Mean-Field Electrodynamics Measurements in a Sodium Experiment

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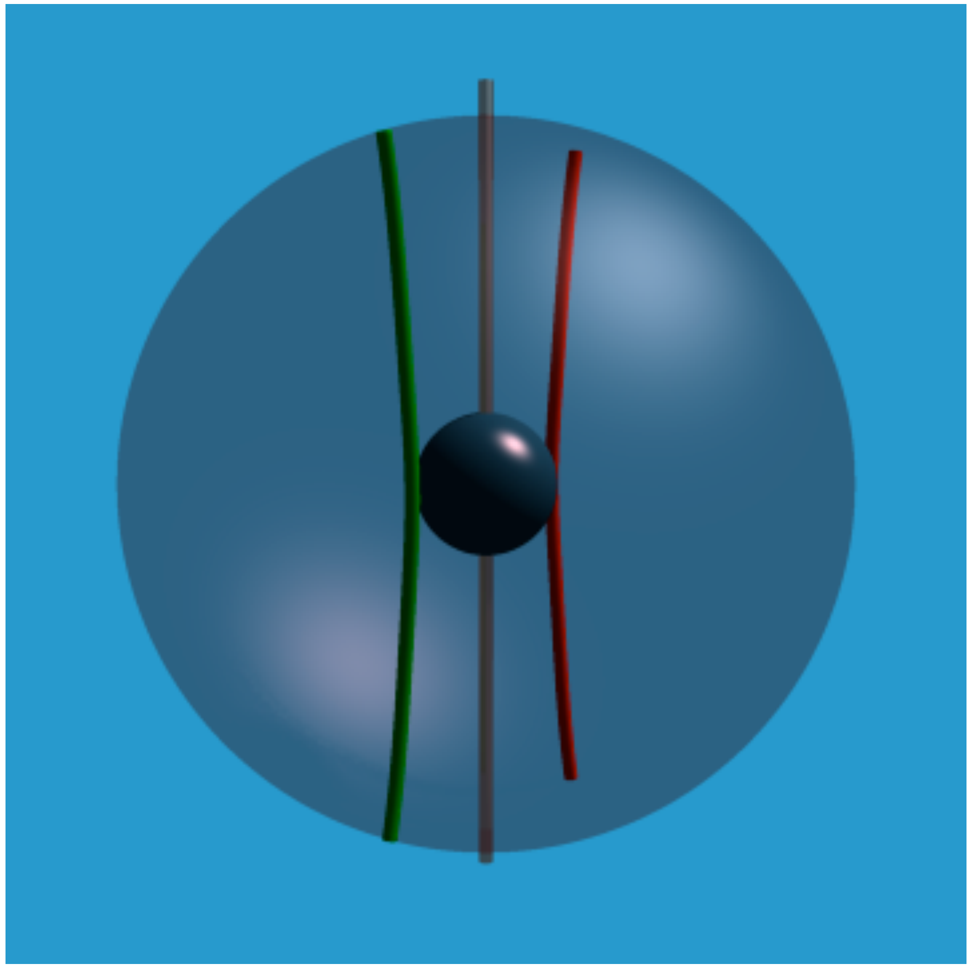
Do Mean-Field Currents Exist in Weakly Magnetized, Turbulent MHD Flows?

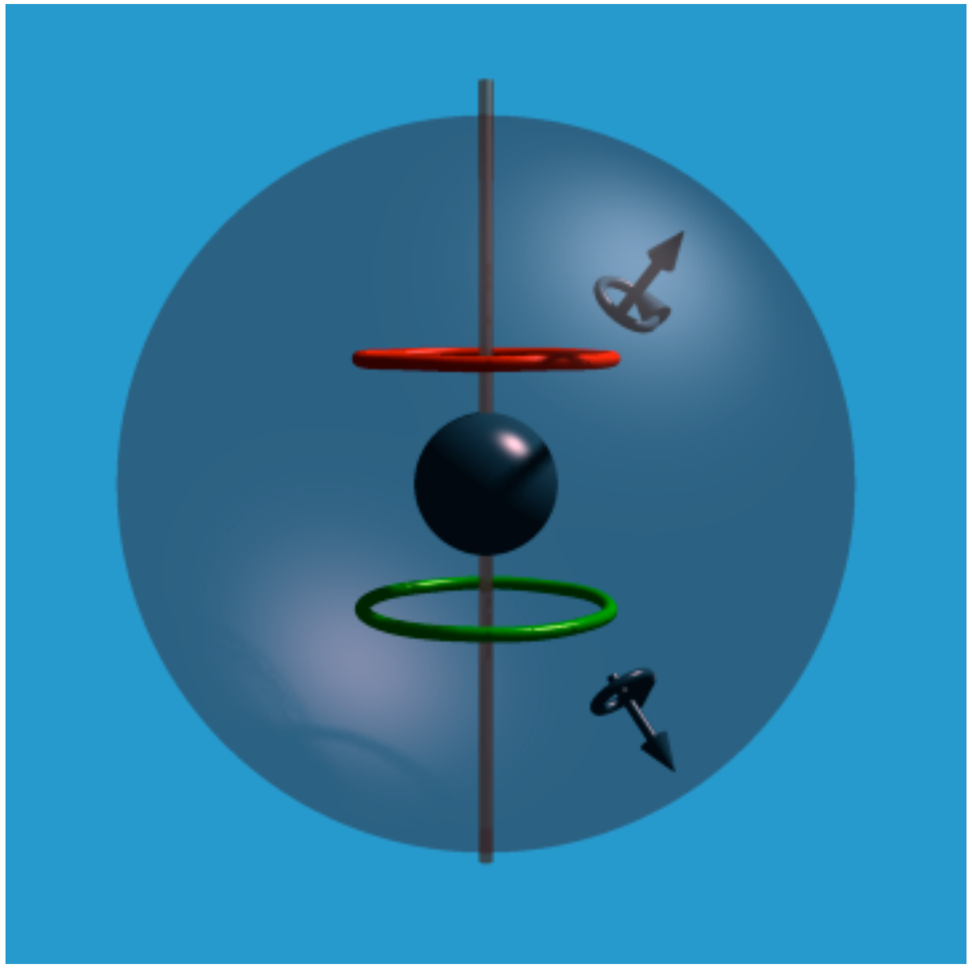
- Not previously measured
- Essential component of the Standard Model of the self-excited dynamo
- understand role of MFED on liquid metal dynamo experiments
- long standing prediction:

$$\begin{aligned}\mathcal{E} &= \left\langle \tilde{\mathbf{v}} \times \tilde{\mathbf{b}} \right\rangle = \alpha \mathbf{B} + \beta \nabla \times \mathbf{B} \text{ with} \\ \alpha &= \frac{1}{3} \int \tilde{\mathbf{v}}(t) \cdot \nabla \times \tilde{\mathbf{v}}(t+\tau) d\tau \text{ and } \beta = \frac{1}{3} \int \tilde{\mathbf{v}}(t) \cdot \tilde{\mathbf{v}}(t+\tau) d\tau\end{aligned}$$

Standard Model of quasi-axisymmetric MHD dynamo requires helical turbulence



The "Ω effect"



The "α effect"

