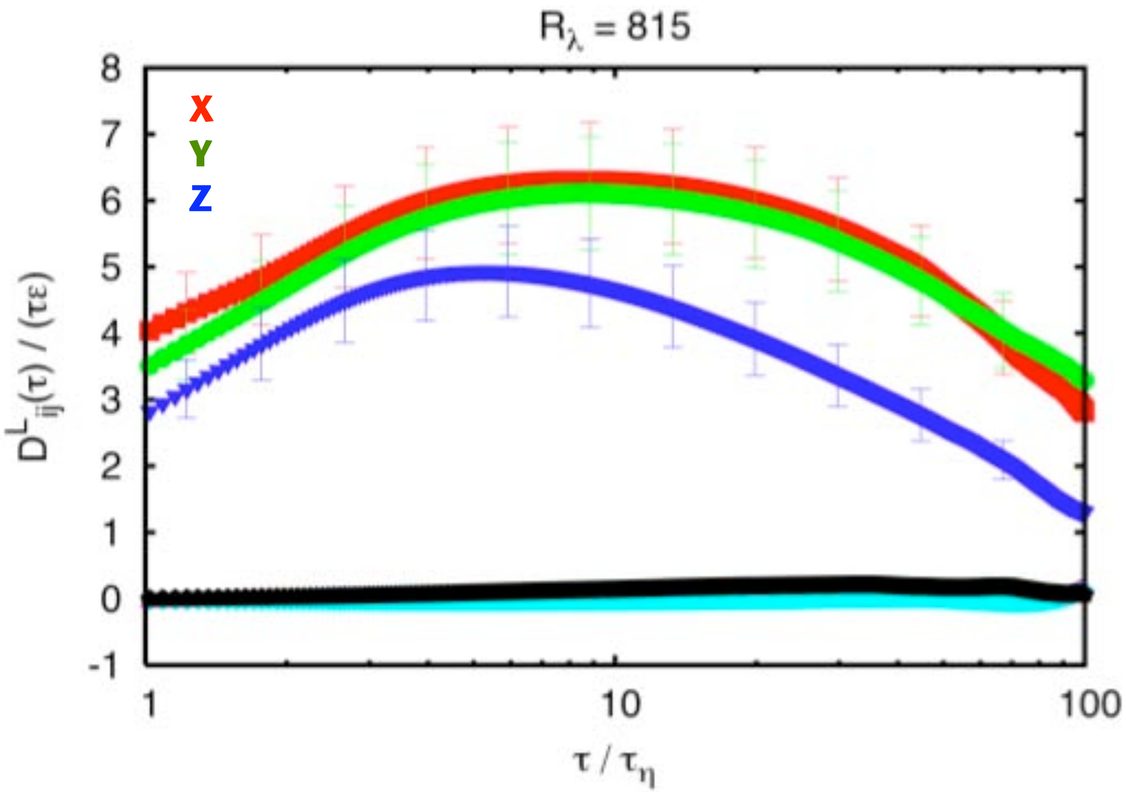
Kolmogorov 'K41'
dimensional argument
($[\varepsilon]=m^2/s^3$)

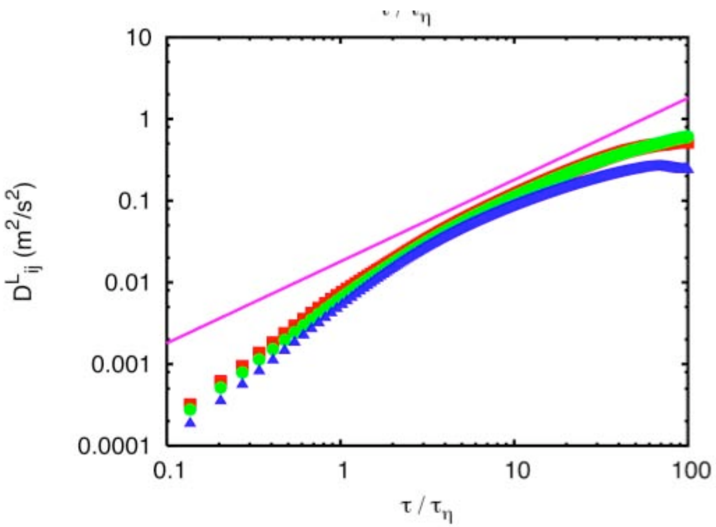
$D^2_L(\tau) = C_0\, \varepsilon\, \tau$

C_0 universal 'constant'
in 'inertial range'

Second order structure function

Nicholas Ouellette PhD Thesis (2006)

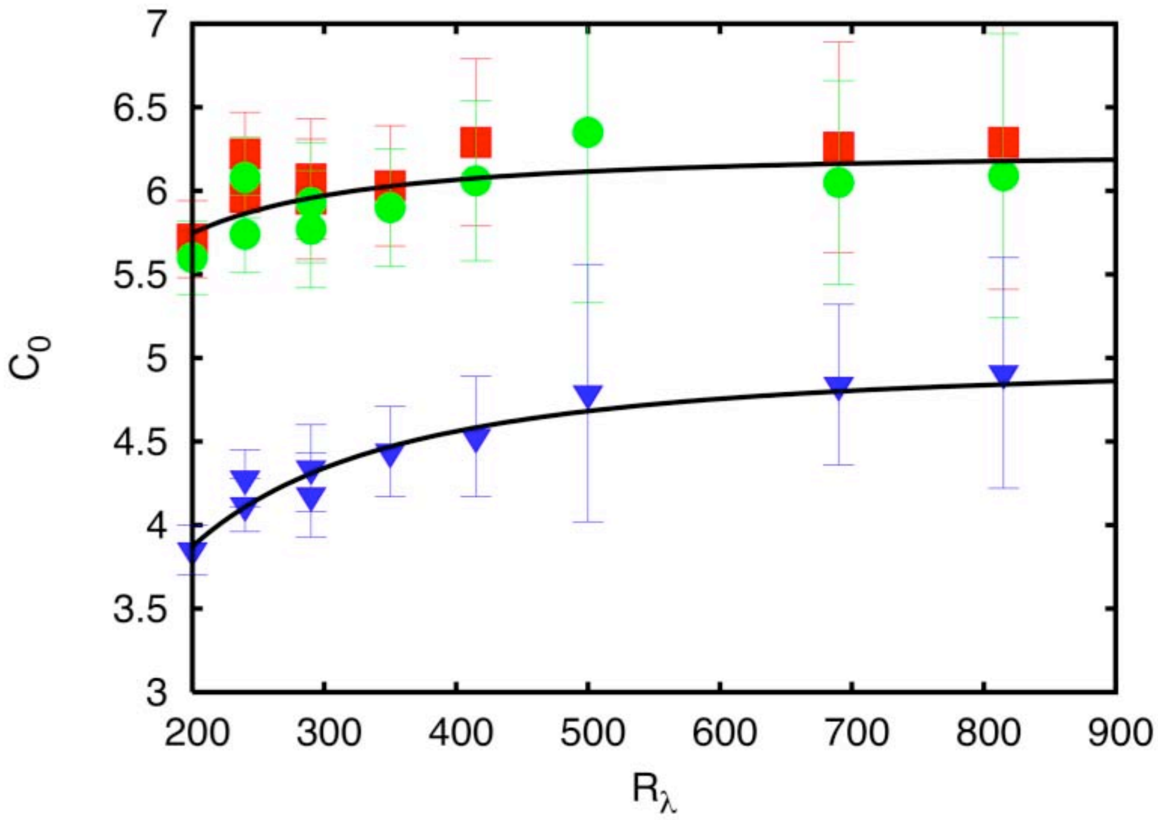




Second order structure function

Nicholas Ouellette PhD Thesis (2006)

X
Y
Z



$$D^2_L(\tau) = C_0 \, \varepsilon \, \tau$$

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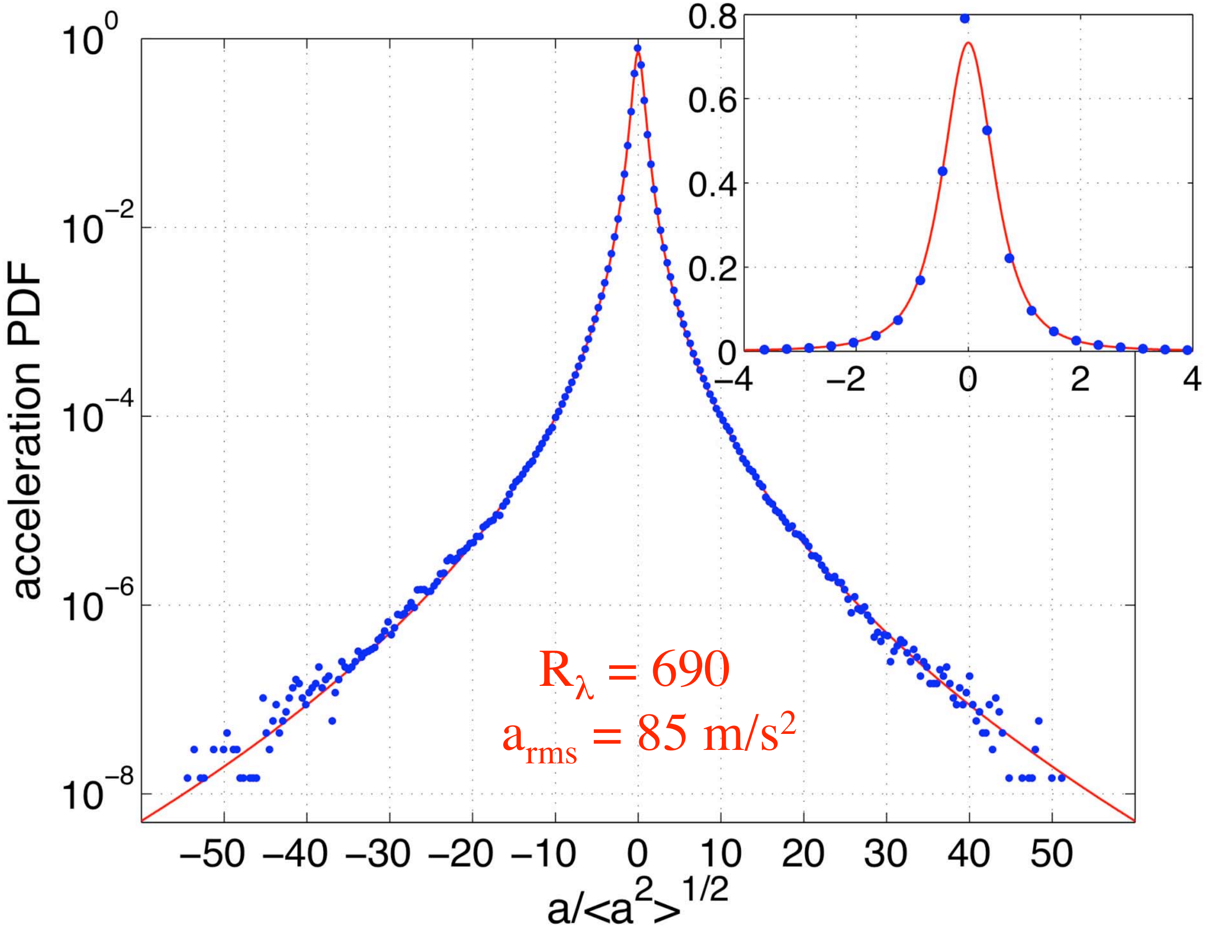
Lagrangian acceleration

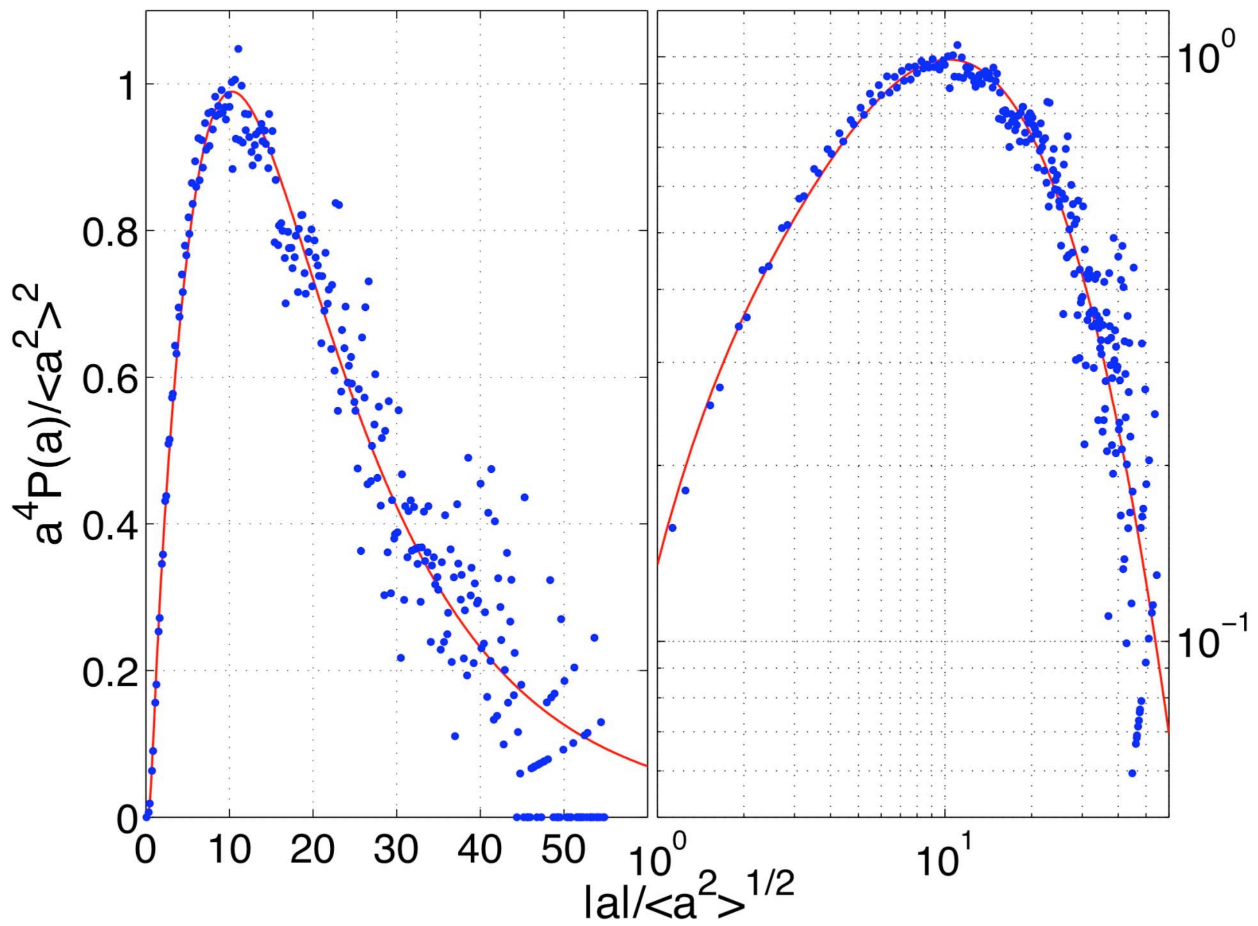
Voth, Satyanarayan, Bodenschatz, *Phys. Fluids*, **10**, 2268, (2000)



