Nonlinear resonances – is it turbulence or not?

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Collobarators: S. Nazarenko, V. L´vov

Vortex <-> Wave

strong T. *versus* weak T.

- V. L´vov, 2006 : „As soon as something is PROVEN, it is regarded as Weak Turbulence"
- Nonlinear resonances:

1. Live between ST and WT
2. Have bad behaviour ;-) (nonlocality, anysotropy, …) (2D-case +…)

Nonlinear waves in 2D-media

2D-waves propagate in two directions

height of the wave $a(\vec{x}, t)$ oscillates as

$$\sin(\vec{k}\vec{x} - \omega t)$$

Now we have

- **wave vector** $\vec{k} = (k_1, k_2)$
- **in resonators (=periodic or zero boundary conditions)** k_1, k_2 are **integers**
- **dispersion function** is $\omega_k = \omega(k_1, k_2)$

Examples

- **1. Capillary waves** (due to surface tension):

$$\omega_k \sim k^{3/2},\ k = |\vec{k}|$$

1.

- **2. Water gravity surface waves** (due to gravity acceleration):

$$\omega_k \sim k^{1/2}$$

2.

- **3. Oceanic planetary waves** (due to the Earth rotation), called also **Rossby waves**:

$$\omega_k \sim k^{-1/2}$$

Physical scales

when discreteness is important?

- Capillary waves: λ < 4cm (desktop experiments)
- Water gravity waves: λ ~ 0.1 - 1000m (laboratory experiments, Chanels, Bays, Seas)
- Oceanic Rossby waves: λ ~ 100-200 km (Oceans)
- Atmospheric Rossby waves: λ ~ 1000-5000 km (Earth Atm.)

Main message: L/λ is important!

Main result

Discrete resonances **DO exist** in many physically relevant 2D-wave systems!

What was the problem?

$$\omega_k = k^{3/2}, \quad k = |\vec{k}| = (m^2+n^2)^{1/2} \quad \Rightarrow \quad k_1^{3/2} + k_2^{3/2} = k_3^{3/2} \quad \Rightarrow$$

$$k_1^3 + 2(k_1k_2)^{3/2} + k_2^3 = k_3^3 \quad \Rightarrow \quad 2(k_1k_2)^{3/2} = k_3^3 - k_1^3 - k_2^3$$

$$4(k_1k_2)^3 = (k_3^3 - k_1^3 - k_2^3)^2 \quad \Rightarrow \quad 16(k_1k_2)^6 = (k_3^3 - k_1^3 - k_2^3)^4$$

Left part has no roots, right part still have them,
After taking 8 times power 2, we get terms of commulative degree 16 and some roots are still left.
Already at this step, for wave numbers ~ 1000, we need to make computations with integers of order

.

$$10^{48}$$

Kinematics and a bit of dynamics

Volume 72, Number 13 PHYSICAL REVIEW LETTERS 28 March 1994

Weakly Nonlinear Theory of Finite-Size Effects in Resonators

Elena A. Kartashova*

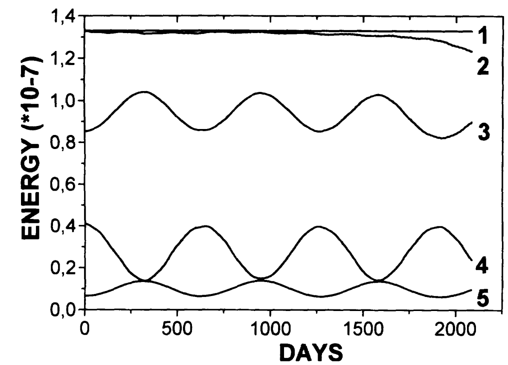
- Not all modes interact resonantly

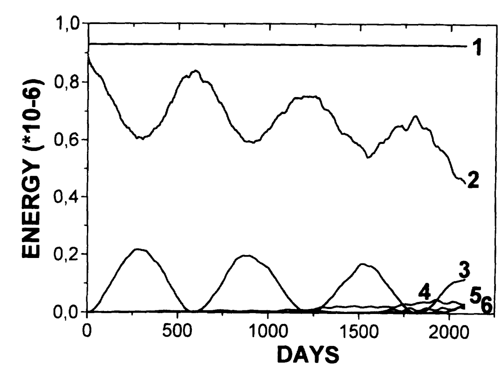
- Resonant modes form small dynamically independent clusters