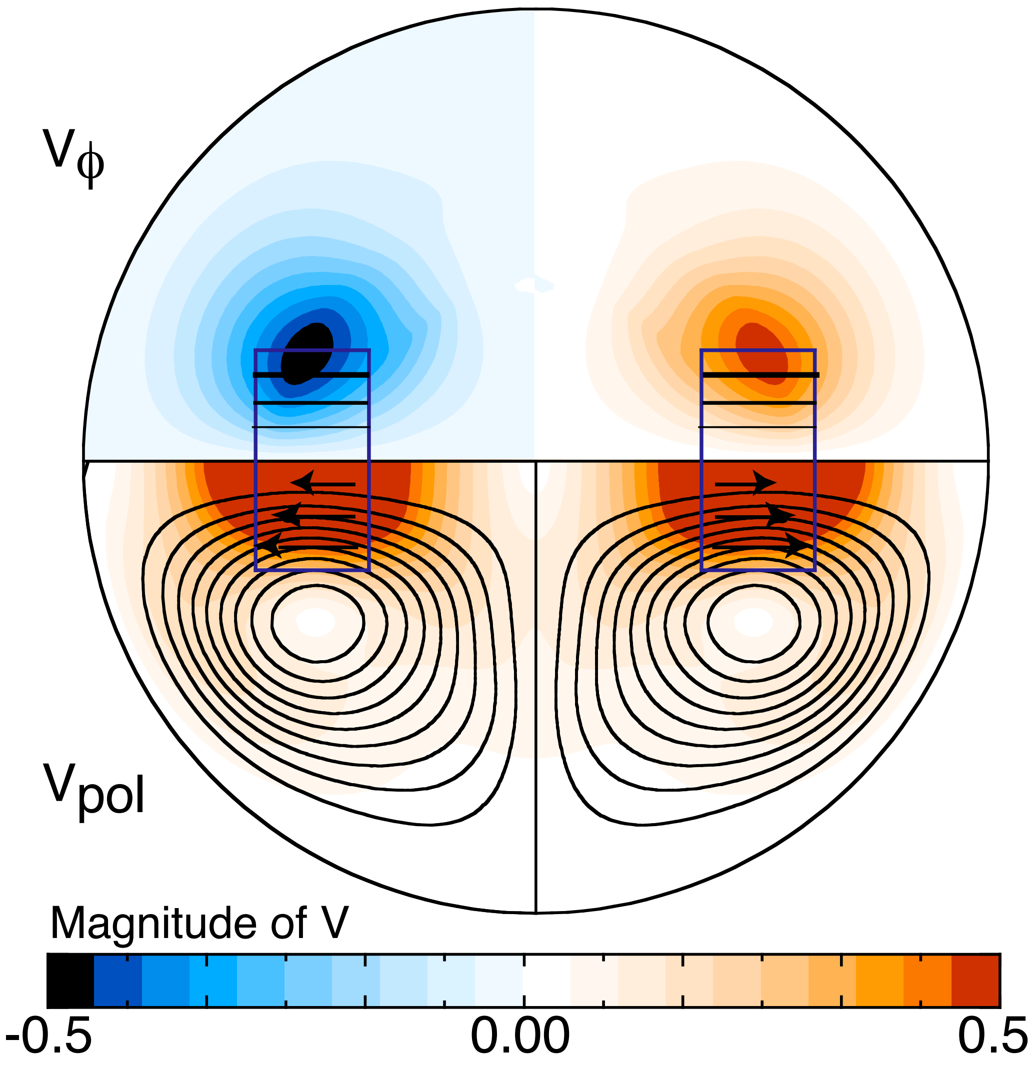
Mean Field Electrodynamics

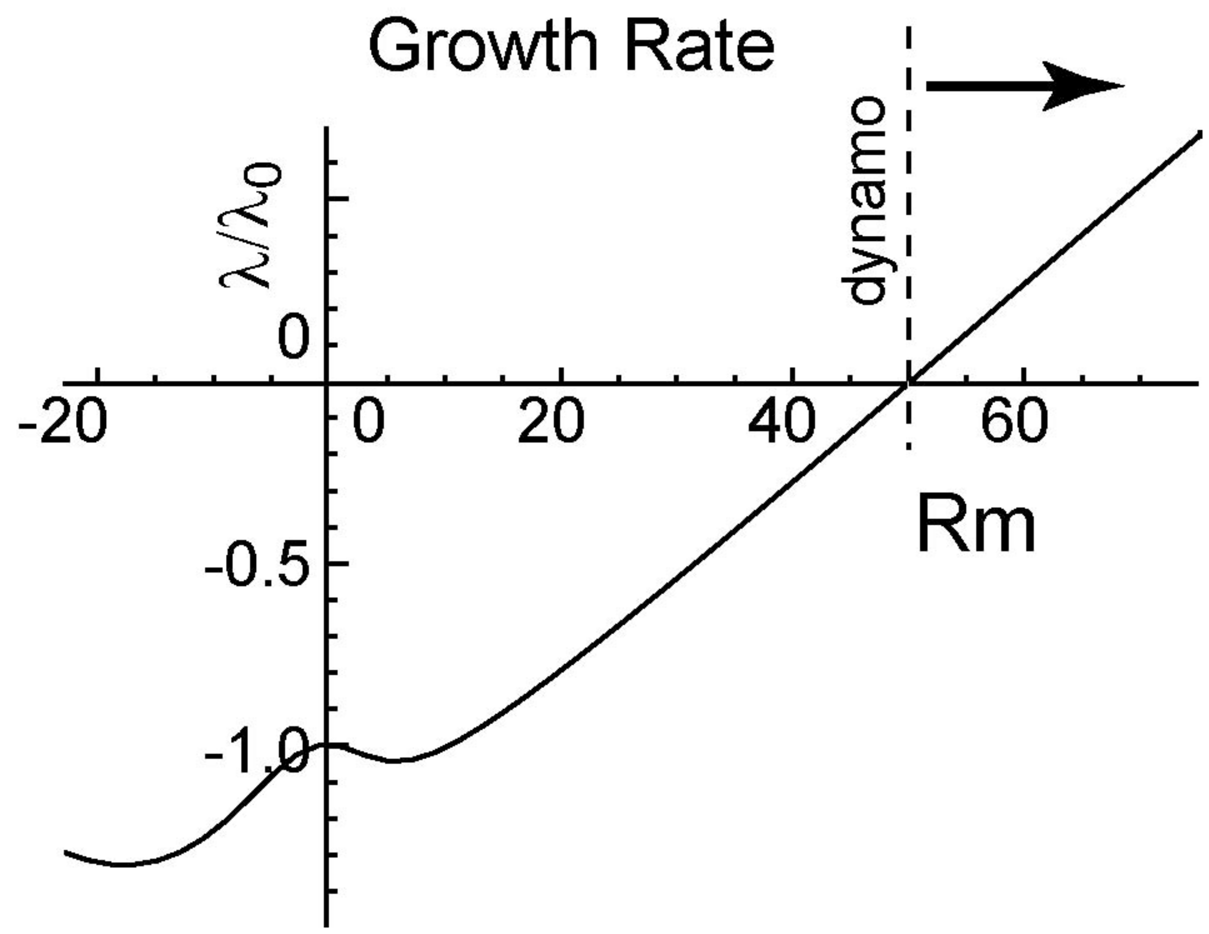
$$\boldsymbol{B} = \langle \boldsymbol{B} \rangle + \widetilde{\boldsymbol{b}}, \quad \boldsymbol{V} = \langle \boldsymbol{V} \rangle + \widetilde{\boldsymbol{v}}$$

$$\langle \boldsymbol{J} \rangle = \sigma \left(\langle \boldsymbol{E} \rangle + \langle \boldsymbol{V} \rangle \times \langle \boldsymbol{B} \rangle + \left\langle \widetilde{\boldsymbol{v}} \times \widetilde{\boldsymbol{b}} \right\rangle \right)$$

$$\begin{aligned} \mathcal{E} &= \left\langle \tilde{\mathbf{v}} \times \tilde{\mathbf{b}} \right\rangle = \alpha \mathbf{B} + \beta \nabla \times \mathbf{B} \text{ with} \\ \alpha &= \frac{1}{3} \int \tilde{\mathbf{v}}(t) \cdot \nabla \times \tilde{\mathbf{v}}(t+\tau) d\tau \text{ and } \beta = \frac{1}{3} \int \tilde{\mathbf{v}}(t) \cdot \tilde{\mathbf{v}}(t+\tau) d\tau \end{aligned}$$

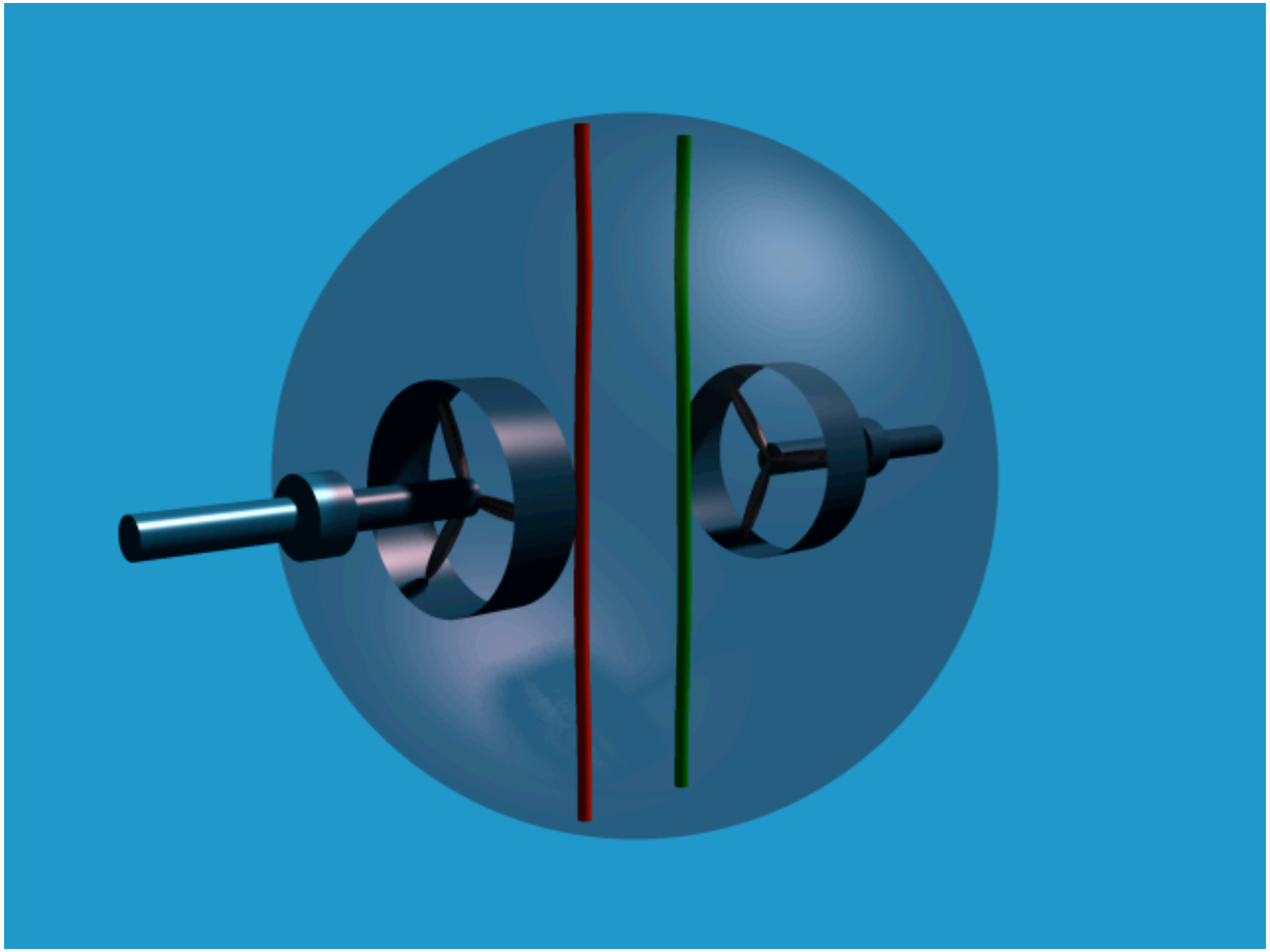
This simplest possible self-exciting flow: a two vortex flow with Rm_{crit}~50



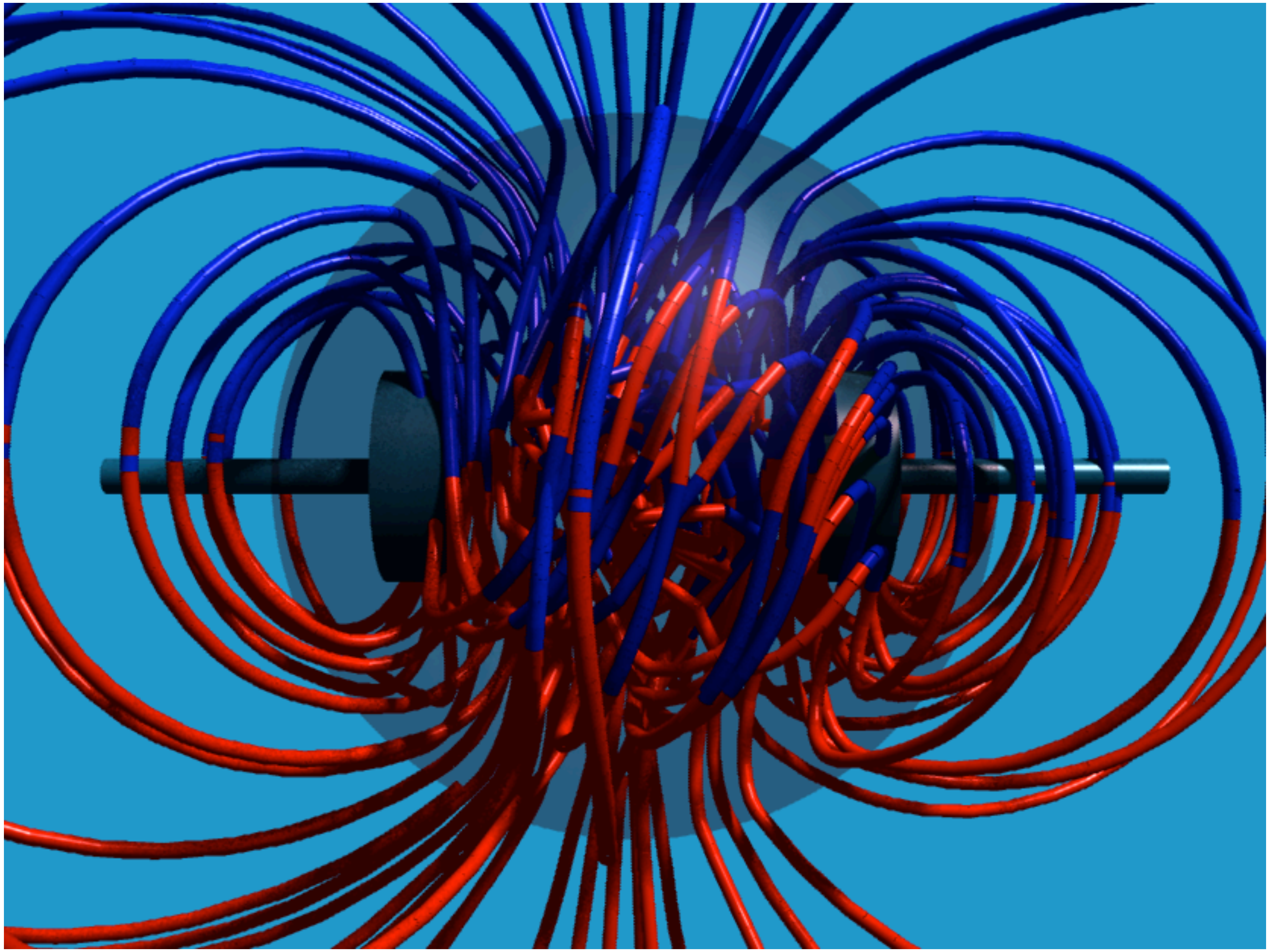


Dudley and James, *Time-dependent kinematic dynamos with stationary flows*, Proc. Roy. Soc. Lond. A. **425** 407 (1989).