Heisenberg-Yaglom scaling : $< a^2 > \propto \epsilon^{3/2} \nu^{-1/2}$

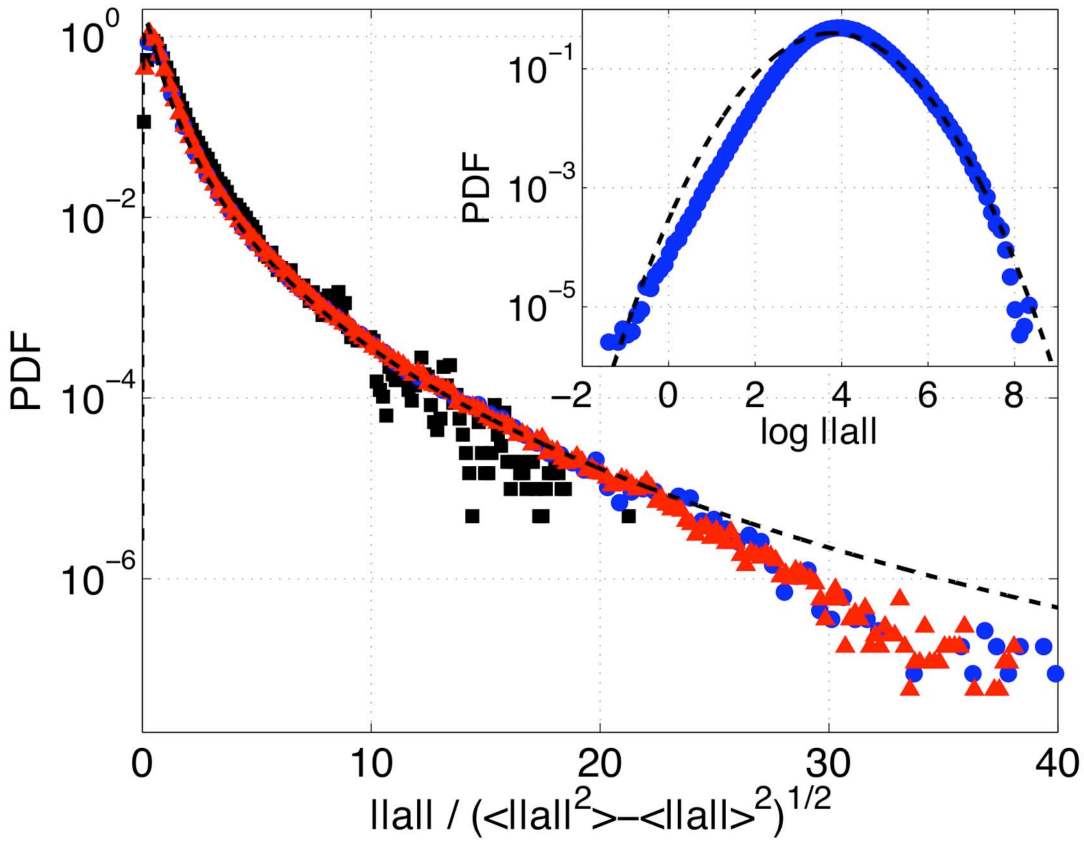
Lagrangian acceleration

Mordant, Crawford, Bodenschatz, *Physica D*, **193**, (2004)



Lagrangian acceleration

Mordant, Crawford, Bodenschatz, *Phys. Rev. Lett.*, **93** (2005)

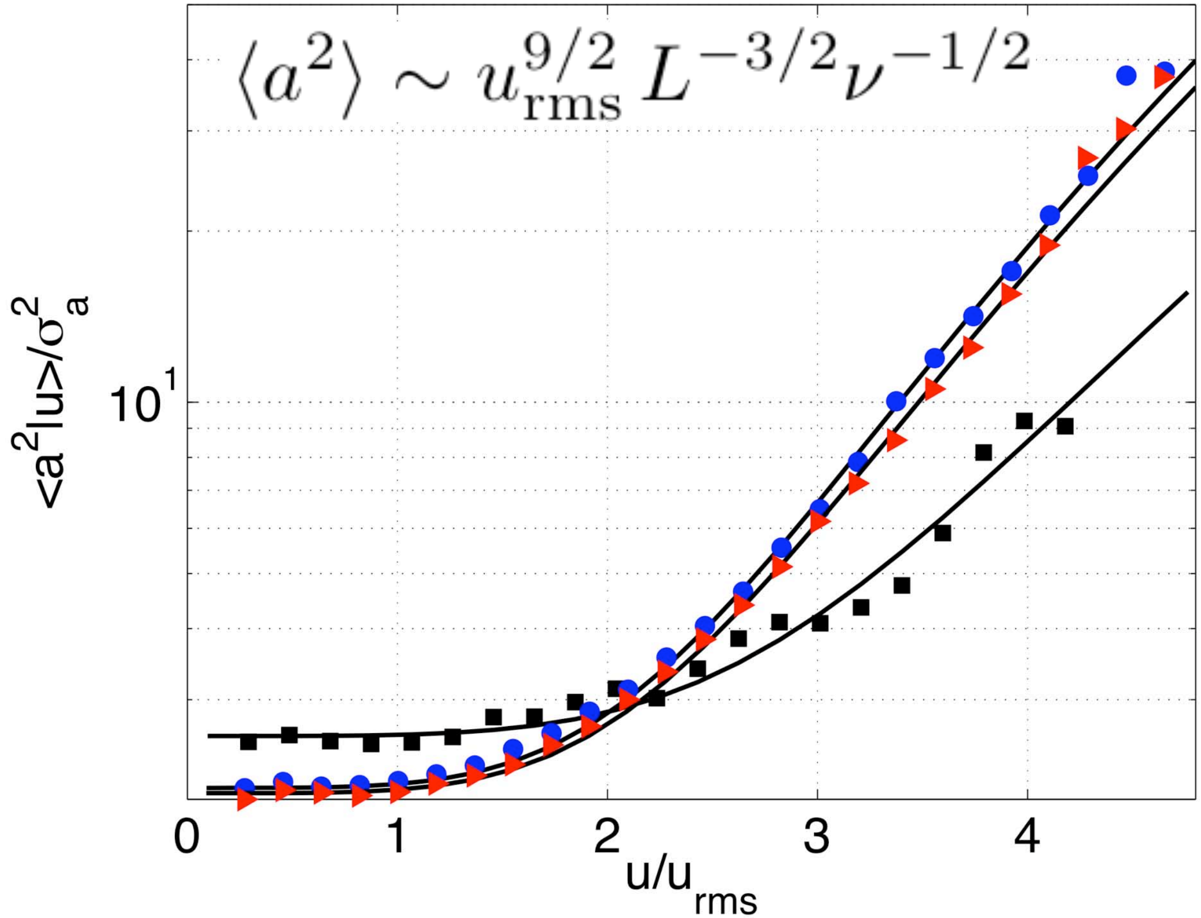


Lognormal acceleration :

$$P(a) = \frac{1}{\sqrt{2\pi a^2 s^2}} \exp\left(-\frac{\ln(a/m)}{2s^2}\right)$$

Lagrangian acceleration

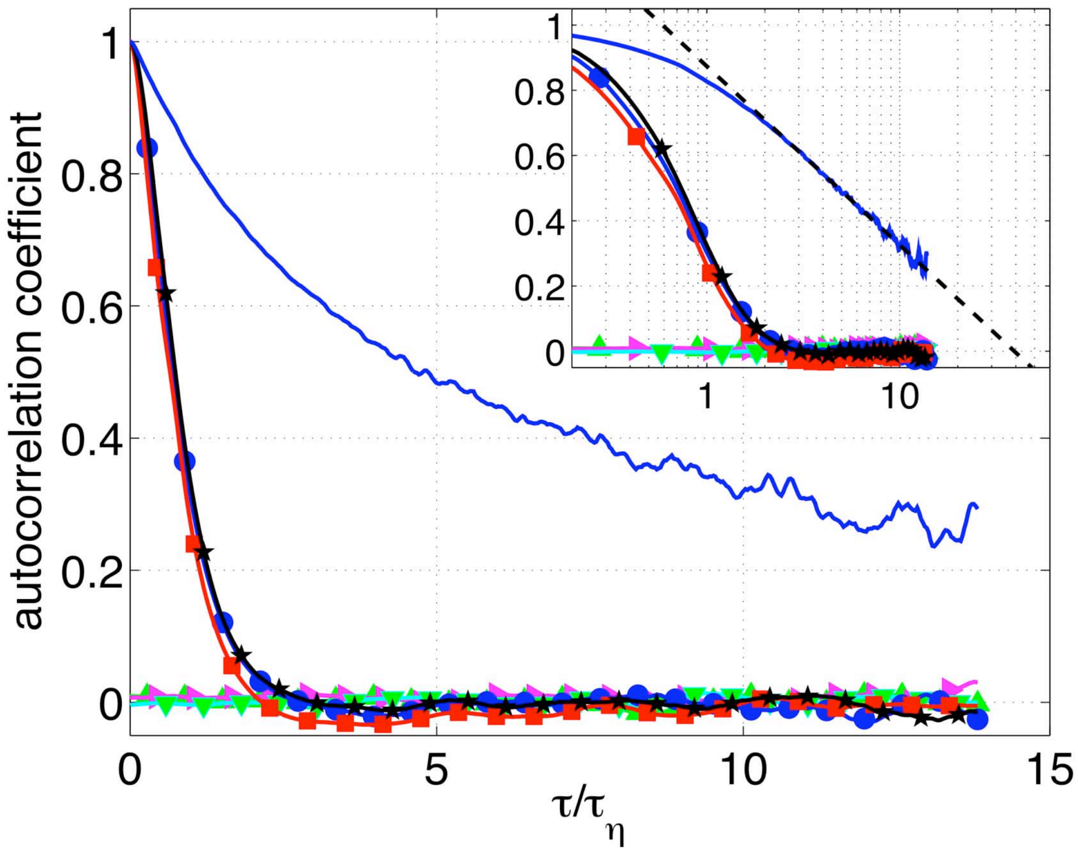
Crawford, Mordant, Bodenschatz, *Phys. Rev. Lett.*, **94** (2005)



Heisenberg-Yaglom scaling : $\langle a^2 \rangle \sim \epsilon^{3/2} \nu^{-1/2}$ and $\epsilon \sim u_{\rm rms}^3 / L$

Lagrangian acceleration

Crawford, Mordant, Bodenschatz, *Phys. Rev. Lett.*, **94** (2005)



Short (Kologorov time) correlation of acceleration direction

***

Long (forcing time) correlation of acceleration amplitude

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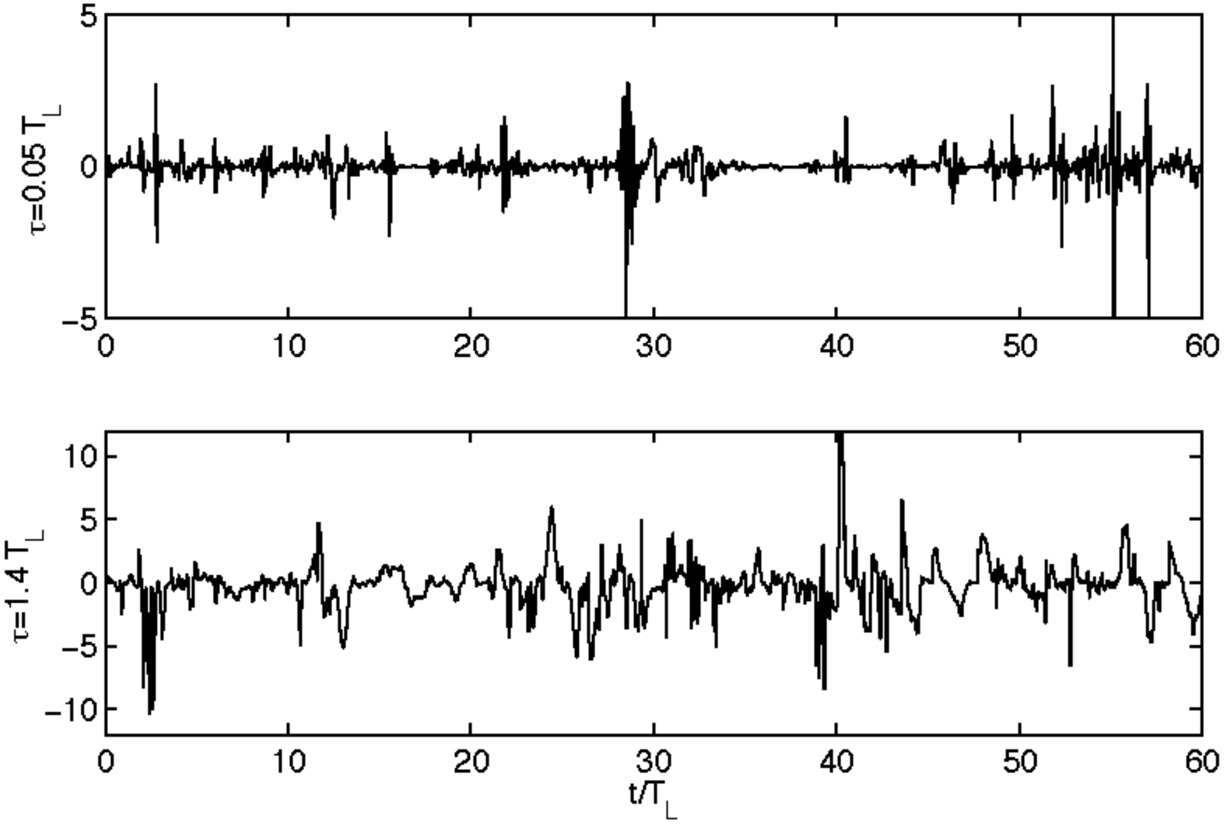
Lagrangian intermittency