- Form of clusters, if any, is defined by boundary conditions

- Resonant interactions are not local in Kolmogorov's sense

Amer. Math. Soc. Transl. (2) Vol. 182, 1998



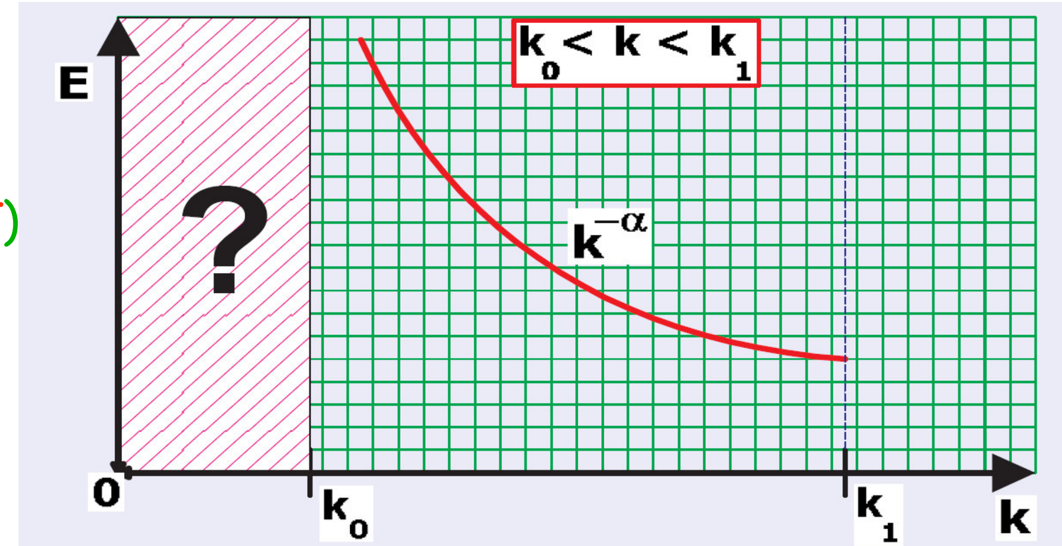


Discrete effects in WT

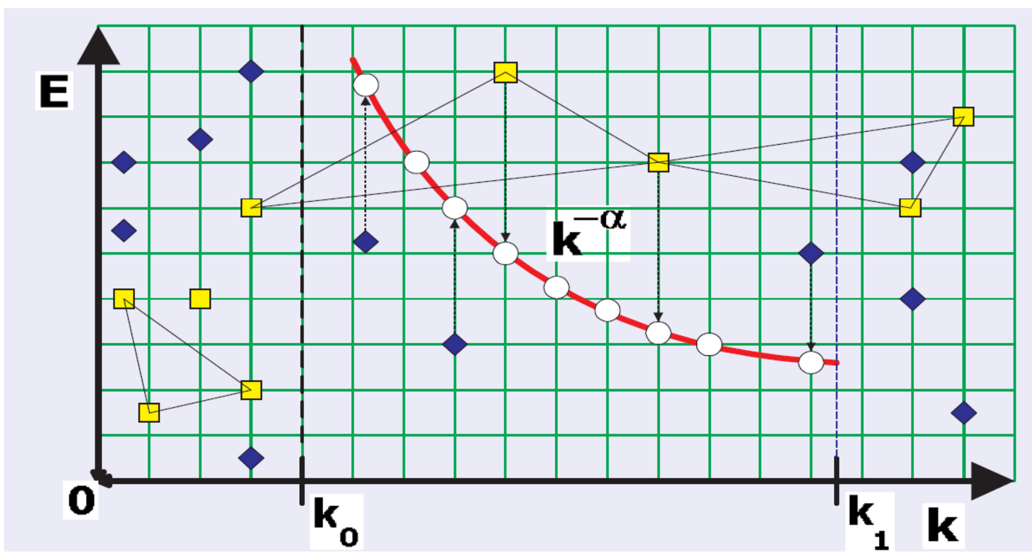
Phillips 1981 („new physics, new mathematics and new intuition is needed…")
Craik 1985
……………………….
Pushkarev 1999 (capillary)
Pushkarev, Zakharov 2000 (frozen WT)
Newell, Nazarenko, Biven 2001
Connaughton, Nazarenko, Pushkarev 2001
……………………….
Zakharov, Korotkevich, Pushkarev 2005 (mesoscopic WT, gravity water w.)
Kartashova 2006 (laminated WT, kinematics)
Lvov, Nazeranko, Pokorni 2006 (gravity water w.)
Nazarenko 2006 (sand-pile model, dynamics)
Denissenko, Lukaschuk, Nazarenko 2007 (lab. experiments)
Tanaka 2001, 2004, 2007 (num. simulations)
Kartashova, L'vov 2008 (dynamics, 3-wave resonances, submitted)