Dynamo is of the stretch-twist-fold type: field line stretching and reinforcement leads to dynamo



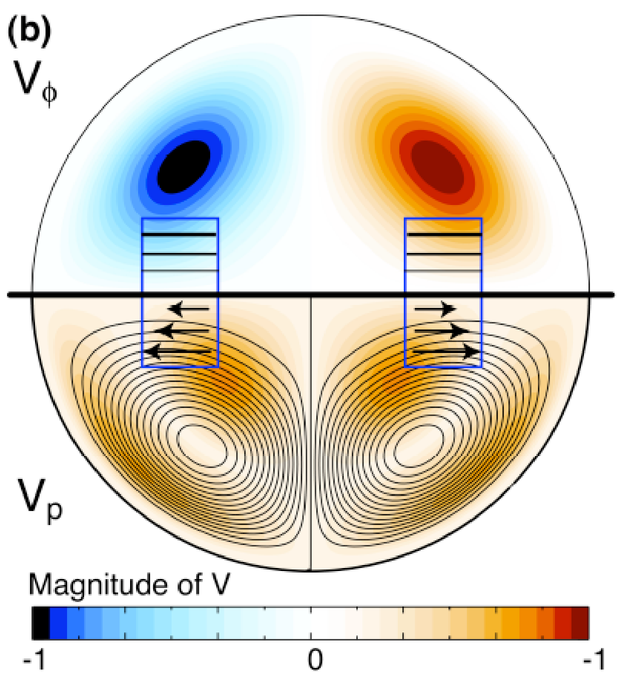
The saturated magnetic eigenmode (from a full 3D, non-linear MHD computation) is an equatorial diplole



Bayliss, Nornberg, Terry and Forest, *Numerical simulations of current generation and dynamo excitation in a mechanically-forced, turbulent flow, Phys. Rev. E,* (2006)

For liquid metals, Re>>Rm

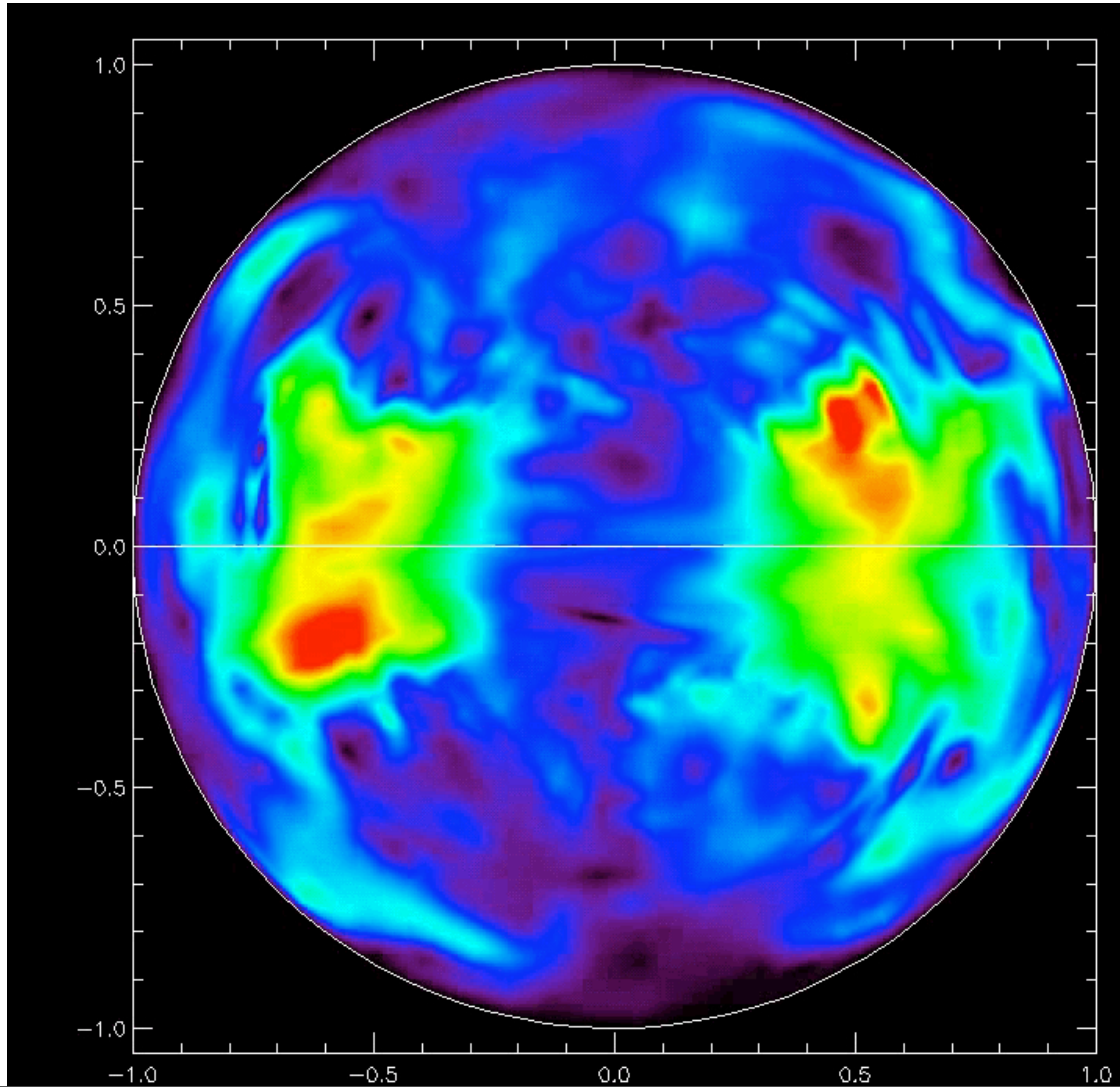
- Direct Numerical Simulations of MHD equations with mechanical forcing
- Re=2200; turbulence for Re>450



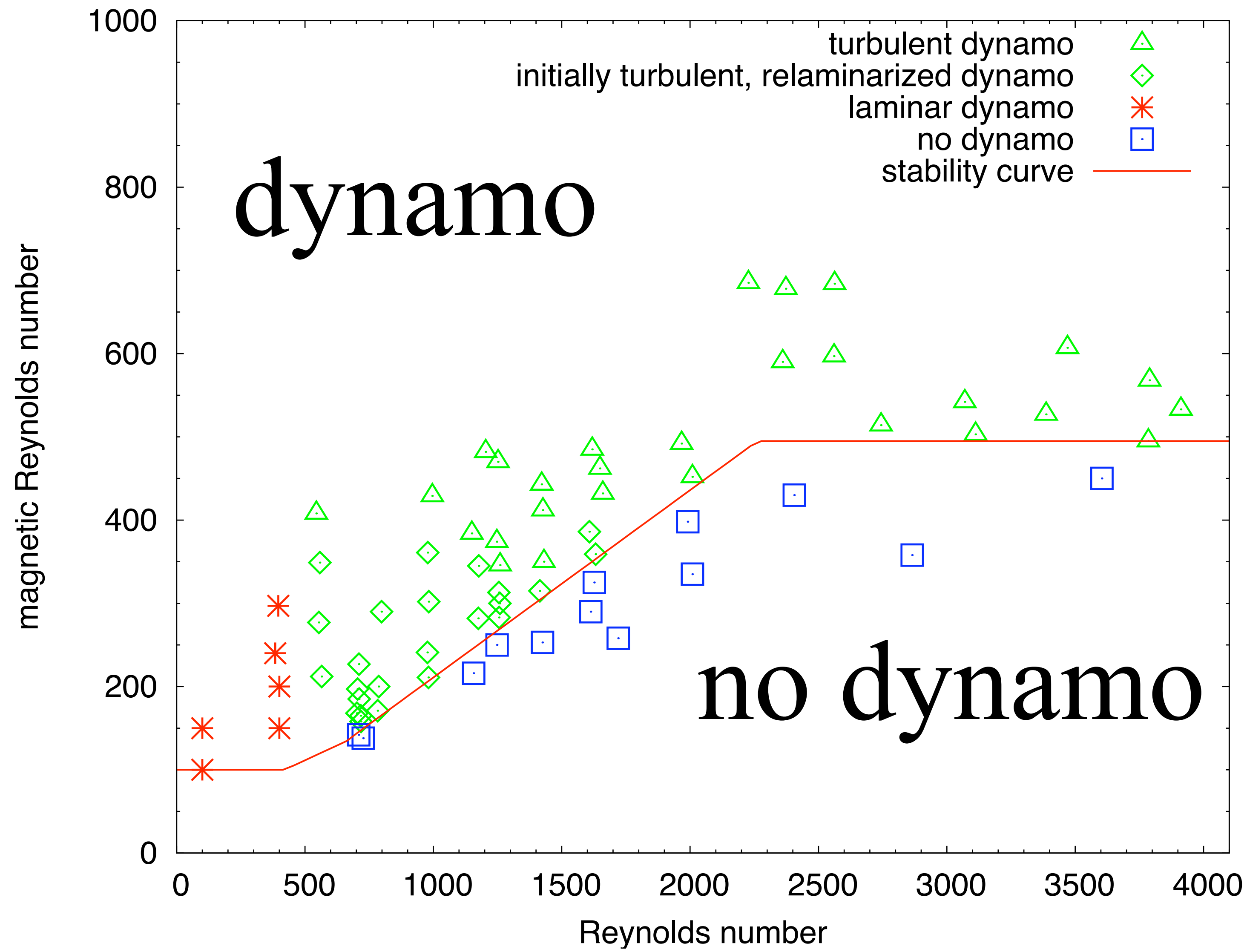
$$F_\phi(\rho, z) = \rho^2 \sin(\pi \rho b)$$

$$F_z(\rho, z) = -\epsilon \sin(\pi \rho c)$$

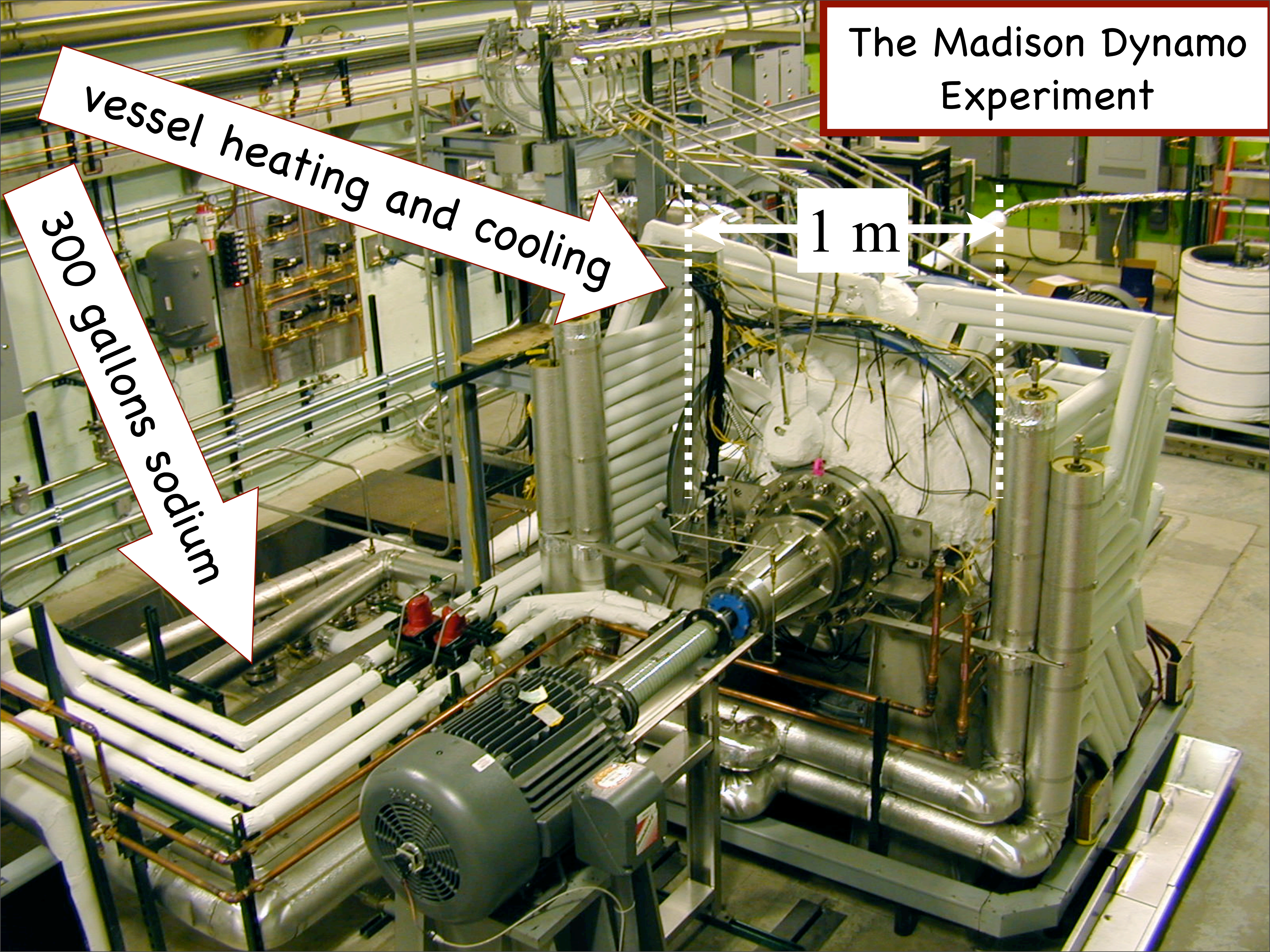
$$0.25a < |z| < 0.55a, \; \rho < 0.3a$$



Turbulence, in the two-vortex dynamo, increases Rm_{crit} by factor of 5



- Recent, fully resolved MHD simulations (no hyperviscosity, no LES) extended to Re~5000
- proper boundary conditions and mechanical forcing term