Mordant, Metz, Michel, Pinton, *Phys. Rev. Lett.* **87**, 214501, (2002)

$$\Delta_\tau v(t) = v(t+\tau) - v(t)$$



Lagrangian intermittency

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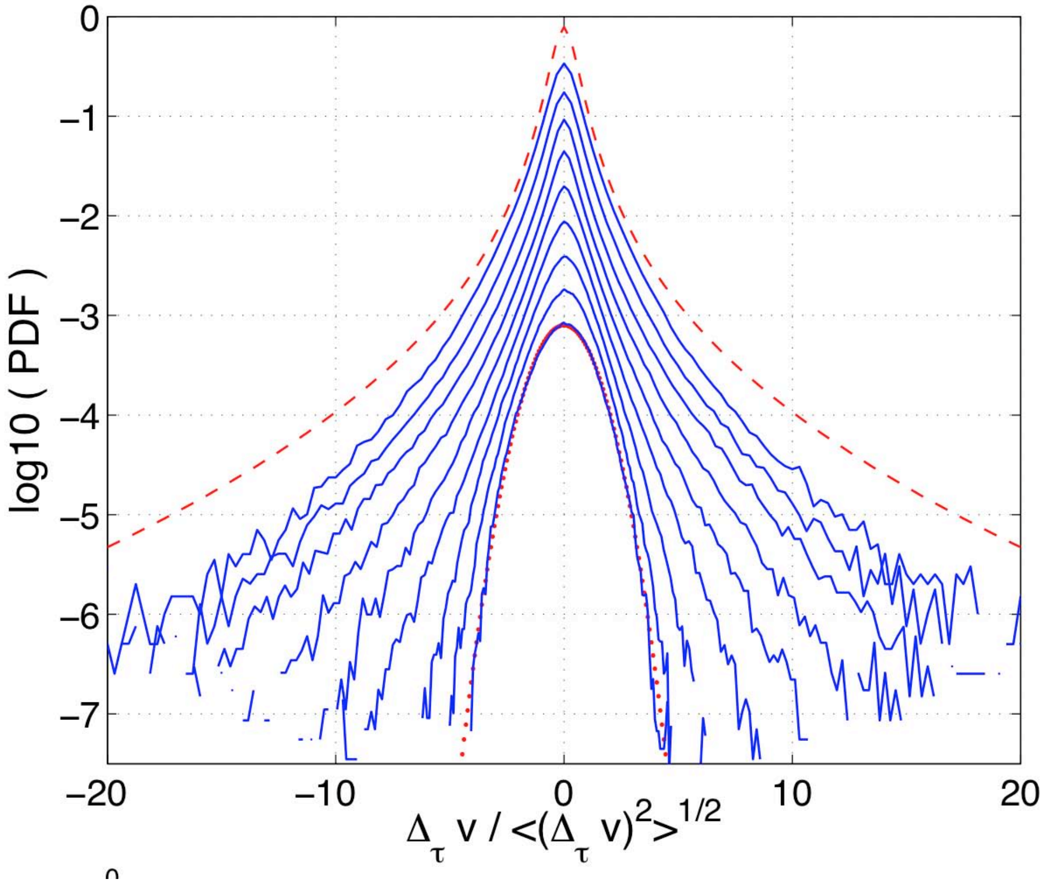
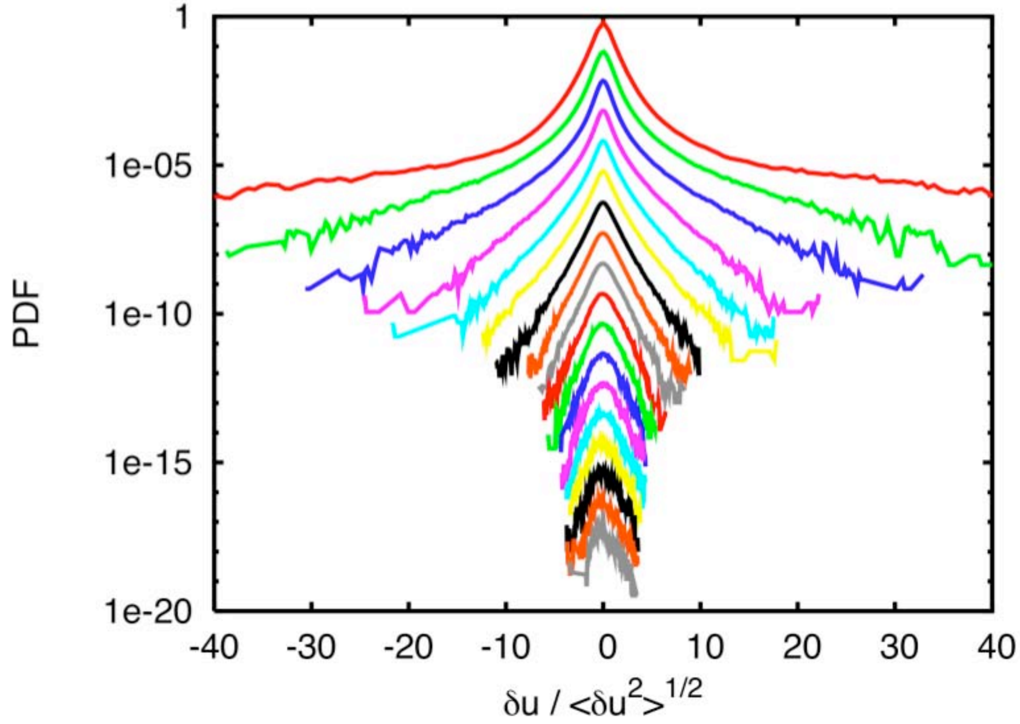


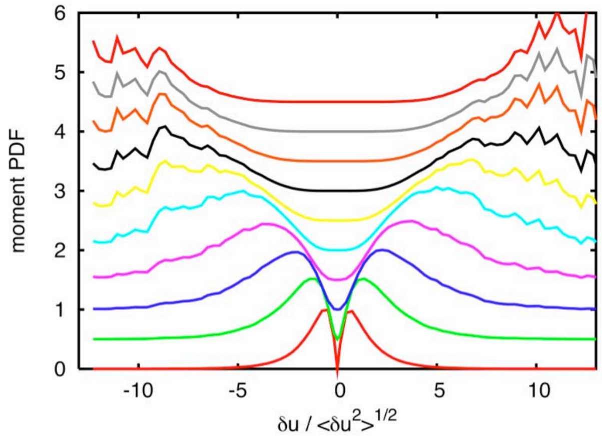
Lagrangian intermittency

Nicholas Ouellette PhD Thesis

$$\Delta_\tau v(t) = v(t+\tau) - v(t)$$

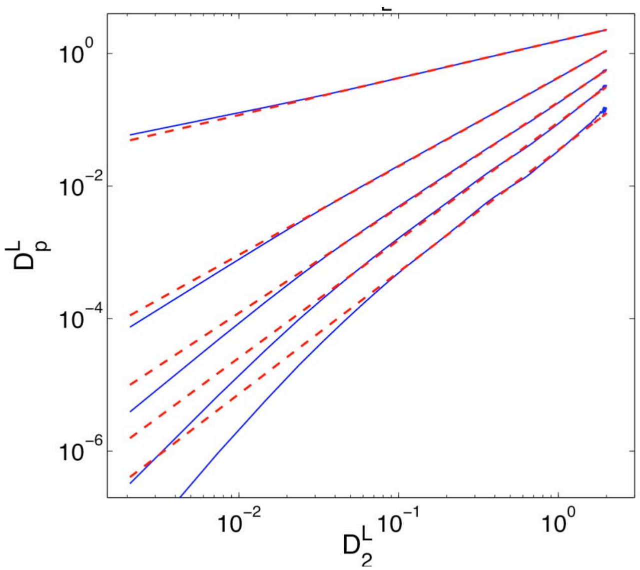
PTV, vK flow, R_λ = 690

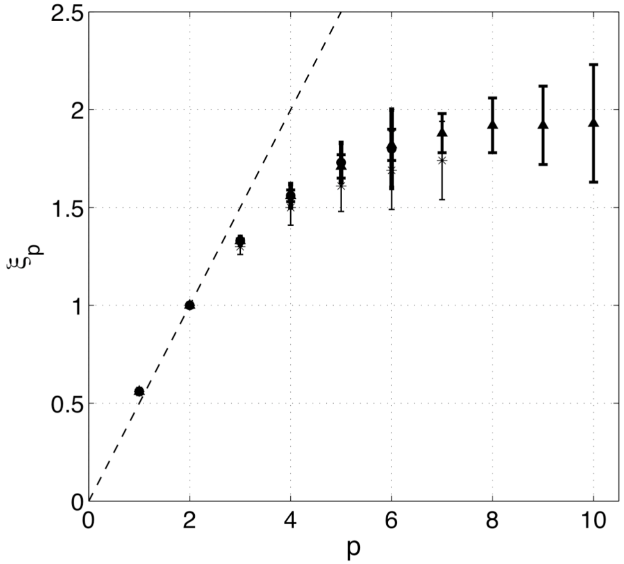




Structure Fct Doppler Acoust

Mordant, Metz, Michel, Pinton, *Phys. Rev. Lett.*, **87**, 214501, (2001)





$$D_p(\tau) = \langle |v(t+\tau) - v(t)|^p \rangle = \int dx\, x^p\, \mathcal{P}_\tau(x)$$

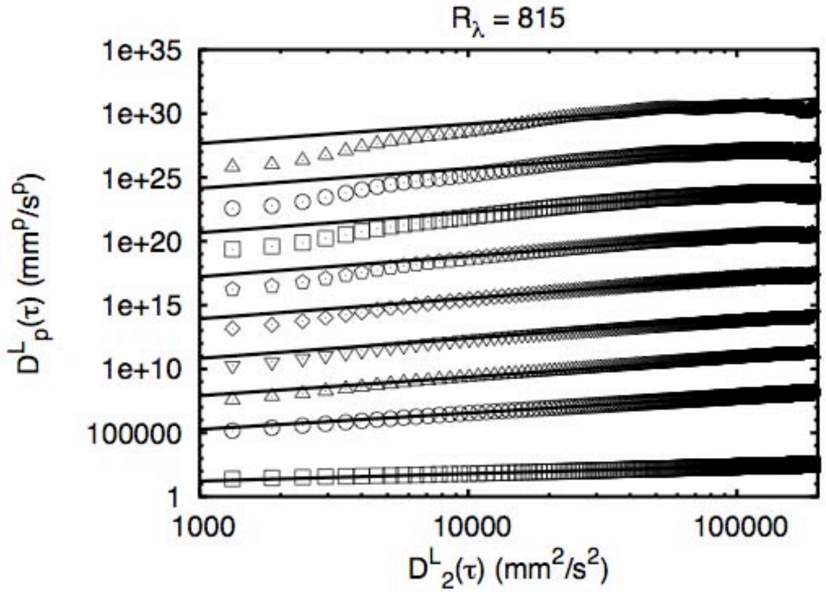
$$D_p(\tau) \neq (\varepsilon\, \tau)^{p/2}$$

ESS anstaz $\Rightarrow$ $D_p(\tau) \propto D_2(\tau)^{\zeta(p)}$

	R_λ=310	R_λ=740	R_λ=1100
ζ(1)	0.56	0.56	0.56
ζ(2)	1	1	1
ζ(3)	1.32	1.33	1.34
ζ(4)	1.54	1.56	1.58
ζ(5)	1.70	1.73	1.75
ζ(6)	1.8	1.85	1.9

Structure Fct / PTV

Xu, Bourgoin, Ouellette, Bodenschartz, *Phys. Rev. Lett.*, (2005))



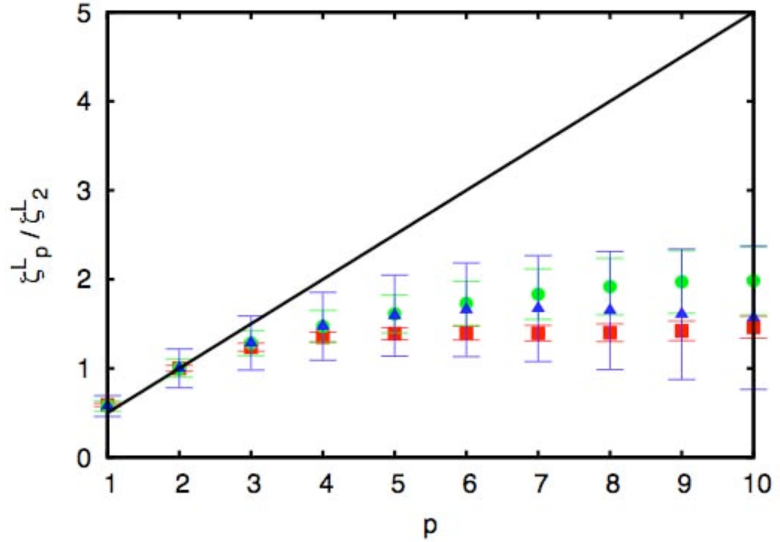


TABLE I: Values of the relative scaling exponents measured in our experiment using ESS. The ESS curves were fit only in the range of times where the second order structure function displayed a K41 scaling range with exponent $\zeta_2^L \approx 1$. For comparison, we included the values measured from the DNS of Biferale *et al.* [8] and the experiment of Mordant *et al.* [9].