Model of laminated turbulence

1 layer:
statistical
(classical WTT)



2 layers:
statistical
and discrete



Numerical methods (kinematics)

1. *Journal of Low Temperature Physics, Vol. 145, Nos. 1–4, November 2006 (© 2006)*
   DOI: 10.1007/s10909-006-9237-1

2. International Journal of Modern Physics C
   Vol. 17, No. 11 (2006) 1579–1596

3. COMMUNICATIONS IN COMPUTATIONAL PHYSICS
   Vol. 2, No. 4, pp. 783-794
   Commun. Comput. Phys.
   August 2007

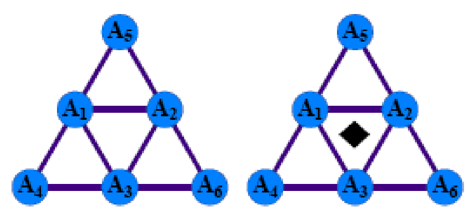
4. Physica A 380 (2007) 66–74

5. Physica A 385 (2007) 527–542

1. EK, *q-class method,* formulation

2.-4. EK+A. Kartashov, implementation

5. EK+G. Mayrhofer, graphs <-> dynamical systems (3-wave resonances)



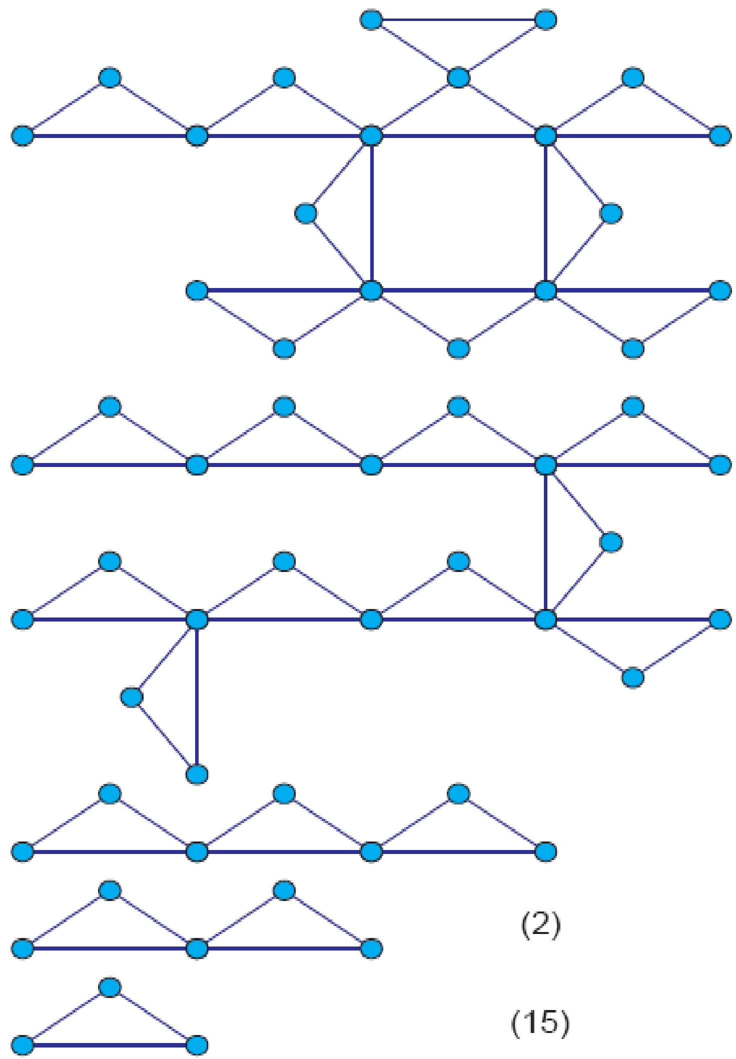
Structure of Resonances

atmospheric planetary waves

Galerkin:
2500 Fourier harmonics
(m, n <= 50)