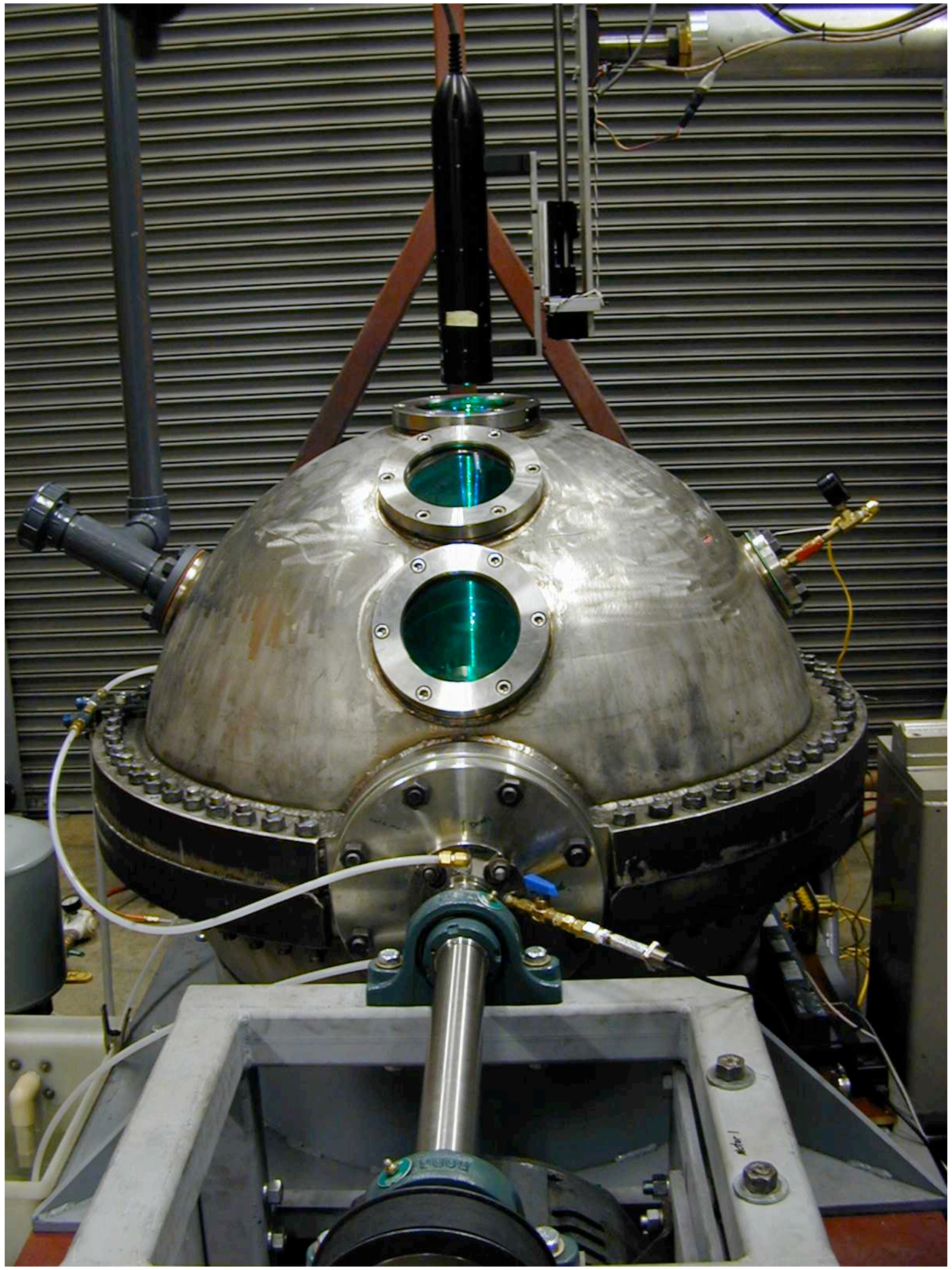

The Madison Dynamo Experiment
vessel heating and cooling
300 gallons sodium
1 m

Dimensionally identical water experiment was used to demonstrate feasibility

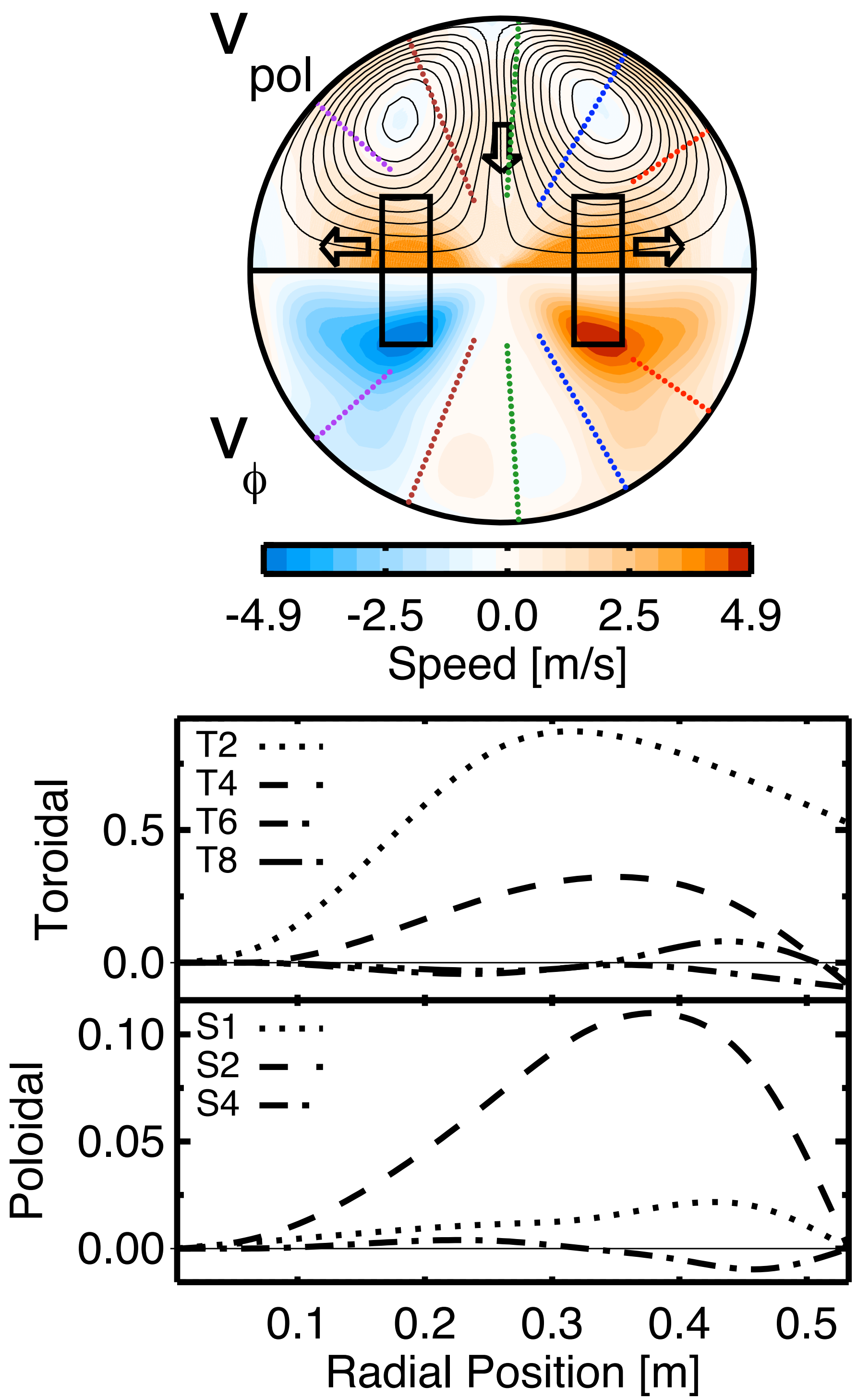
- Laser Doppler velocimetry is used to measure vector velocity field
- Measured flows are used as input to MHD calculation
- Full scale, half power

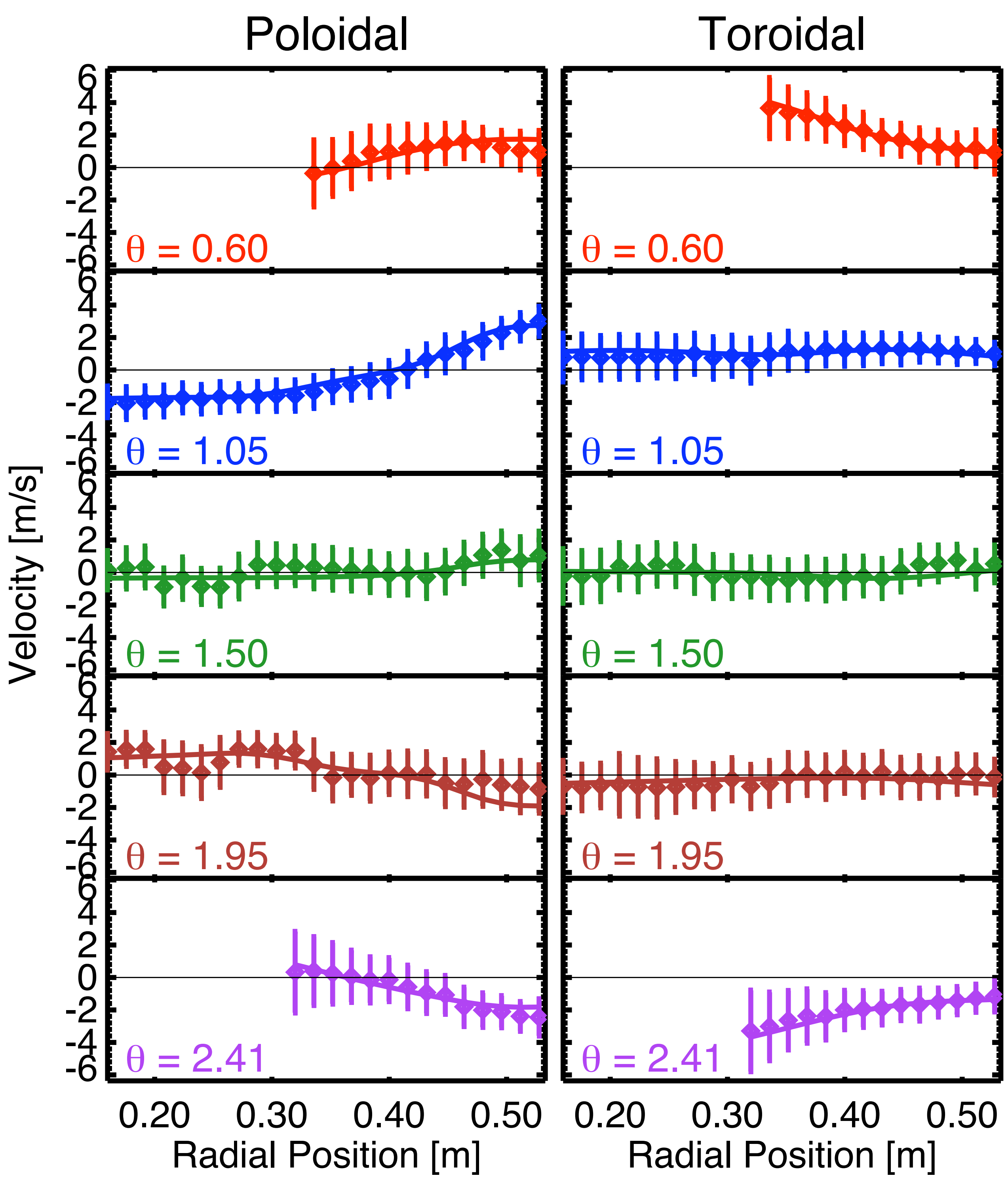
	Sodium	Water
Temperature	$110^\circ C$	$50^\circ C$
viscosity	$0.65\times10^{-6}\ \mathrm{m}^2\,\mathrm{sec}^{-1}$	$0.65\times10^{-6}\ \mathrm{m}^2\,\mathrm{sec}^{-1}$
mass density	$0.925\ \mathrm{gm\ cm}^{-3}$	$0.988\ \mathrm{gm\ cm}^{-3}$
resistivity	$10^{-7}\ \Omega\mathrm{m}$	