	R_λ	ζ_1^L/ζ_2^L	ζ_3^L/ζ_2^L	ζ_4^L/ζ_2^L	ζ_5^L/ζ_2^L	ζ_6^L/ζ_2^L	ζ_7^L/ζ_2^L	ζ_8^L/ζ_2^L	ζ_9^L/ζ_2^L	ζ_{10}^L/ζ_2^L
	200	0.59 ± 0.02	1.24 ± 0.03	1.35 ± 0.04	1.39 ± 0.07	1.40 ± 0.08	1.39 ± 0.09	1.40 ± 0.10	1.42 ± 0.11	1.46 ± 0.12
	690	0.58 ± 0.05	1.28 ± 0.14	1.47 ± 0.18	1.61 ± 0.21	1.73 ± 0.25	1.83 ± 0.28	1.92 ± 0.32	1.97 ± 0.35	1.98 ± 0.38
	815	0.58 ± 0.12	1.28 ± 0.30	1.47 ± 0.38	1.59 ± 0.46	1.66 ± 0.53	1.67 ± 0.60	1.65 ± 0.66	1.61 ± 0.73	1.57 ± 0.80
Ref. [8]	284			1.7 ± 0.05	2.0 ± 0.05	2.2 ± 0.07				
Ref. [9]	740	0.56 ± 0.01	1.34 ± 0.02	1.56 ± 0.06	1.8 ± 0.2					

Euler vs. Lagrange

M. Borgas. *Phil. Trans. Roy. Soc. London*, **A342**, 379, (1993)

$$D_p(\tau) \sim \langle \epsilon_\tau^{p/2} \rangle \tau^{p/2} \sim \tau^{p/2+\alpha^L(p/2)}$$

$$S_p(\ell) \sim \langle \epsilon_\ell^{p/3} \rangle \ell^{p/3} \sim \ell^{p/3+\alpha^E(p/3)}$$

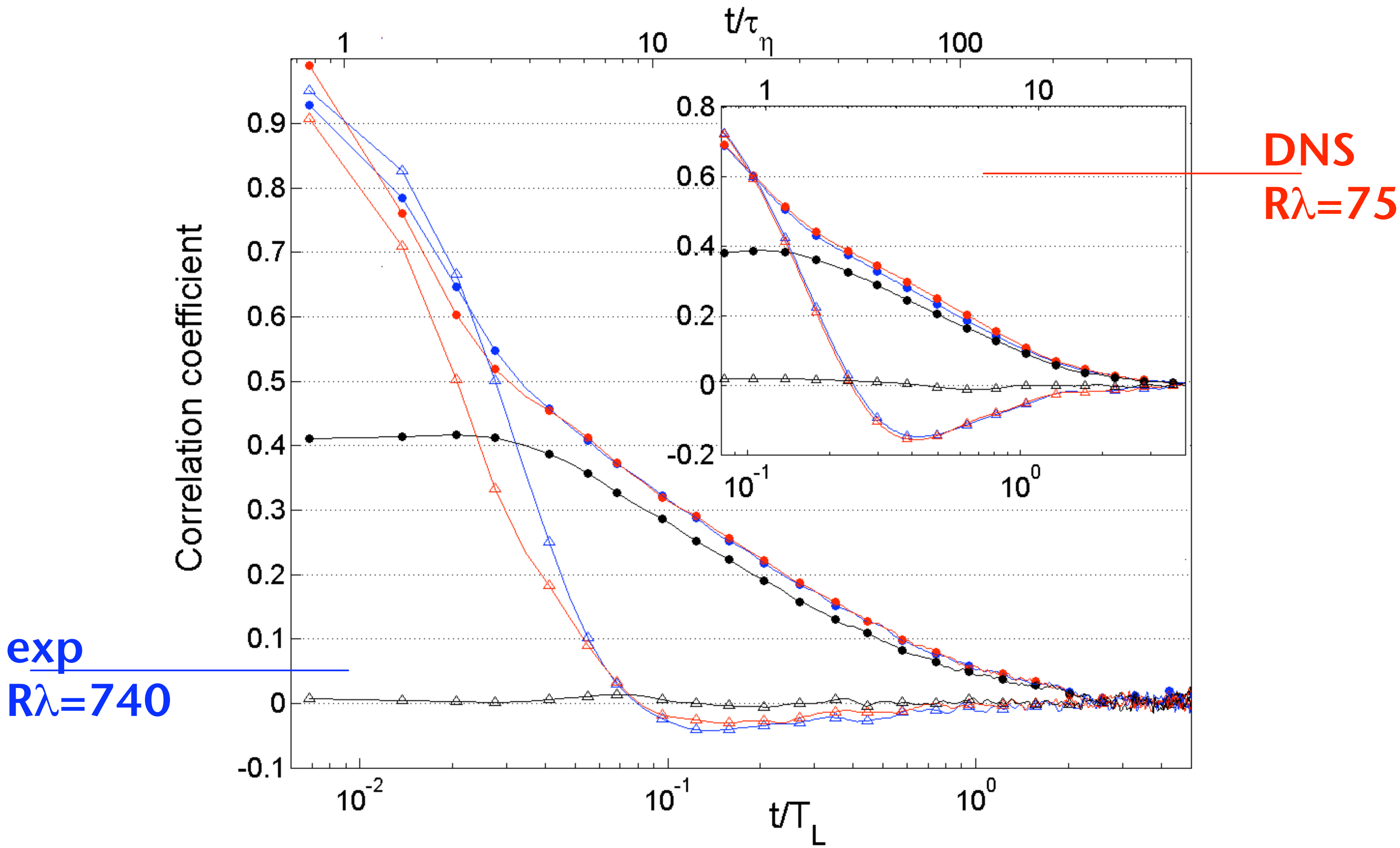
$$\text{Richardson} \quad \ell^2 \sim \tau^3$$

$$\boxed{\lambda_L^2/\lambda_E^2 = (3/2)^3 \simeq 3.5}$$

$\lambda_E^2 \approx 0.022$ $\lambda_L^2 \approx 0.085$ $\lambda_L^2/\lambda_E^2 \approx 3.7 \pm 0.5$

Correlation of velocity increments

- $\Delta u_{\tau 0}(t) = v(t+\tau_0)-v(t)$ -> $C(t)= < u_{\tau 0}(t') \, u_{\tau 0}(t'+t) >_{t'}$



MRW model

Bacry, Delour, Muzy, *Phys. Rev. E*, **64**, (2001).

stochastic equation for the velocity increments

$$d_t u = -\gamma(u)u + \xi(t)$$

'K41' theory : $\xi(t)$ is δ-correlated noise,

Model, from observations :

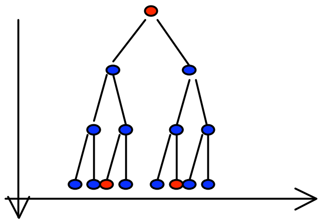
$$\xi(t) = e^{\omega(t)}G(t)$$

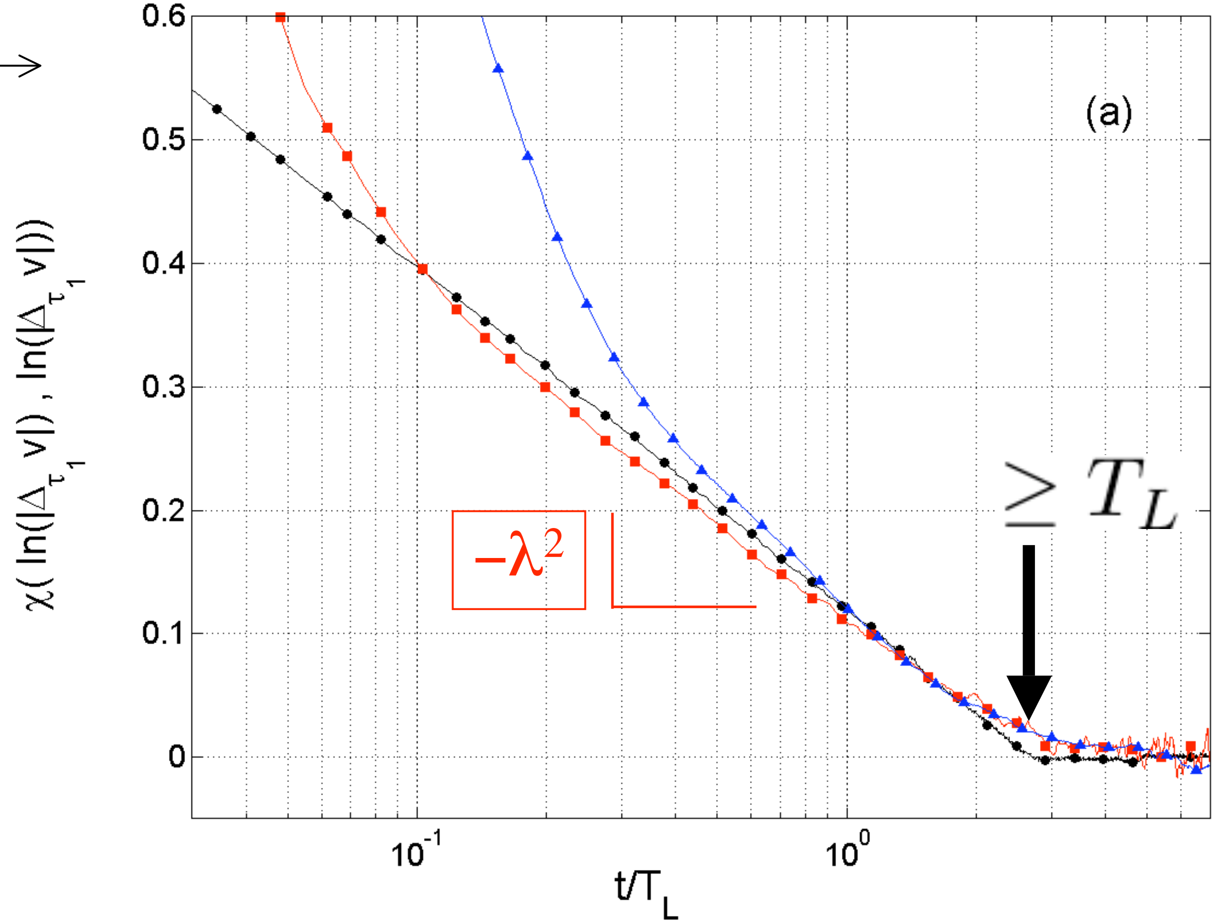
G(t) : gaussian , white in time, and :

$$\langle \omega(t)\omega(t+\Delta t)\rangle_t = -\lambda^2 \log(\Delta t / T_L)$$

A Lagrangian Random Walk

Mordant, Delour, Lévêque, Arnéodo, Pinton, *Phys. Rev. Lett.* **89**, 254502 (2002).

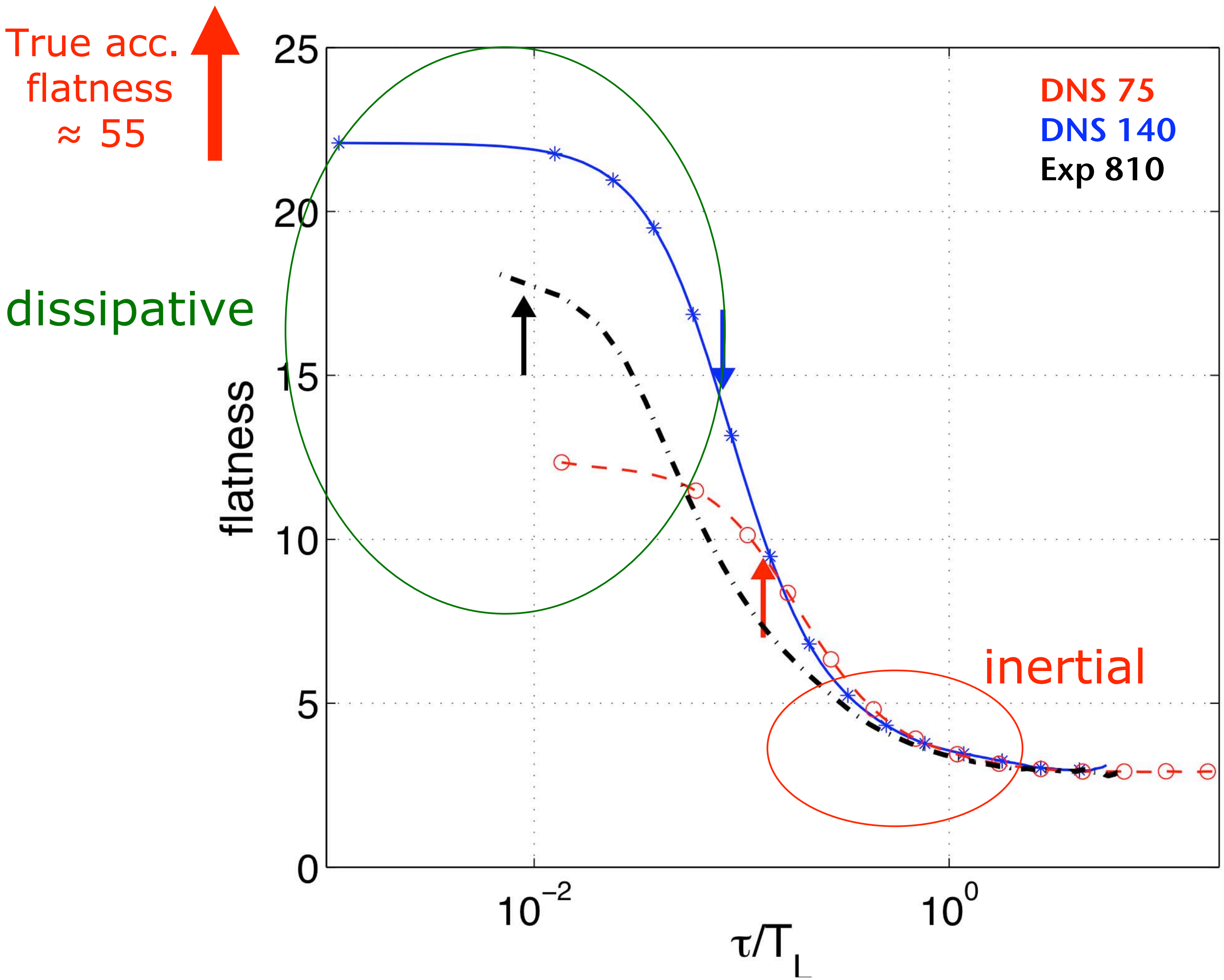




$$< \log| u_{\tau 0}(t')| \;-< \log| u_{\tau 0}| >)(\log| u_{\tau 0}(t'+t)| \;-< \log| u_{\tau 0}| >_{t'} \propto -\lambda^2 \log(t)$$

From inertial to dissipative

Chevillard, Roux, Lévêque, Mordant, Arnéodo, Pinton, *Phys. Rev. Lett.*, **91**, 214502, (2003)