Clipping:
Only 128 are important

On-line software:
Mayrhofer
Raab
Schreiner



Quasi-resonances

To apply this reasoning to a real physical problem we have answer the question about quasi-resonances:

$$\begin{cases} \omega(k_1) + \omega(k_2) - \omega(k_3) = \delta > 0 \\ k_1 + k_2 = k_3 \end{cases}$$

δ is called resonance width

PRL 98, 214502 (2007) PHYSICAL REVIEW LETTERS week ending 25 MAY 2007

Exact and Quasiresonances in Discrete Water Wave Turbulence

Elena Kartashova*

Interplay of three values should be regarded:

1) Local low boundary,

2) Global low boundary

3) Accuracy of numerical simulations or laboratory experiments

Stop to model turbulence – try to model a real phenomenon ;-)

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Model of Intraseasonal Oscillations in Earth's Atmosphere

Elena Kartashova[1,2,*] and Victor S. L'vov[1,†]

$$\partial \triangle\psi / \partial t + 2\,\partial\psi/\partial\lambda = -J(\psi, \triangle\psi)$$

All clusters in m,n <=21
4 triads,
3 butterflies,
1 cluster of 6 connected triads

$$\begin{cases} N_1\dot{A}_1 = 2Z(N_3 - N_2)A_3A_2^*, \\ N_2\dot{A}_2 = 2Z(N_1 - N_3)A_1^*A_3, \\ N_3\dot{A}_3 = 2Z(N_2 - N_1)A_1A_2\,, \end{cases}$$

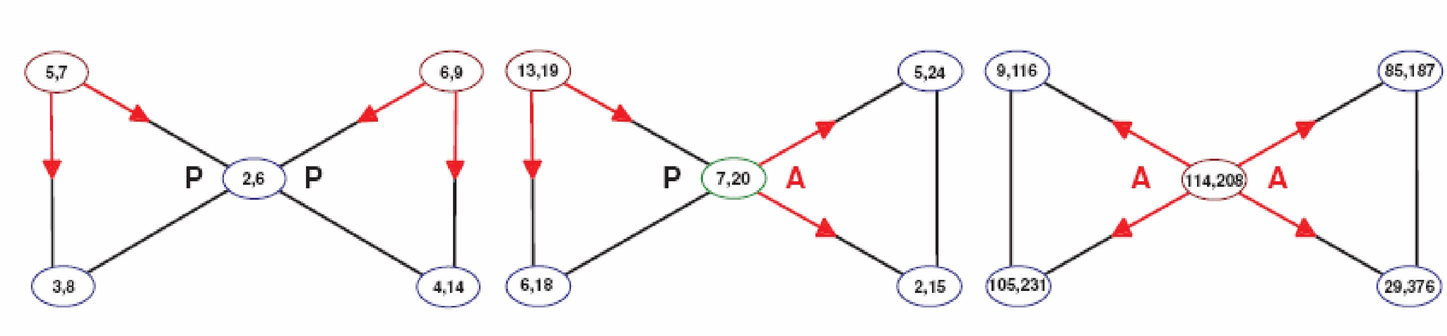
Main result – all periods are of the range of 30 to 80 days :

Intra-seasonal oscillations in the Earth atmosphere

Discovered some 40 years ago, in contradiction to Lorenz (predictability is possible to 10 days only), no reasonable explanation was known

Further studies

- General dynamics of clusters (with V. L´vov)



- Transition from discrete to statistical regime (with V. L´vov and S. Nazarenko), dynamics

- Classification of 4-wave resonances, kinematics

- Transcendental dispersion function, kinematics