$$\longrightarrow \quad Rm = \frac{\mu_0 a V}{\eta} = 4\pi\, a(m)\, V(m/s)$$

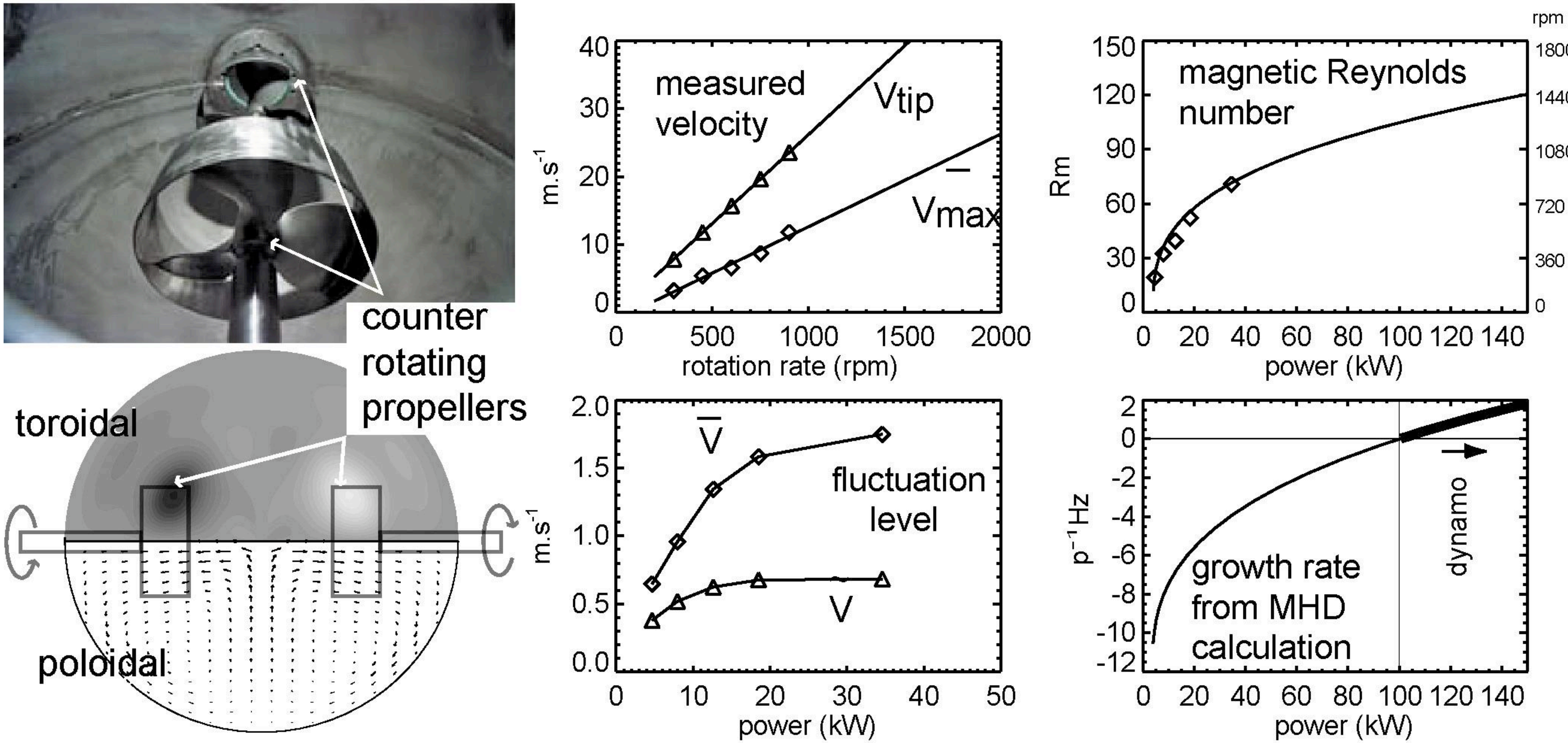


LDV measurements provide data for a reconstruction of the mean velocity field





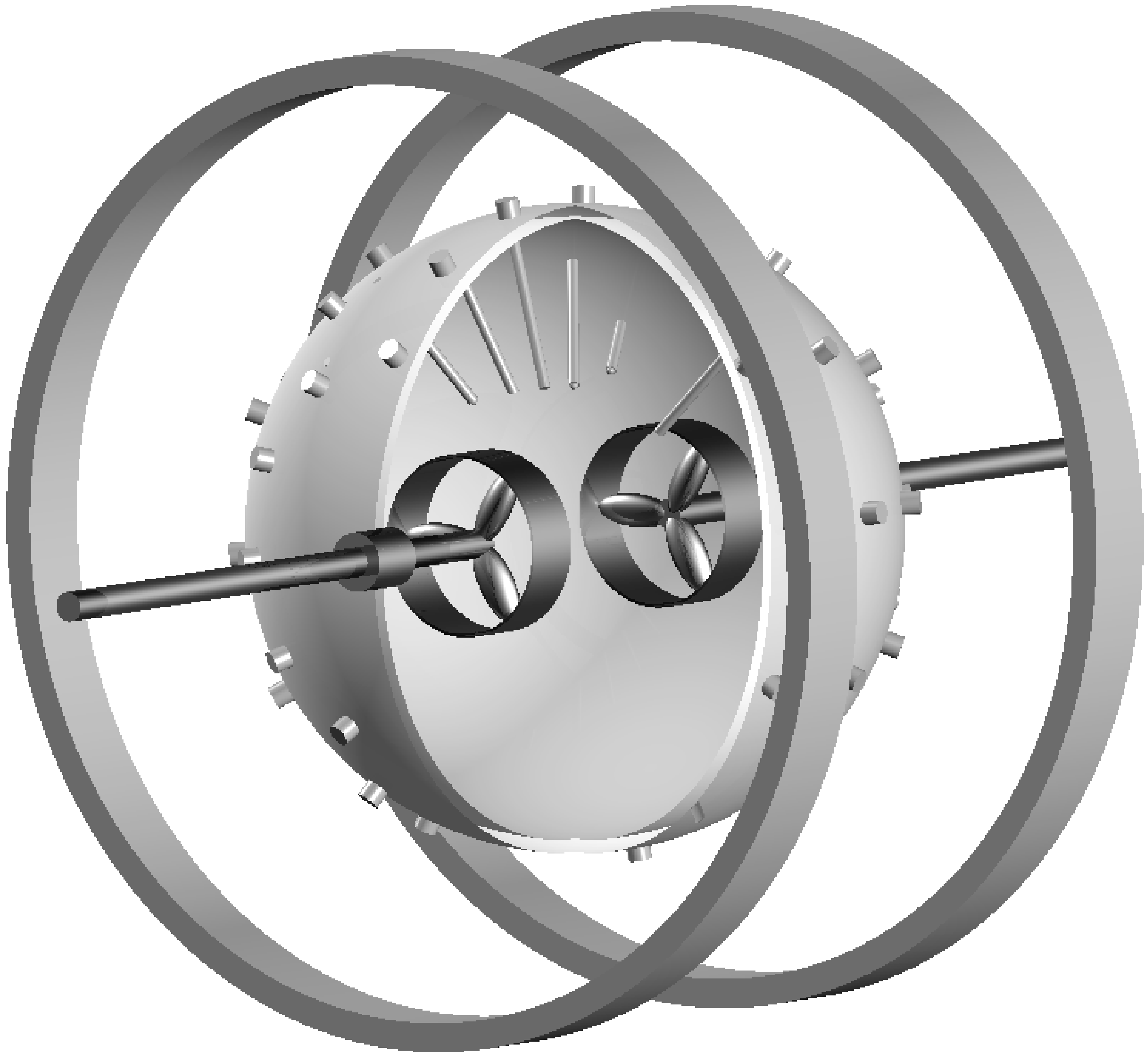
Velocity fields can be generated in water which lead to dynamo action



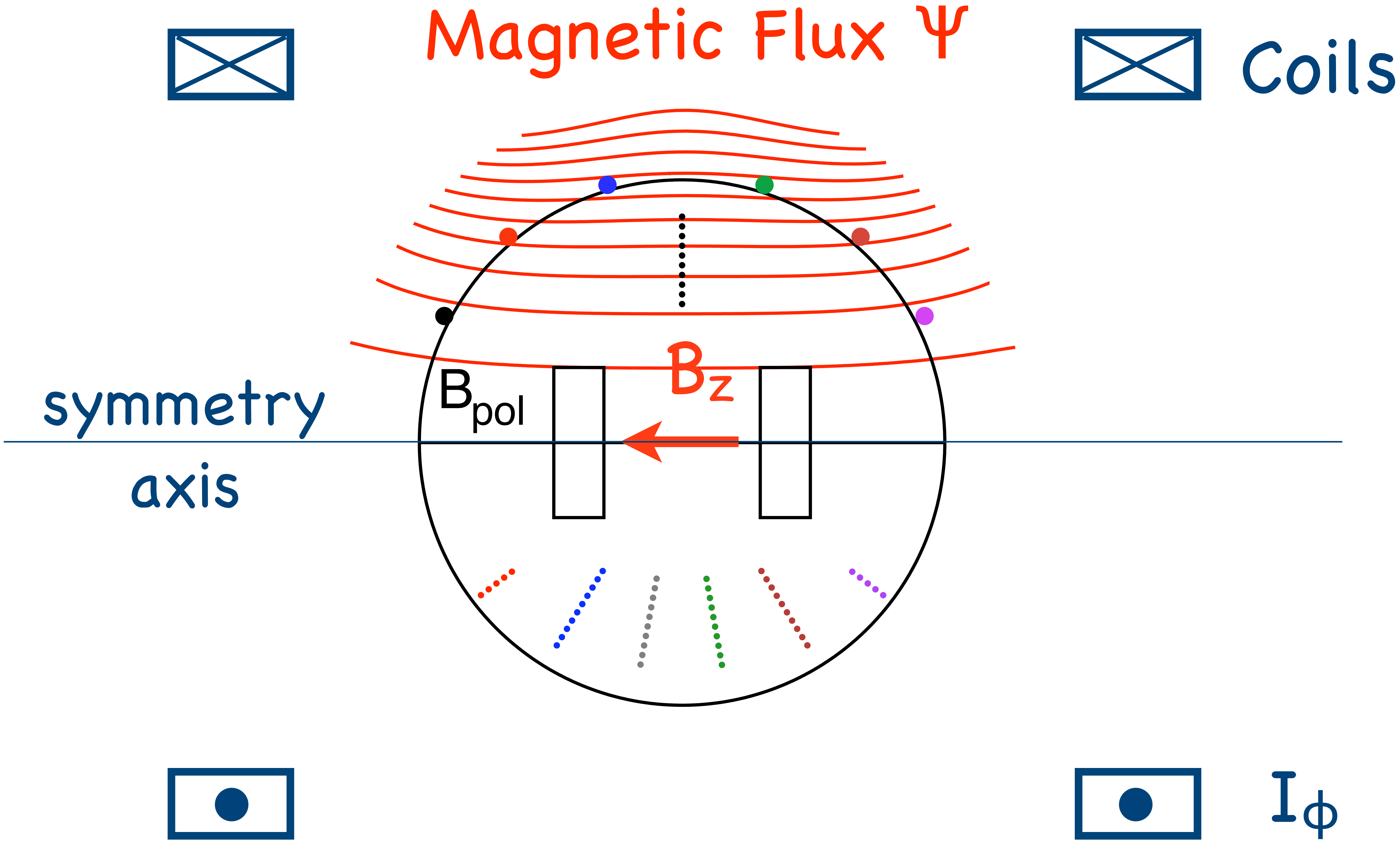
$a=0.5$ m, $\sigma=10^7$ mhos

Magnetic field is measured both internally and externally; external magnetic fields can be applied to probe experiment