General exp-dns agreement

ICTR collaborative paper, http://ictr.cineca.it

dns1:Biferale 2005

dns2: Yeung 2006

dns3: Berg 2006

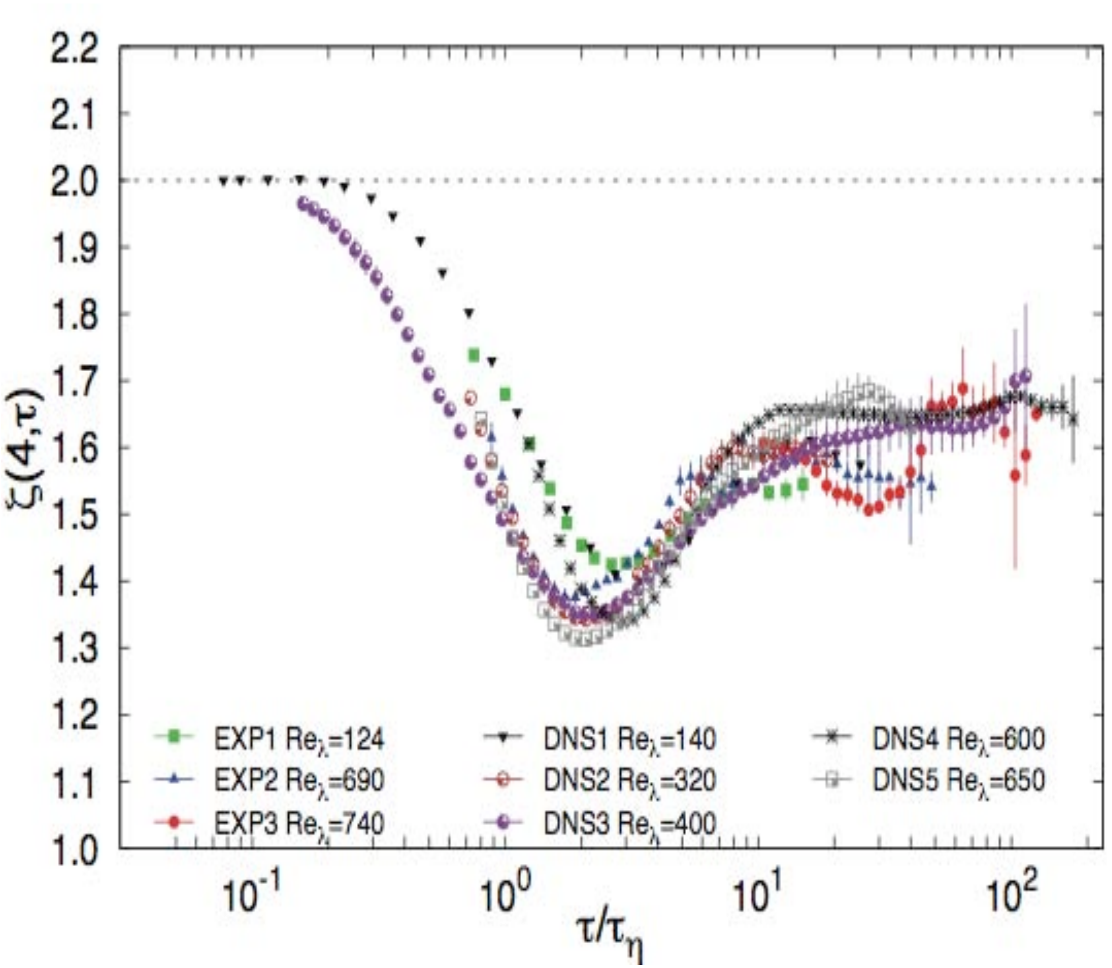
dns4: Shaw 2003

dns5: Lévêque 2004

exp1: Xu 2006

exp2: Mordant 2001

exp3: LaPorta 2001



From inertial to dissipative

Chevillard, Roux, Lévêque, Mordant, Arnéodo, Pinton, *Phys. Rev. Lett.*, **91**, 214502, (2003)

$$\delta_\tau v(t) = \beta(\tau/T)\delta_T v \qquad \mathcal{P}\left(\delta_\tau v\right) = \int \frac{d\beta}{\beta}\mathcal{G}\left(\frac{\delta_\tau v}{\beta}\right)\mathcal{P}\beta$$

Multifractal formalism

$$\text{I.R.} \quad \beta \sim (\tau/T)^h \quad \text{with} \quad \mathcal{P}_{IR}(h,\tau/T) \sim (\tau/T)^{1-\mathcal{D}(h)}$$

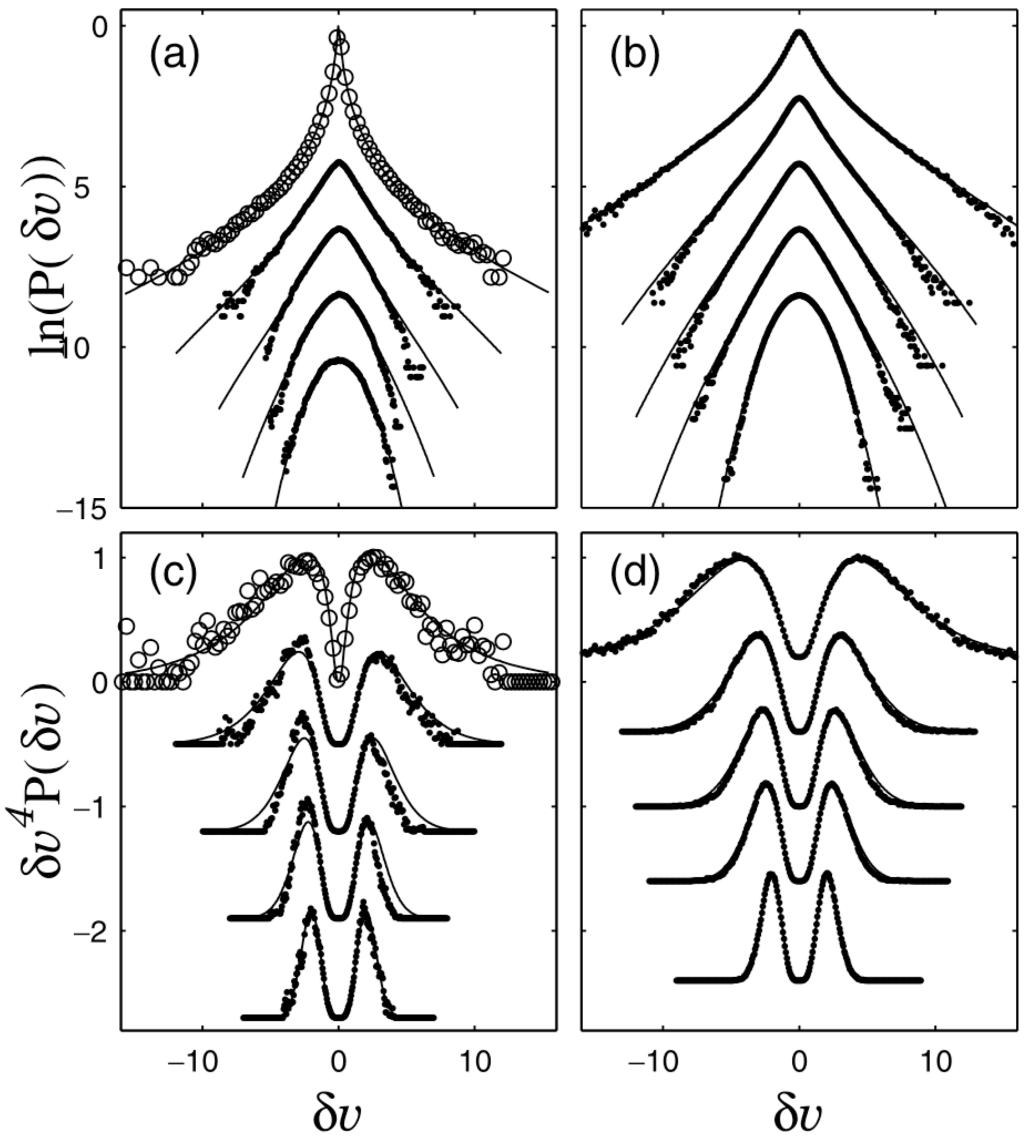
$$\text{D.R.} \quad \beta \sim \tau/T \quad \text{with} \quad \mathcal{P}_{DR}(h,\tau/T) \sim (\tau_\eta(h)/T)^{1-\mathcal{D}(h)}$$

$$\frac{\tau_\eta(h)}{T} = Re^{\frac{-1}{2h+1}} \quad \text{from} \quad Re(\text{at } \tau_\eta) = 1$$

$$\mathcal{P}\left(\delta_\tau v\right) = \int_{h\in I.R.} \cdots \; + \int_{h \in D.R.} \cdots$$

unified multifractal description

Chevillard, Roux, Lévêque, Mordant, Arnéodo, Pinton, *Phys. Rev. Lett.*, **91**, 214502, (2003)

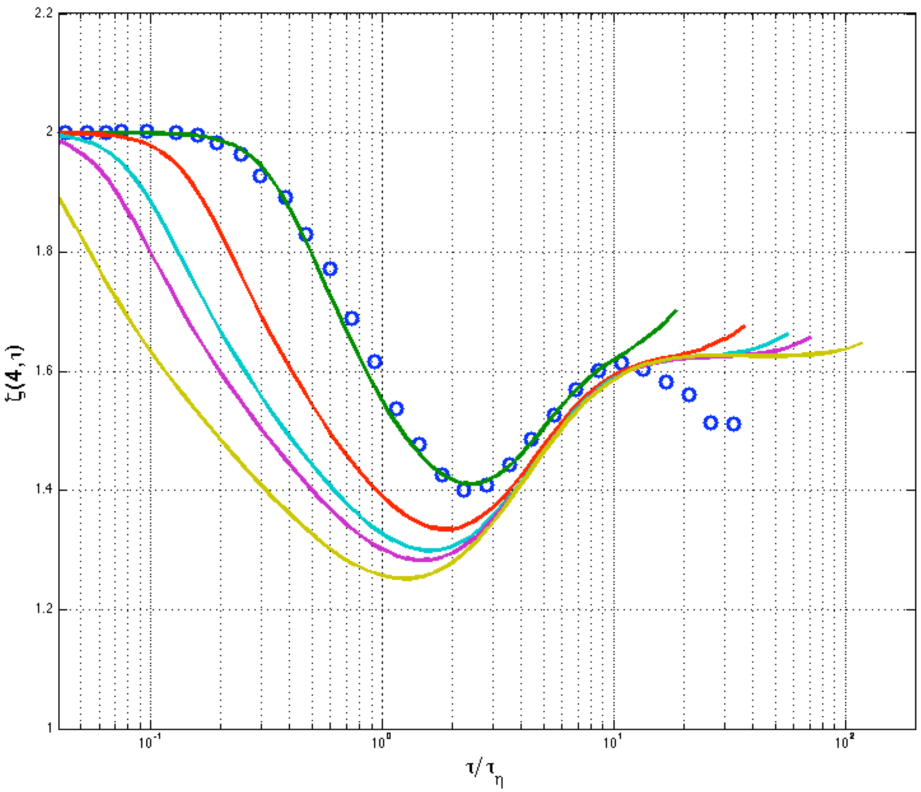


unified multifractal description

$$\beta\left(h;\frac{\tau}{T},\mathrm{Re}\right)=\frac{(\frac{\tau}{T})^{h}}{[1+(\frac{\tau}{\tau_\eta(h)})^{-\delta}]^{(1-h)/\delta}},$$

delta=4,
Rstar=15, Rlambda=150 350 600 800 1500

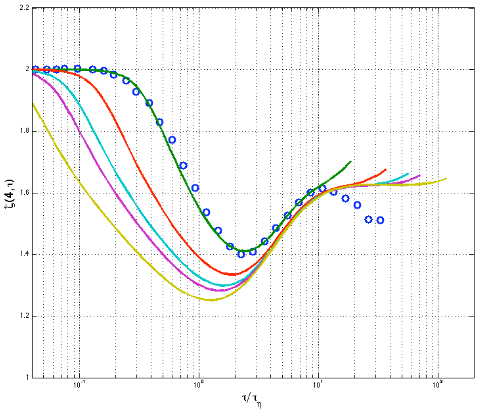
local slope $d\ln S^4/d\ln S^2$



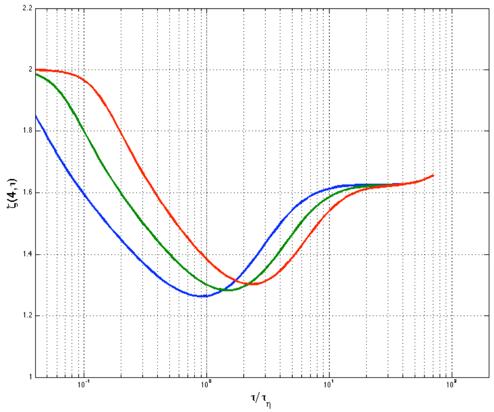
τ/τ_η

multifractal parameters

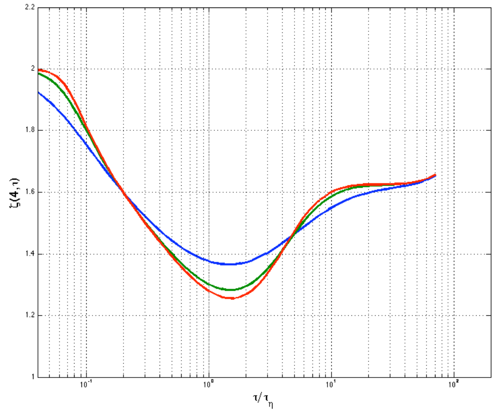
delta=4,
Rstar=15, Rlambda=150 350 600 800 1500