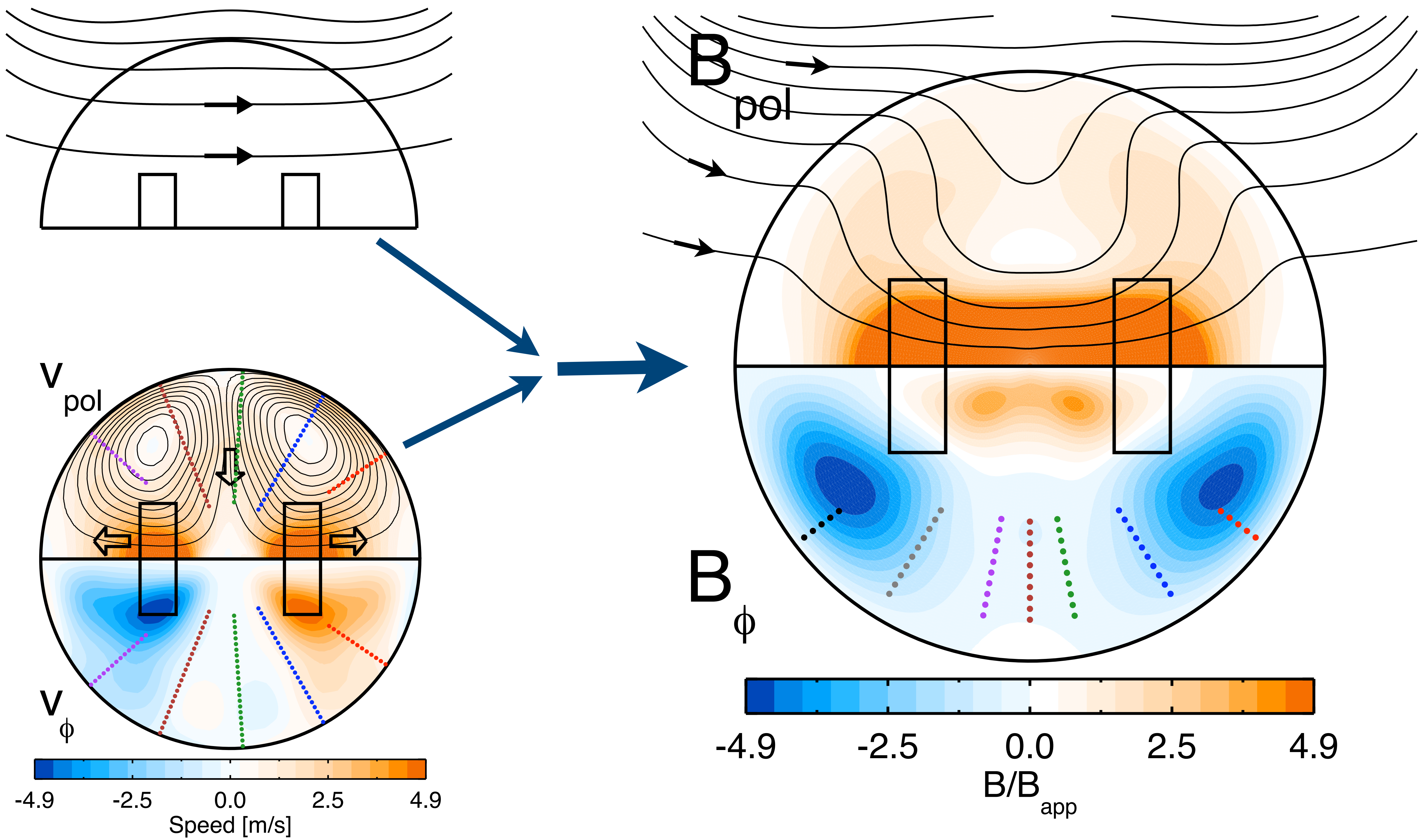
- $B_z \leq 100$ gauss
- Measure
  - surface probes
    - $B_r(a,\theta,\phi)$
    - Y_{lm} for $l \leq 6$, $|m| \leq 4$
  - Internal Probes
    - $B_\phi(r,\theta_p)$, 6 arrays
    - $B_z(r,\theta=\pi/2)$



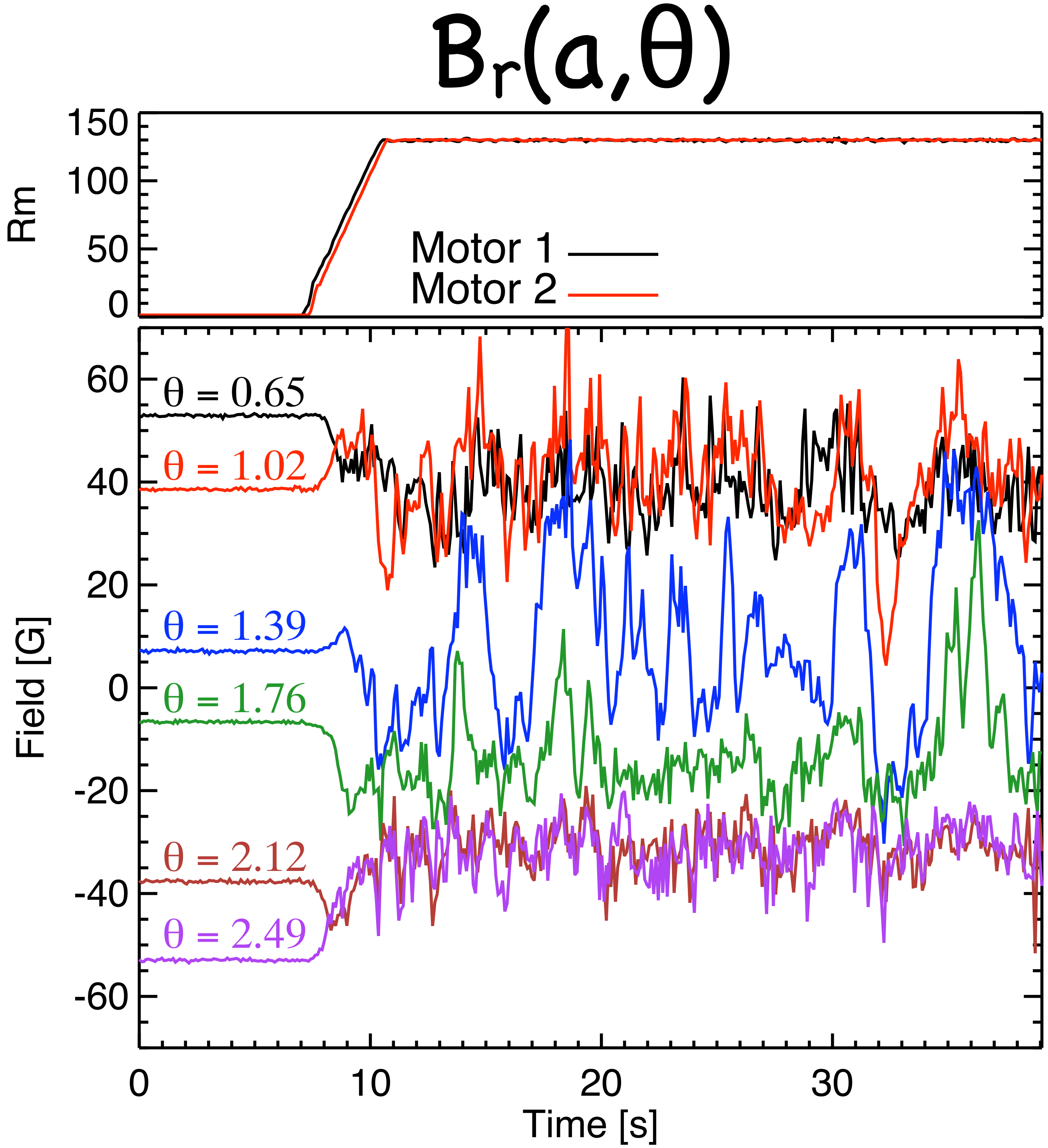
Experiment: apply axisymmetric poloidal seed field to sphere and measure induced magnetic fields

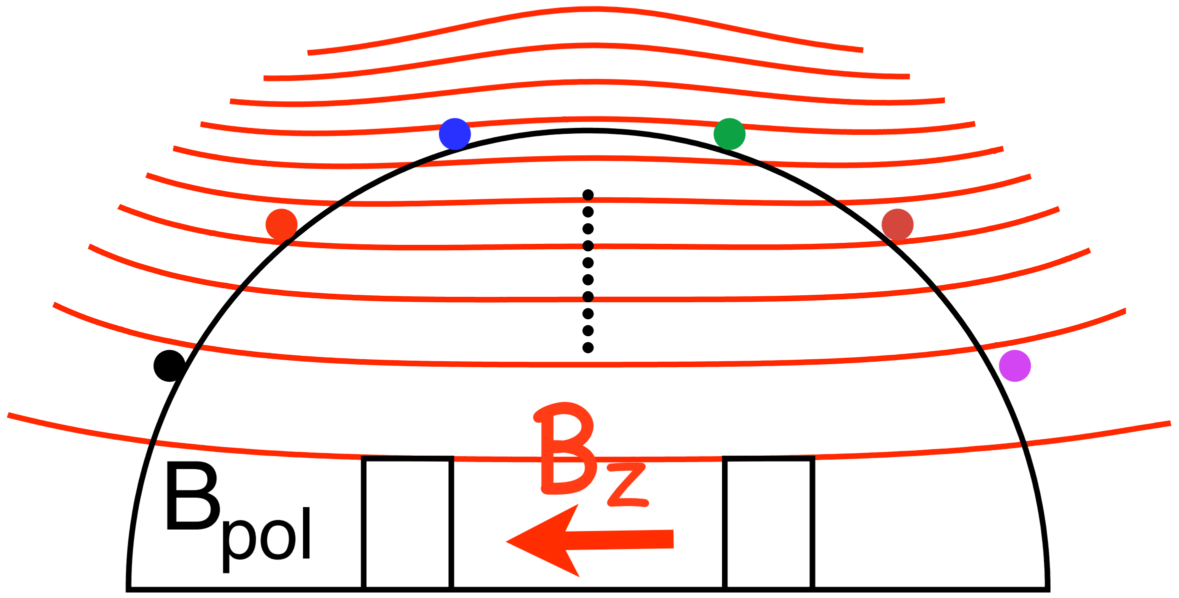


Predicted total magnetic fields

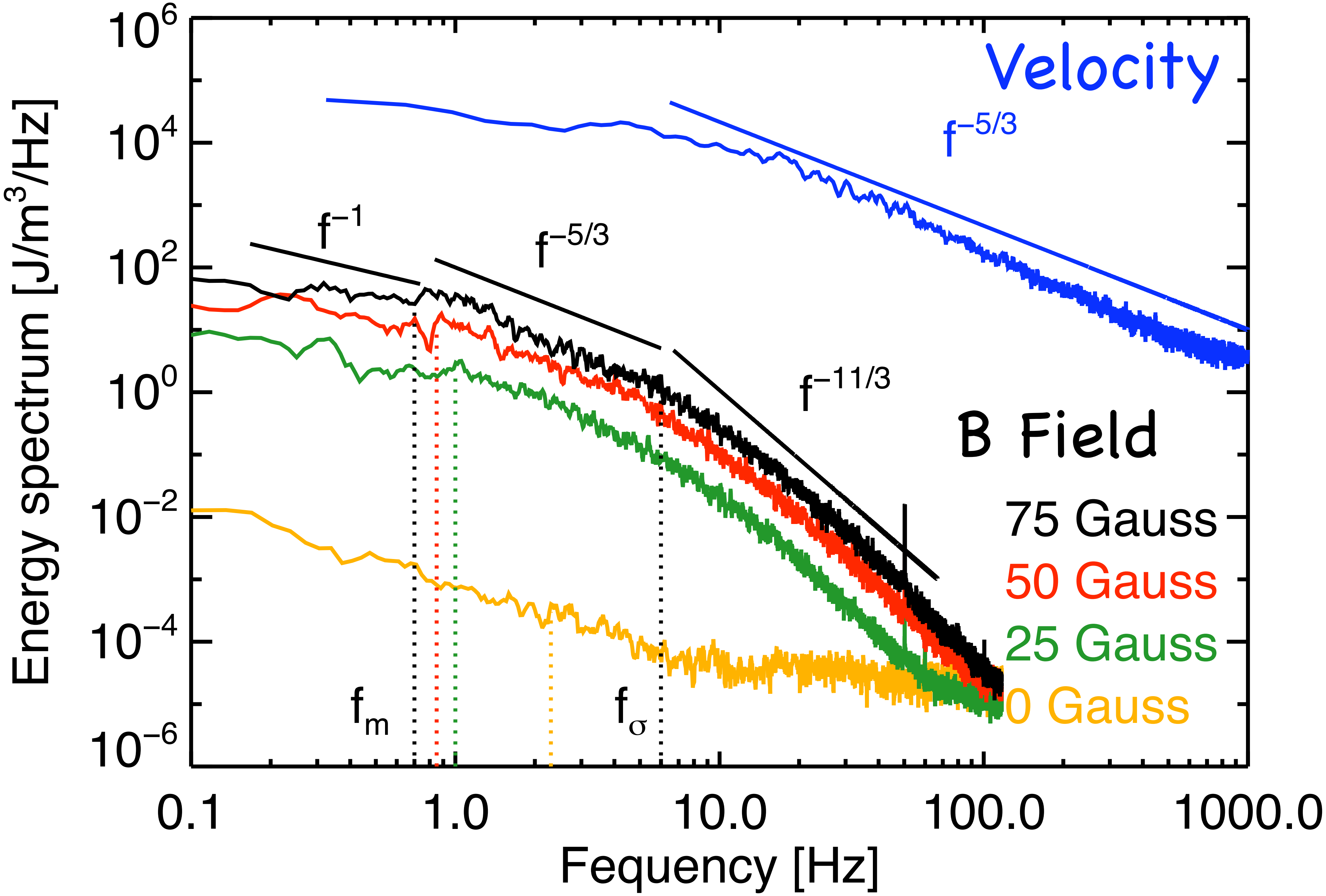


Large scale (mean) and small scale (turbulent) magnetic fields are generated by liquid sodium flows





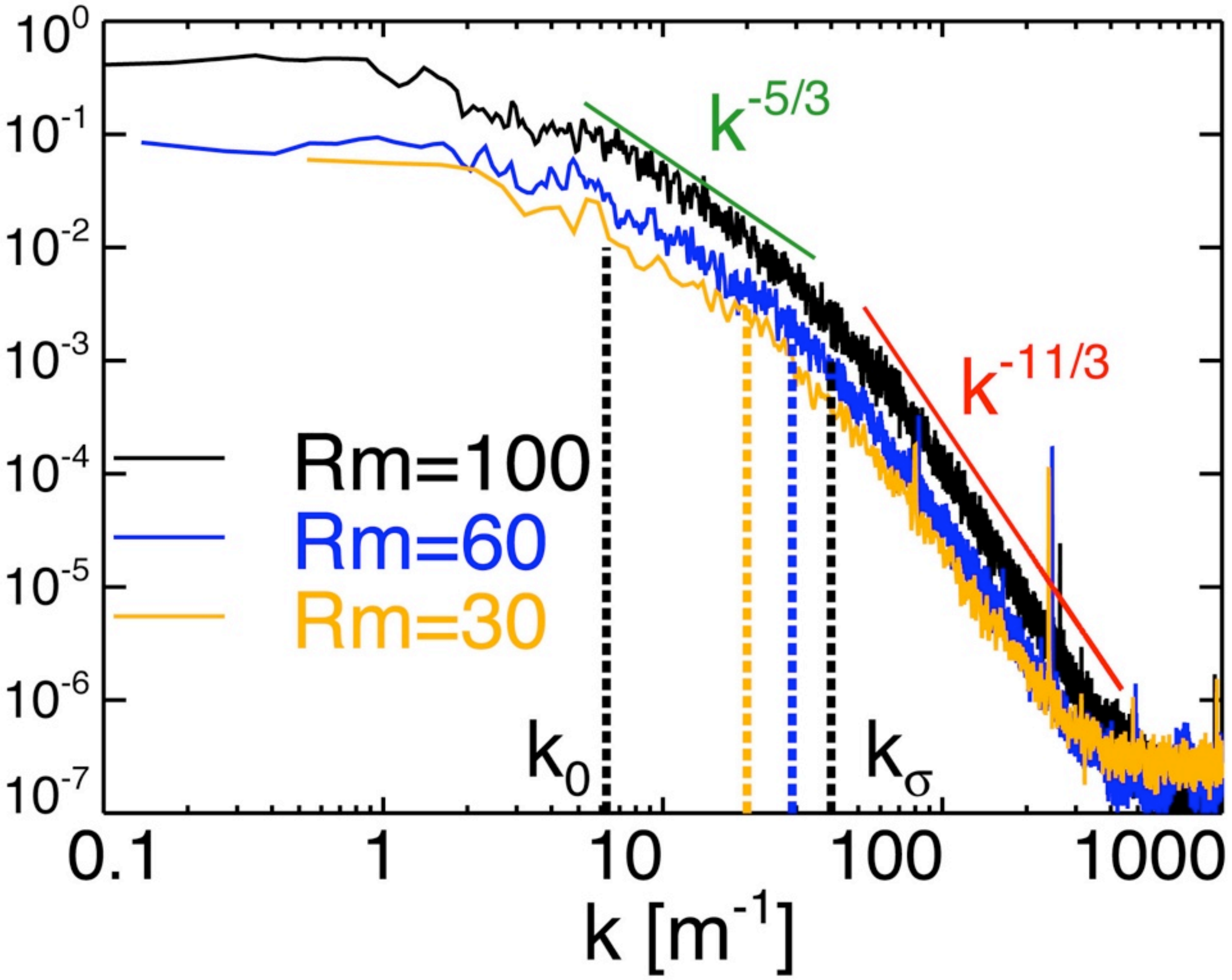
Spectra are turbulent: the turublent magnetic energy is much smaller than the kinetic energy



Nornberg, Spence, Bayliss, Kendrick, and Forest, *Measurements of the magnetic field induced by a turbulent flow of liquid metal,* Phys. Plasmas **13** 055901 (2006).

Internal magnetic fluctuations are consistent with passive advection of B by Kolmogorov turbulence

Wavenumber Spectrum

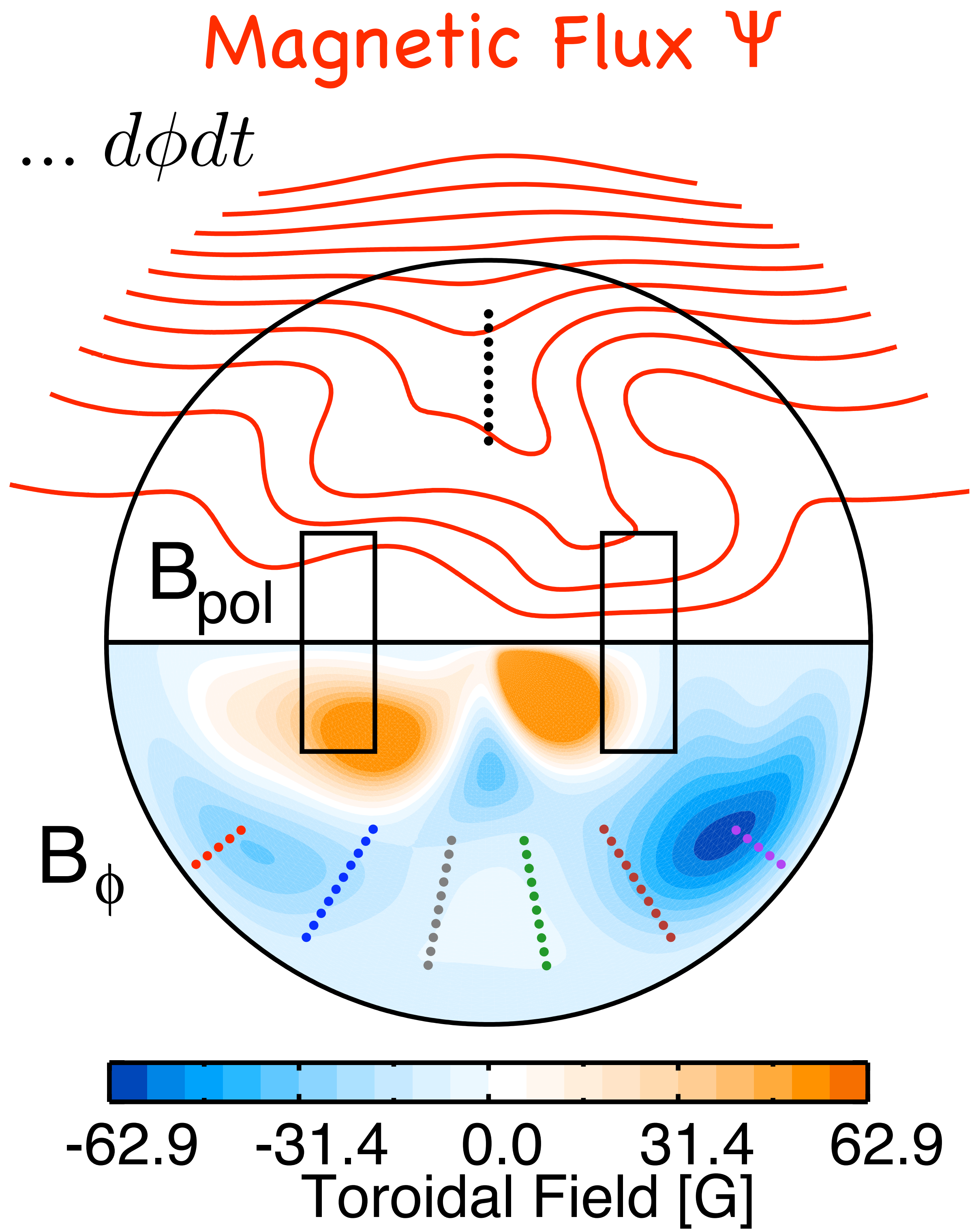


- Limited inertial range exists
- Inertial range scales with Rm: $k_\sigma \propto Rm\, k_0$