delta=4, Rlambda=800, Rstar=7 15 30

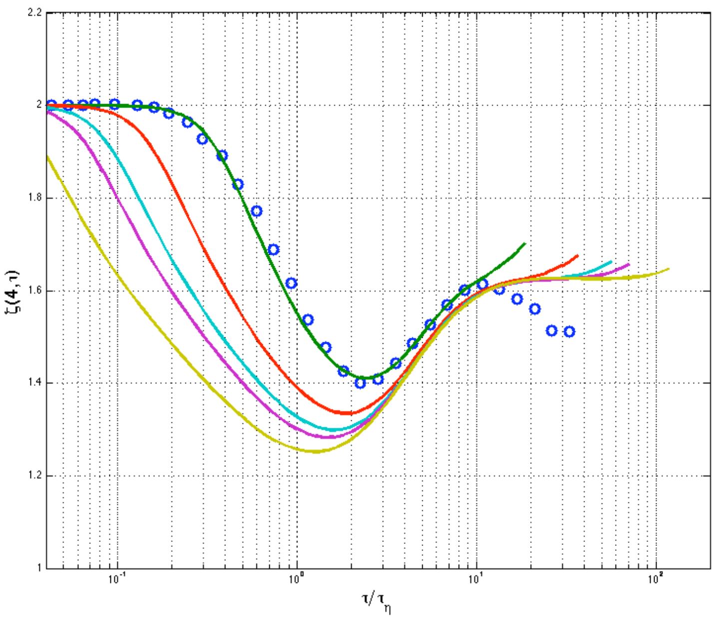


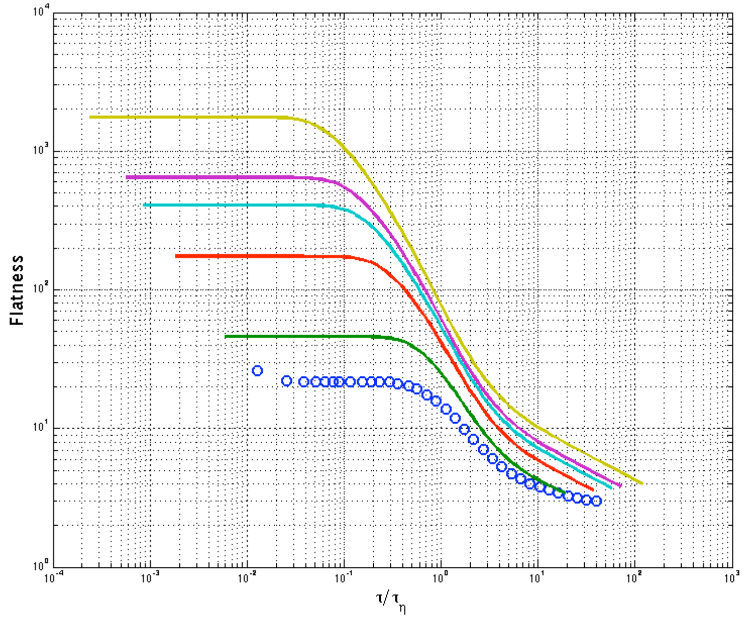
Rstar=15, Rlambda=800, delta=2 4 6



perspective for fitting …

beta=4,
Rstar=15, Rlambda=150 350 600 800 1500





alternatives …

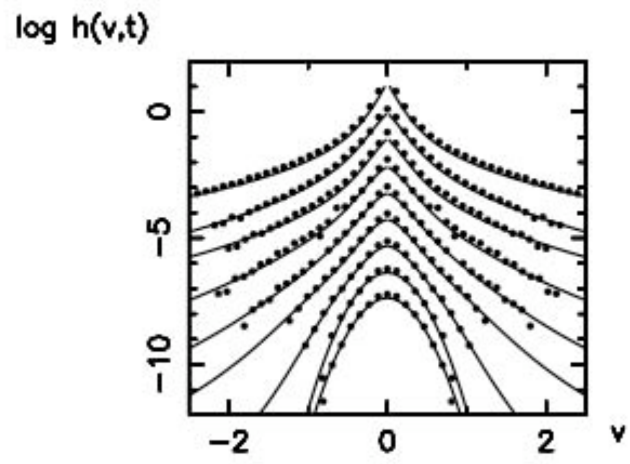


FIG. 1. A comparison of probability distributions $1/\sqrt{\overline{u(t)^2}}\,h[u/\sqrt{\overline{u(t)^2}}, t]$, Eq. (17), for various values of ν with the experimentally obtained ones determined by the Lyon group [6] [the curves are shifted (from above: $t = 0.15$, 0.3, 0.6, 1.2, 2.5, 5.0, 10.0, 20.0, and 40.0 ms) and fitted by the values $\nu = 1.71$, 1.48, 1.36, 1.26, 1.14, 1.06, 0.94, 0.70, and 0.70].

PHYSICAL REVIEW LETTERS

Statistics of Lagrangian Velocities in Turbulent Flows

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Institute for Theoretical Physics, University of Münster, Wilhelm-Klemm-Strasse 9, 48149 Münster, Germany
(Received 23 July 2002)

1) Motivations

2) Measurement methods

3) “K41” measurements (2nd order qty)

4) Acceleration

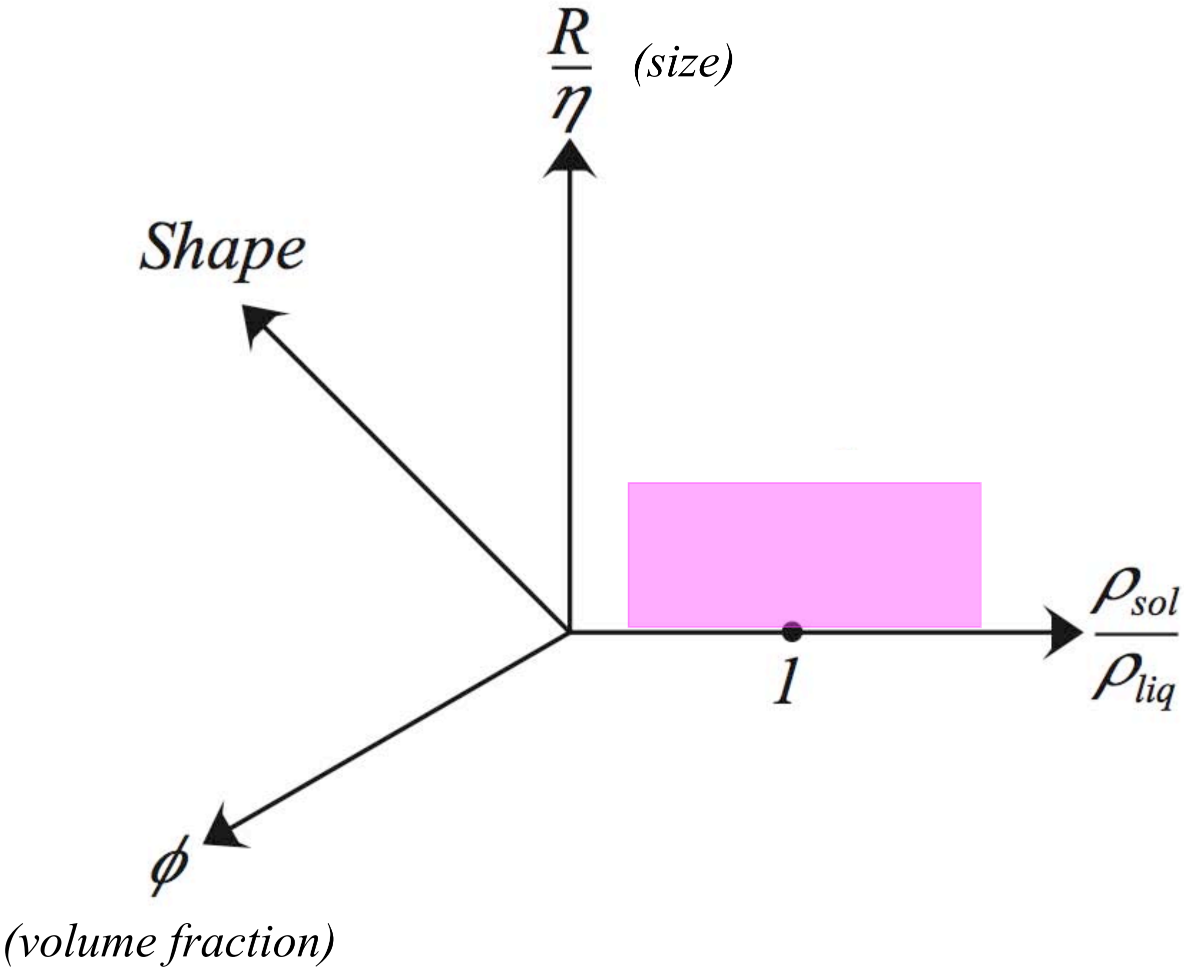
5) Intermittency (higher orders)

6) [2-points: dispersion, Multipoints: gradients]

7) **Note 1 : Inertial particles**

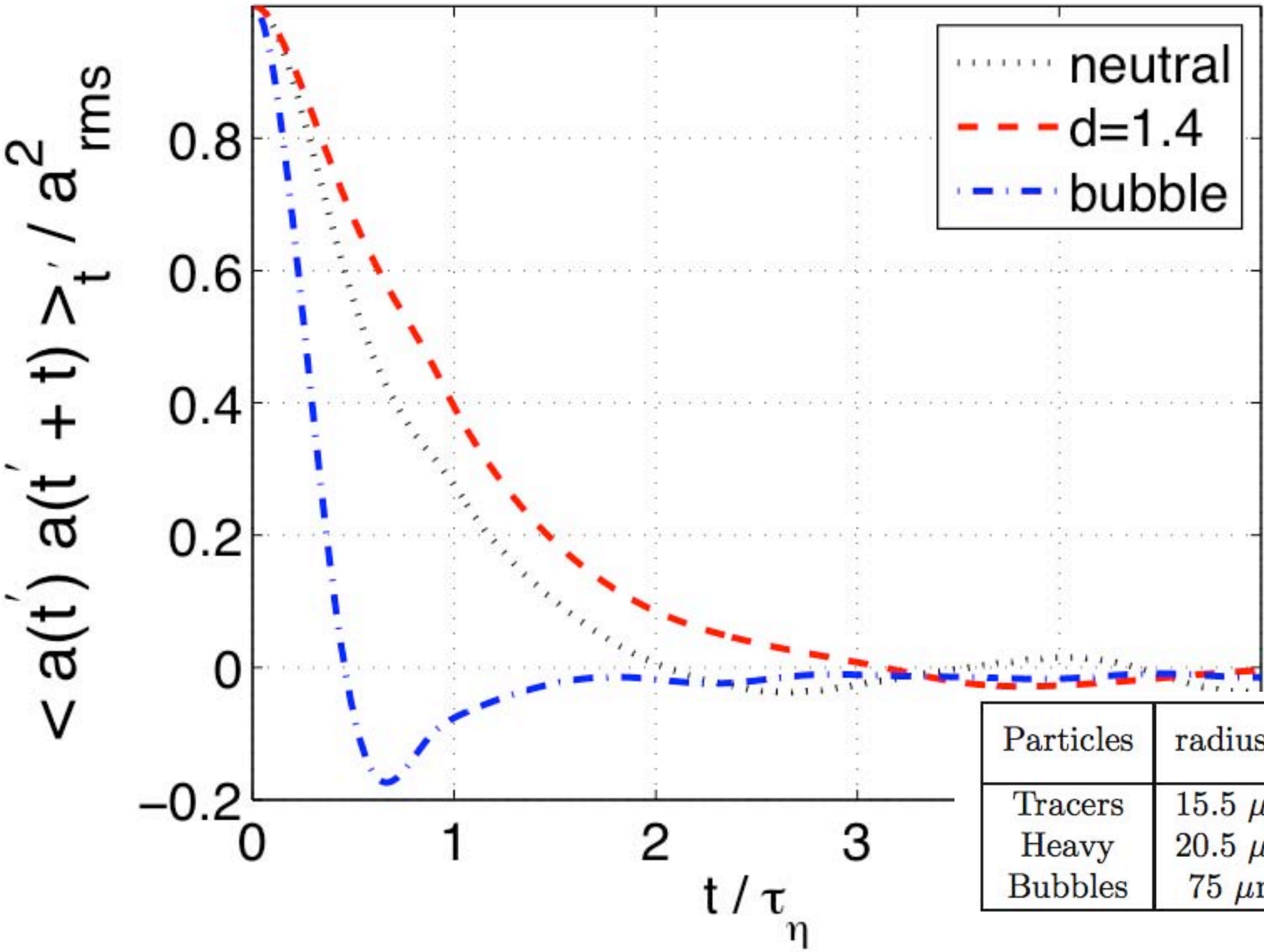
8) Note 2 : Convection

Beyond fluid tracers…



Inertial particles : dynamics

Volk, Verhille, Mordant, Pinton, *EPL preprint*, arXiv 0708.3350



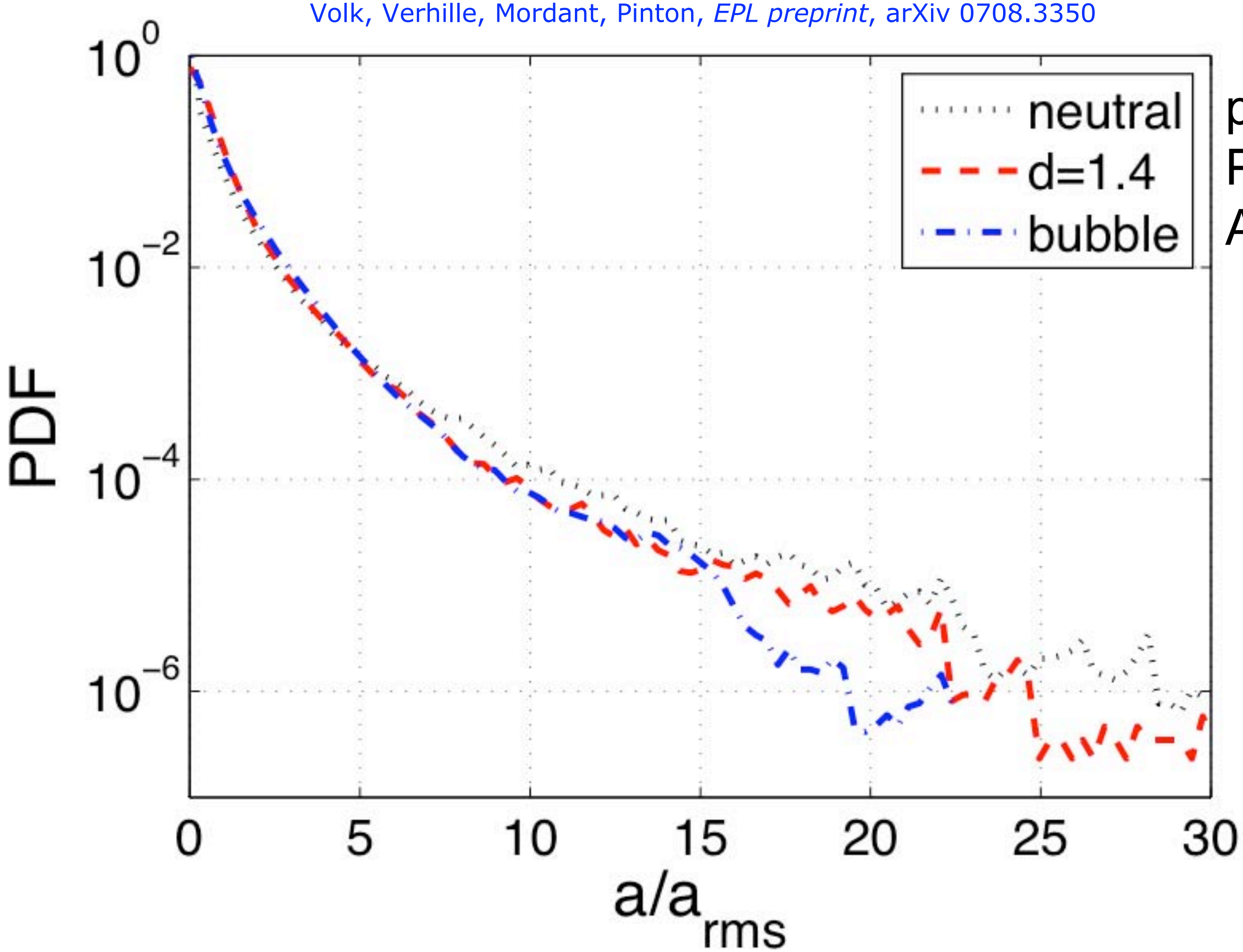
polyS, 31 μm
PMMA, 43 μm
Air, 150 μm

Particles	radius a	$\beta = \frac{3\rho_f}{\rho_f + 2\rho_p}$	$\frac{\tau_{corr}}{\tau_\eta}$	a_0
Tracers	15.5 μm	0.96	0.5	6.4 ± 1
Heavy	20.5 μm	0.79	0.9	4.3 ± 1
Bubbles	75 μm	2.99	0.25	26 ± 5

Table 2: Parameters of the particles at $R_\lambda = 850$ ($\eta = (\nu^3/\epsilon)^{1/4} = 17\ \mu m$ and $\tau_\eta = \sqrt{\nu/\epsilon} = 0.26\ 10^{-3}$ s). ρ_p and ρ_f are the densities of the particles and fluid, and τ_{corr} is defined as the half-width at mid amplitude of the acceleration autocorrelation function.

Inertial particles : acceleration PDFs

Volk, Verhille, Mordant, Pinton, *EPL preprint*, arXiv 0708.3350

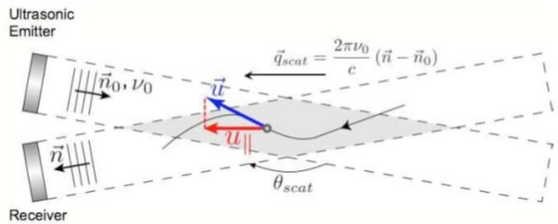


polyS, 31 µm
PMMA, 43 µm
Air, 150 µm

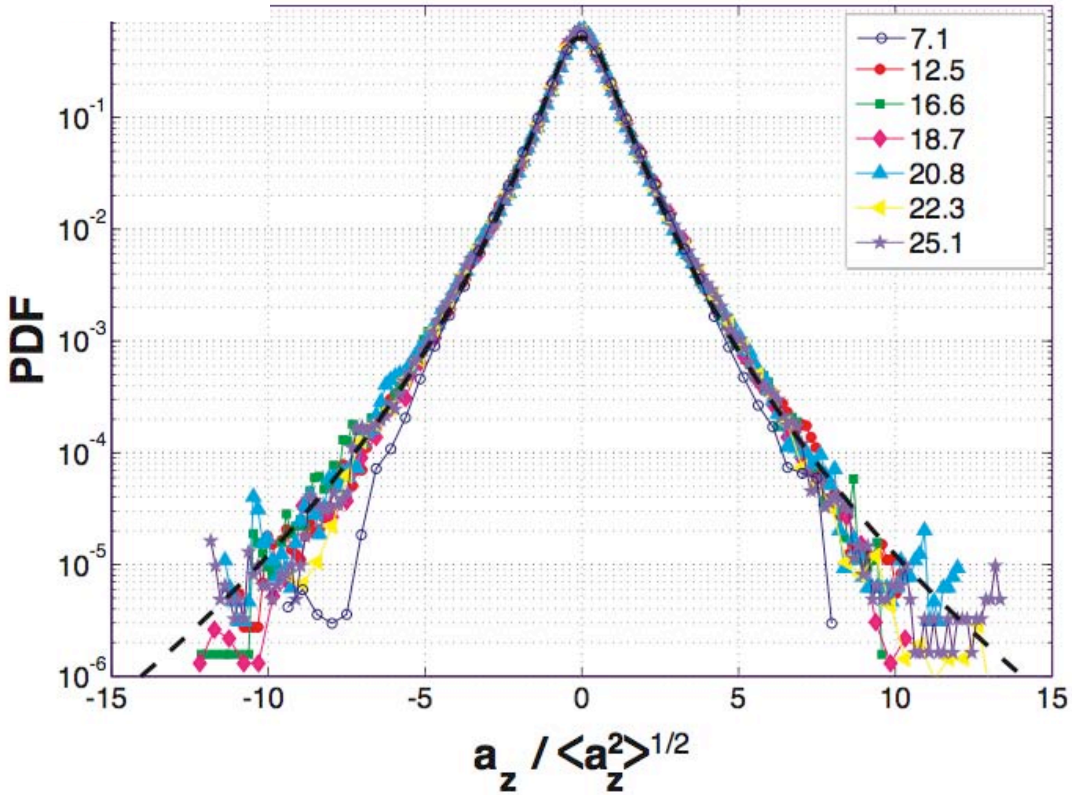
	d/η	ρ_p/ρ_f	St	$3\rho_f/(\rho_f+2\rho_p)$
Heavy	3	1.4	0.2	0.8
Neutrally boyant	2	1.06	0.01	~ 1
Air bubbles	5	0	1.4	3

Neutrally buoyant, varying size

Qureshi, Bourgoin, Baudet, Gagne, *Phys. Rev. Lett*. **99**, 184502 (2007)

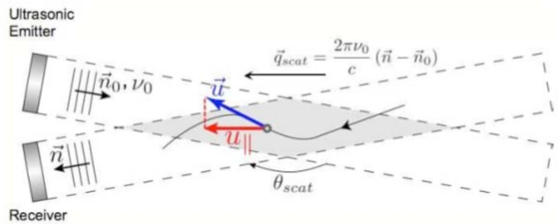


Air jet, soap bubbles filled with He



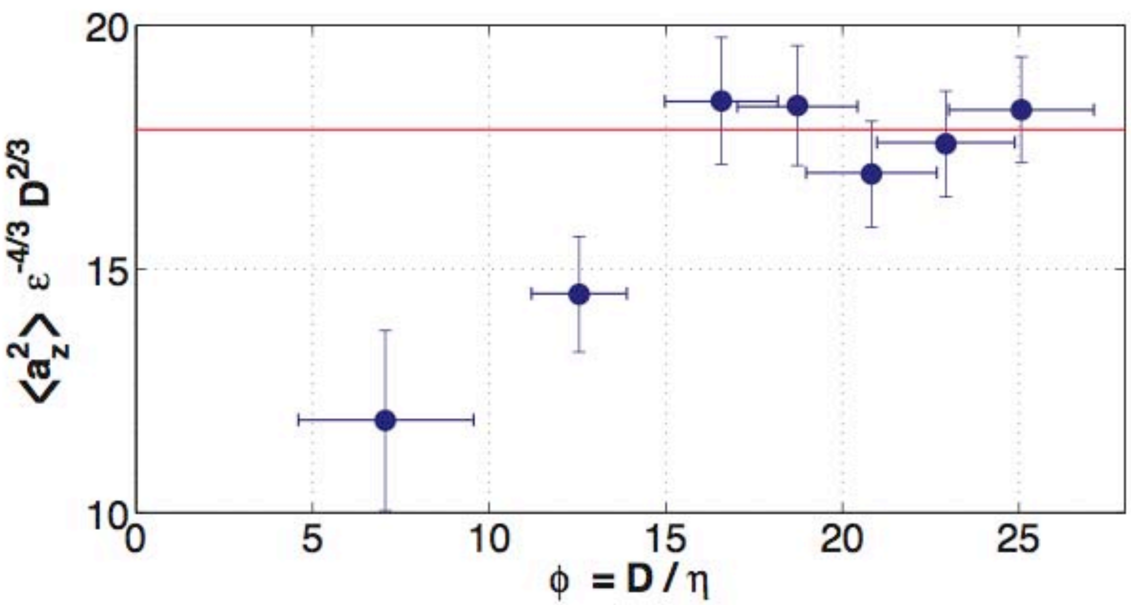
Neutrally buoyant, varying size

Qureshi, Bourgoin, Baudet, Gagne, *Phys. Rev. Lett*. **99**, 184502 (2007)



$$\left\langle a_z^2 \right\rangle_{\text{particle}}(D) \propto \frac{S_2^P(D)}{D^2},$$

$$\left\langle a_z^2 \right\rangle_{\text{particle}}(D) = a_0' \epsilon^{4/3} D^{-2/3}$$



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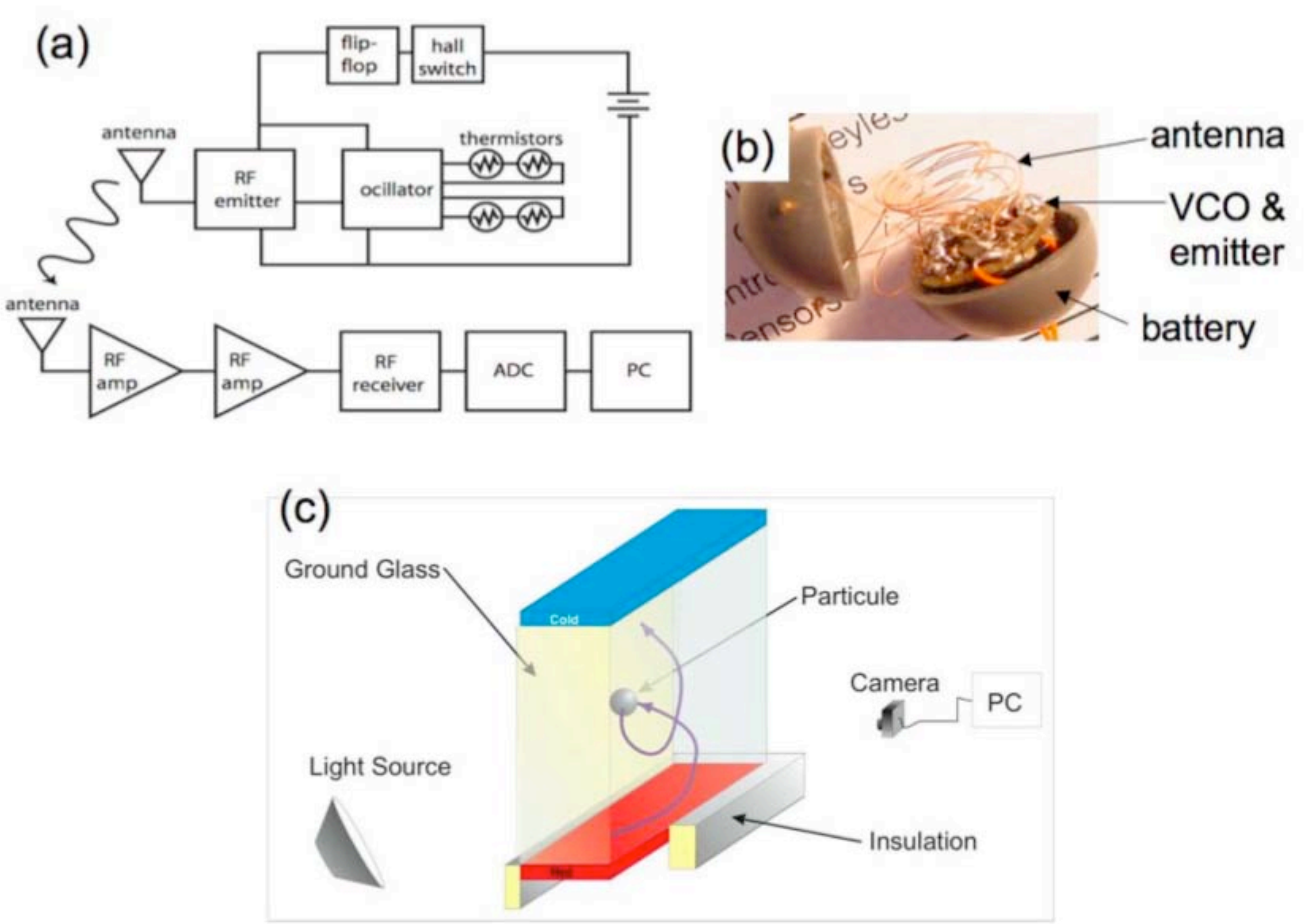
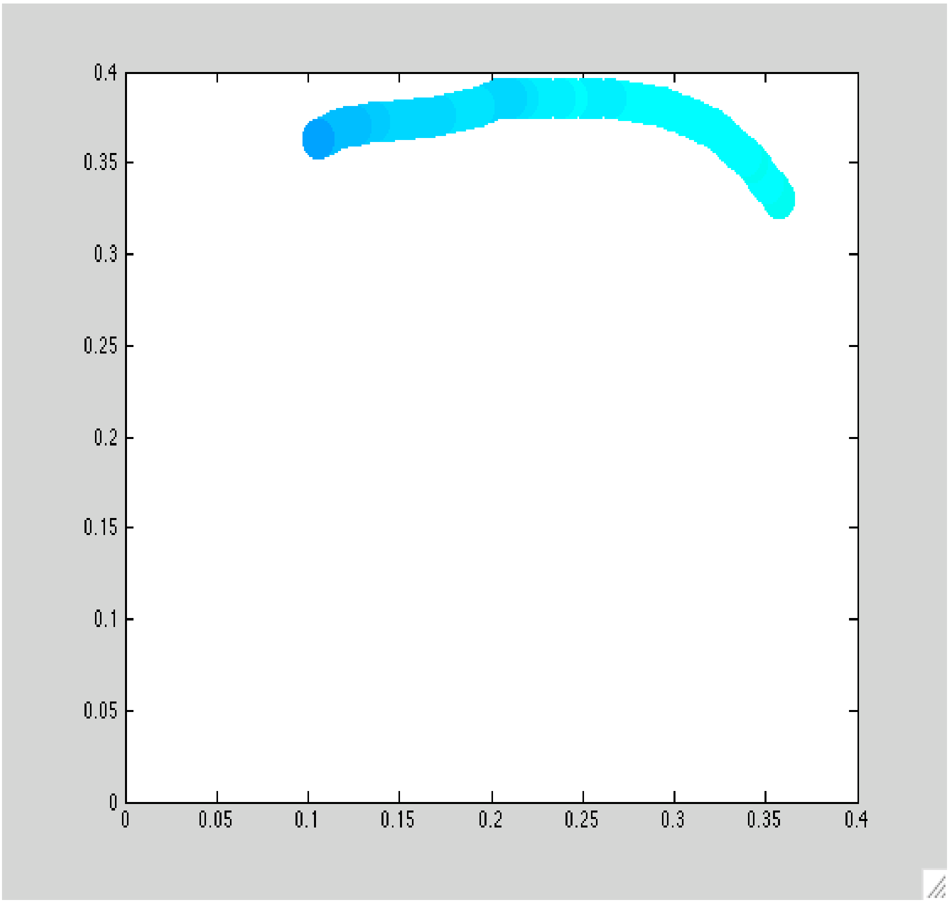
Rayleigh Bénard Convection

Gasteuil, Gibert, Shew, Metz, Pinton, *Rev. Sci. Instr.* **78,** 065105 (2007)



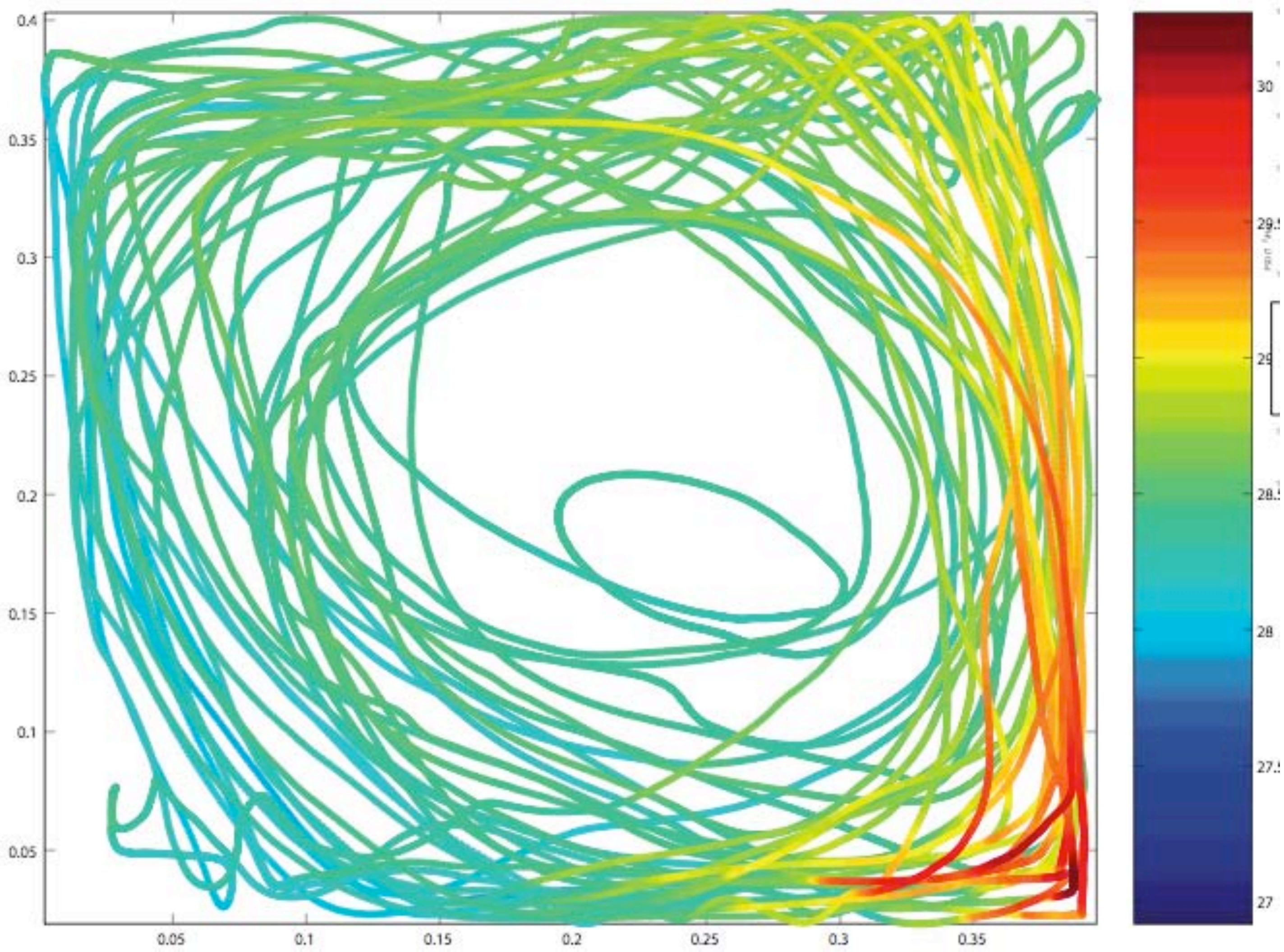
Rayleigh Bénard Convection

Y. Gasteuil, M. Gibert, W. Shew, P. Metz, J.-F. Pinton, *Rev. Sci. Instr.* **78**, 065105 (2007)



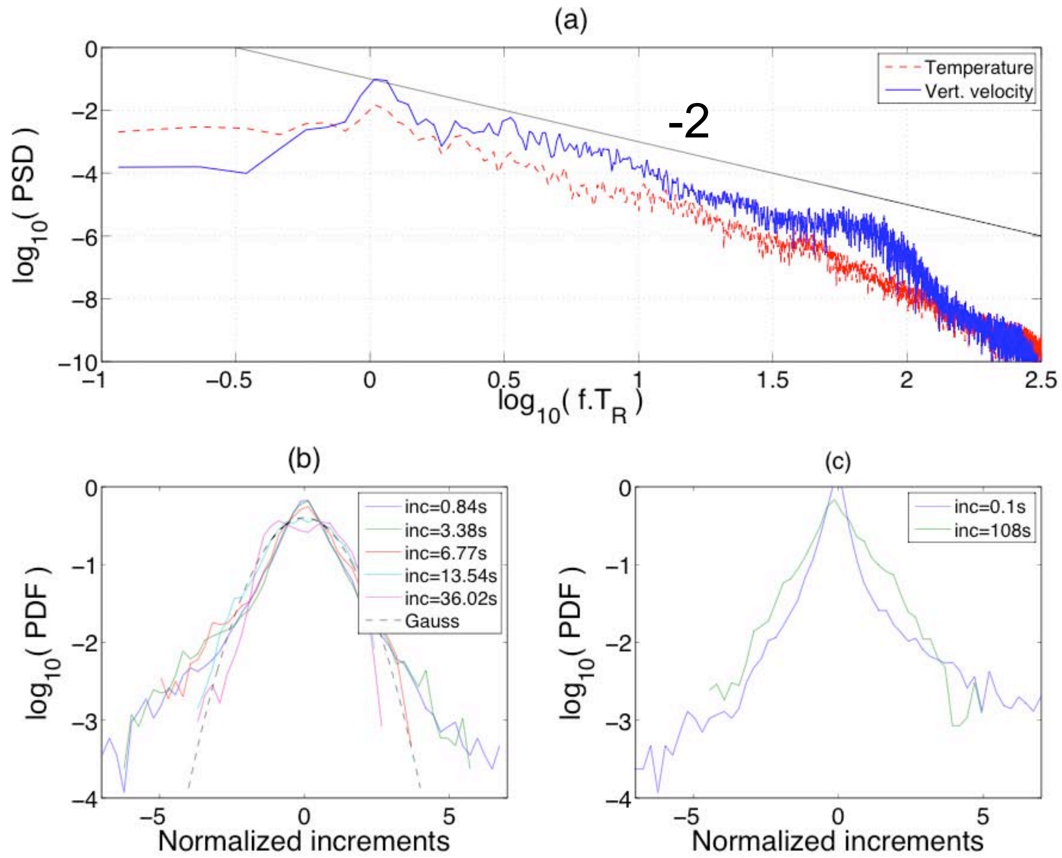
Rayleigh Bénard Convection

Gasteuil, Gibert, Shew, Metz, Pinton, *Rev. Sci. Instr.* **78**, 065105 (2007)



Rayleigh Bénard Convection

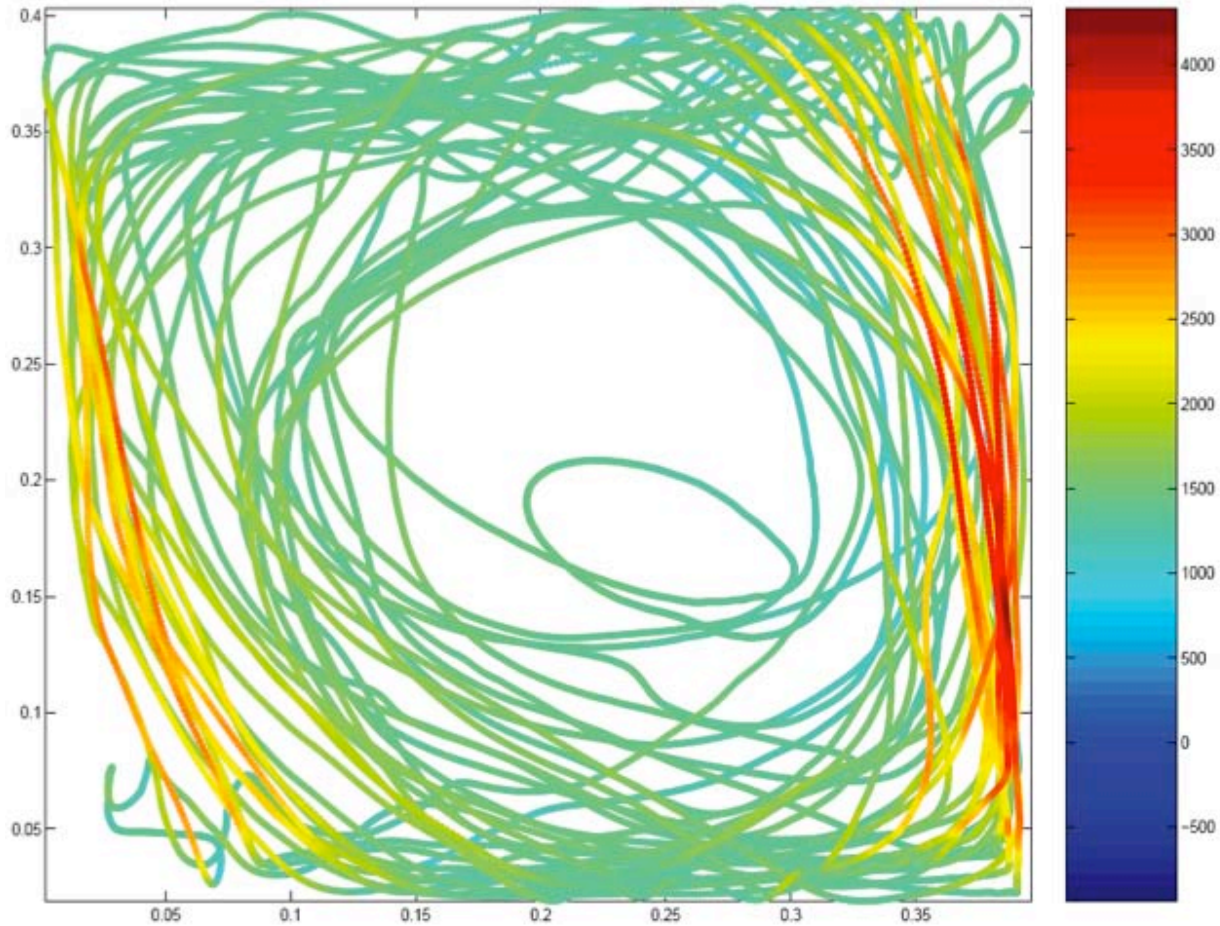
Gasteuil, Gibert, Shew, Chilla, Castaing, Pinton, *Phys. Rev. Lett.* **99,** 234302 (2007)



→ tracer motion is Lagrangian

Rayleigh Bénard Convection

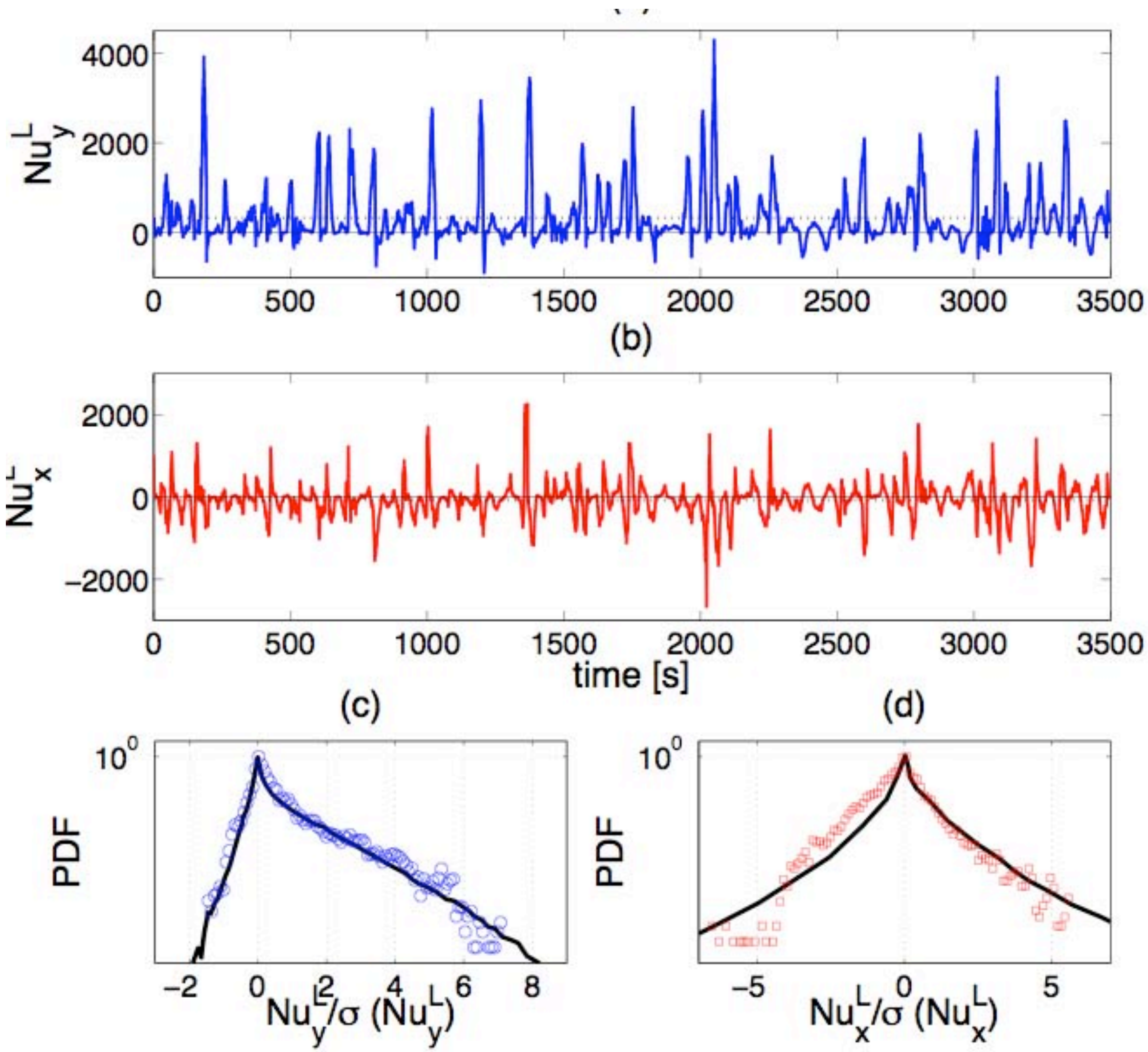
Gasteuil, Gibert, Shew, Chilla, Castaing, Pinton, *Phys. Rev. Lett.* **99,** 234302 (2007)



$$Nu^L(T) = 1 + \frac{H}{\kappa \Delta T} v_z(t)\theta(t)$$

Rayleigh Bénard Convection

Gasteuil, Gibert, Shew, Chilla, Castaing, Pinton, *Phys. Rev. Lett.* **99,** 234302 (2007)



Self-similarity of plumes

Gasteuil, Gibert, Shew, Chilla, Castaing, Pinton, *Phys. Rev. Lett.* **99,** 234302 (2007)