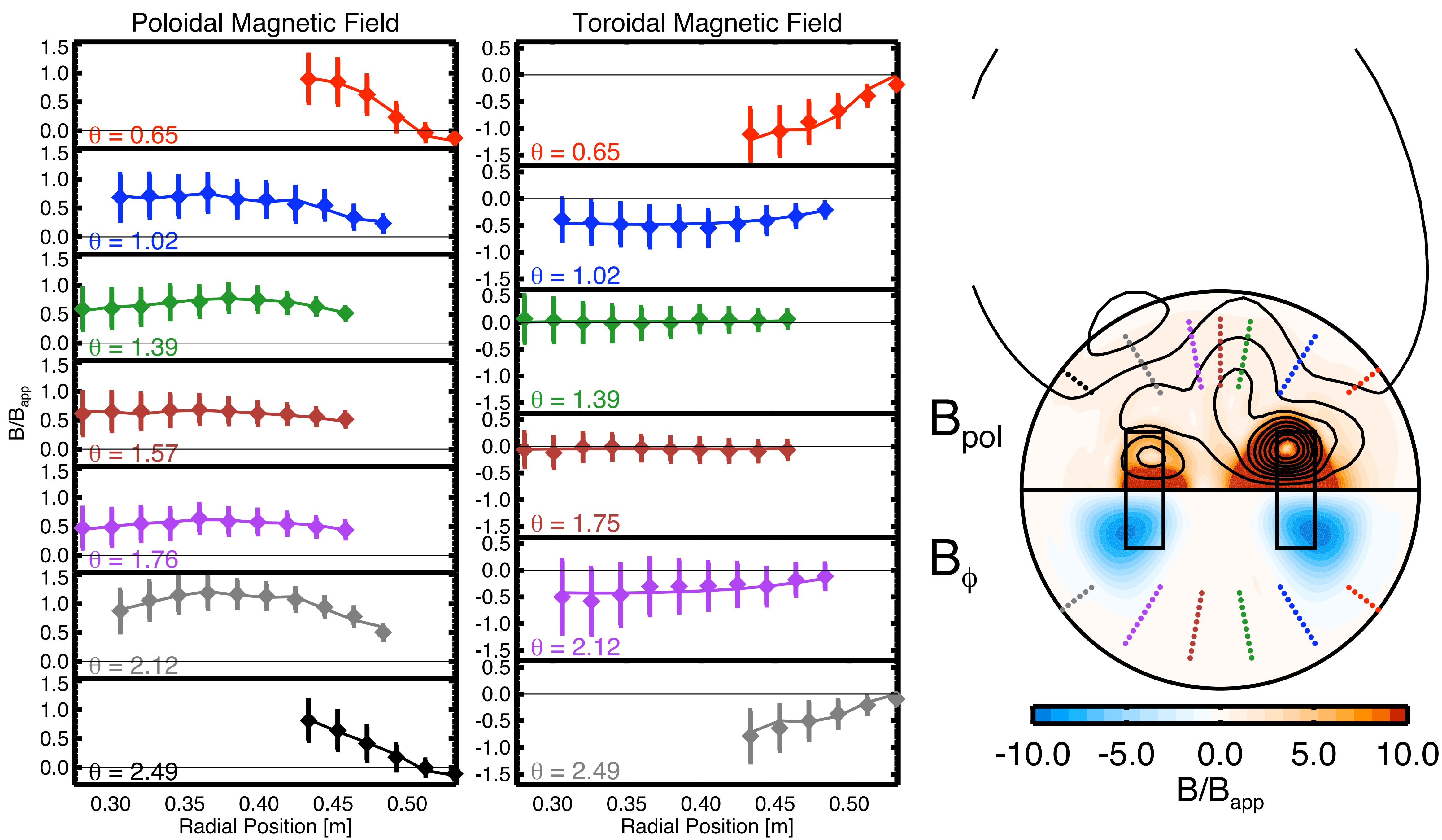
Nornberg, Spence, Bayliss, Kendrick, and Forest, *Measurements of the magnetic field induced by a turbulent flow of liquid metal,* Phys. Plasmas **13** 055901 (2006).

The time-averaged, axisymmetric part of the magnetic field shows poloidal flux expulsion and a strong Ω effect

$$\langle \ldots \rangle \equiv \frac{1}{2\pi T} \int^{T} \int_0^{2\pi} \ldots \, d\phi dt$$

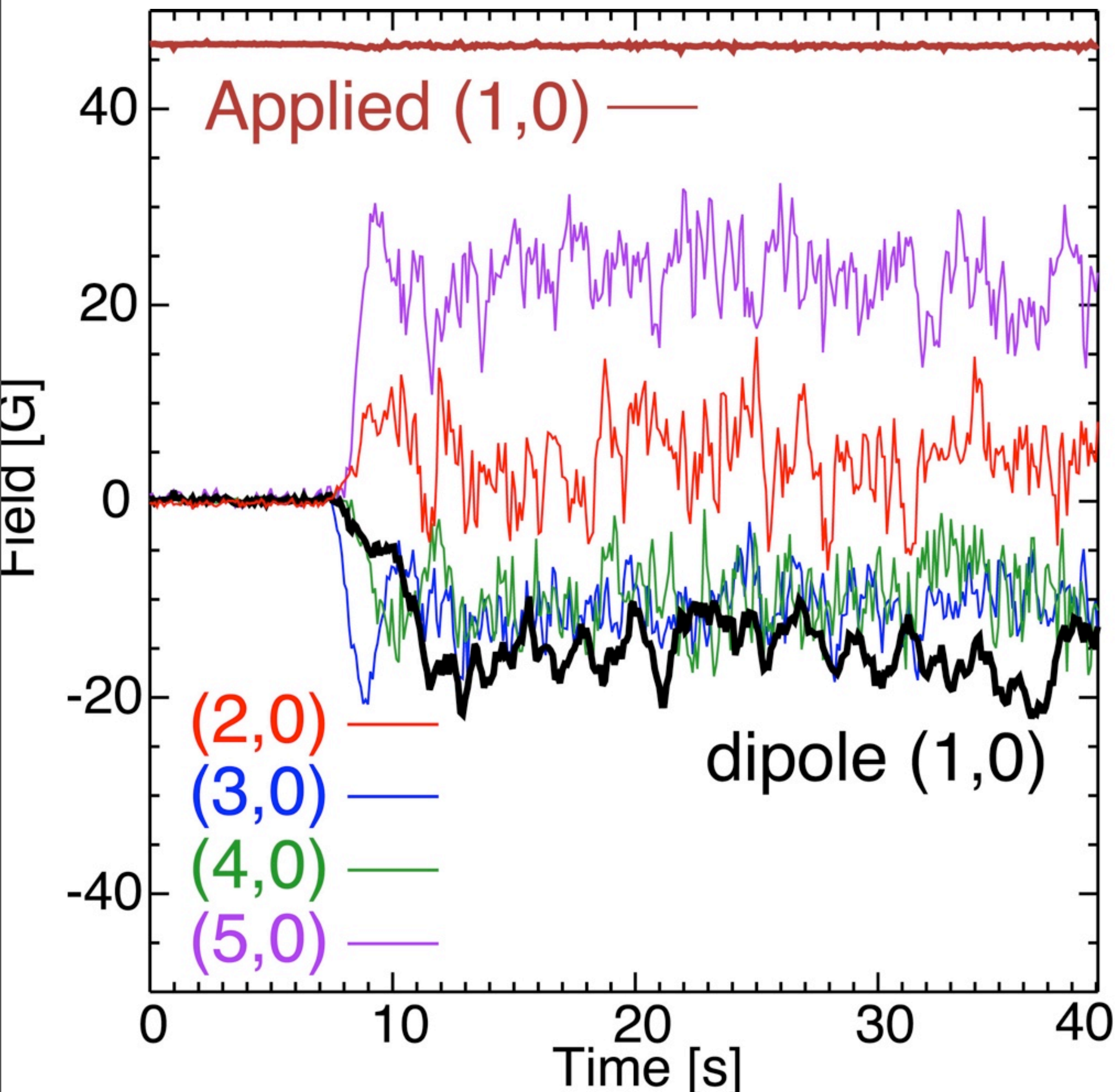


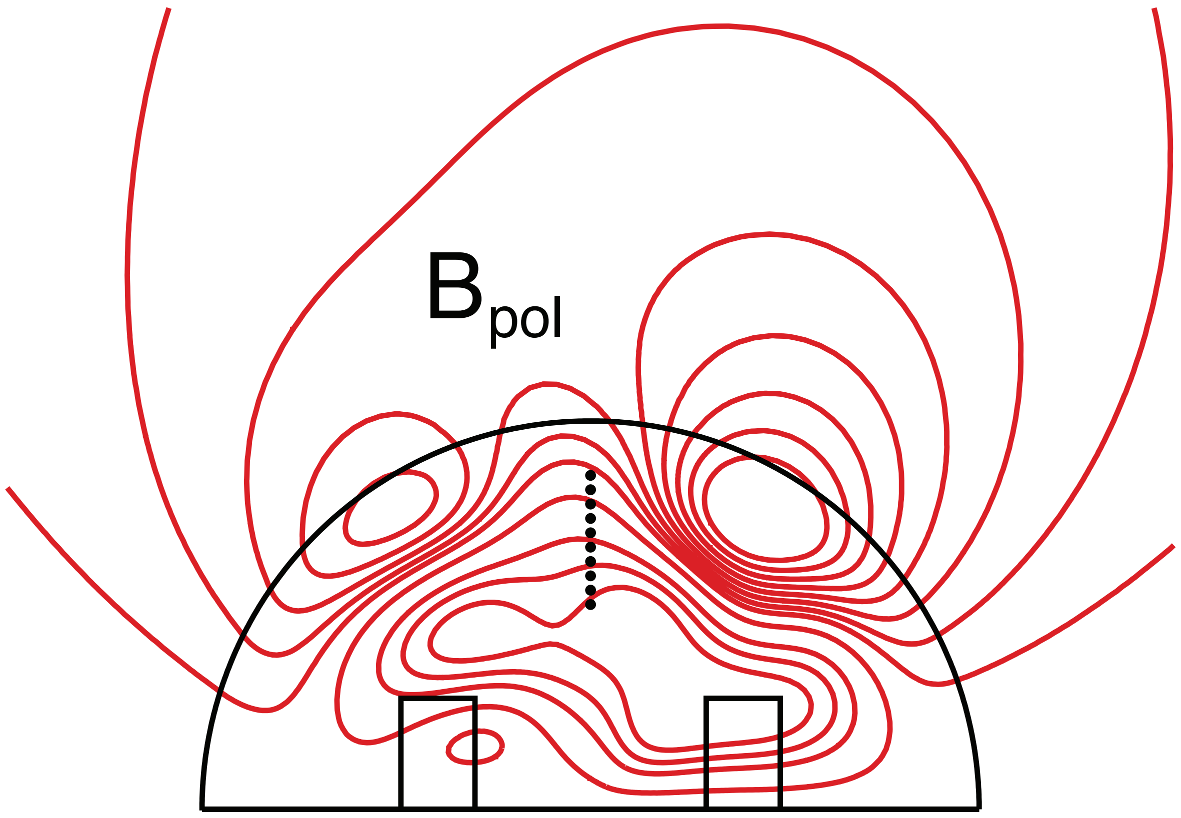
Magnetic field is reconstructed from magnetic field measurements at discrete positions



The mean induced magnetic field has a dipole moment

components of Y_{lm}





Impossible to reconstruct with axisymmetric flows!

Spence, Nornberg, Jacobson, Kendrick, and Forest, *Observation of a turbulence-induced large-scale magnetic field*, Phys. Rev. Lett. **96** 055002 (2006).

Theorem: For a stationary, axisymmetric flow and magnetic field, no dipole moment can exist for the current distribution inside the experiment (even with externally applied fields)

Use cylindrical coordinates (s, Z, ϕ) and stream functions for velocity and magnetic fields:

$$\vec{v} = \nabla\Phi \times \nabla\phi + v_\phi \hat{\phi} \quad (1)$$

$$\vec{B} = \nabla\Psi \times \nabla\phi + B_\phi \hat{\phi} \quad (2)$$

The dipole moment $\mu_z = \int s J_\phi d^3x$ is generated by toroidal currents:

$$J_\phi = \sigma \vec{v} \times \vec{B} \cdot \hat{\phi} \quad (3)$$

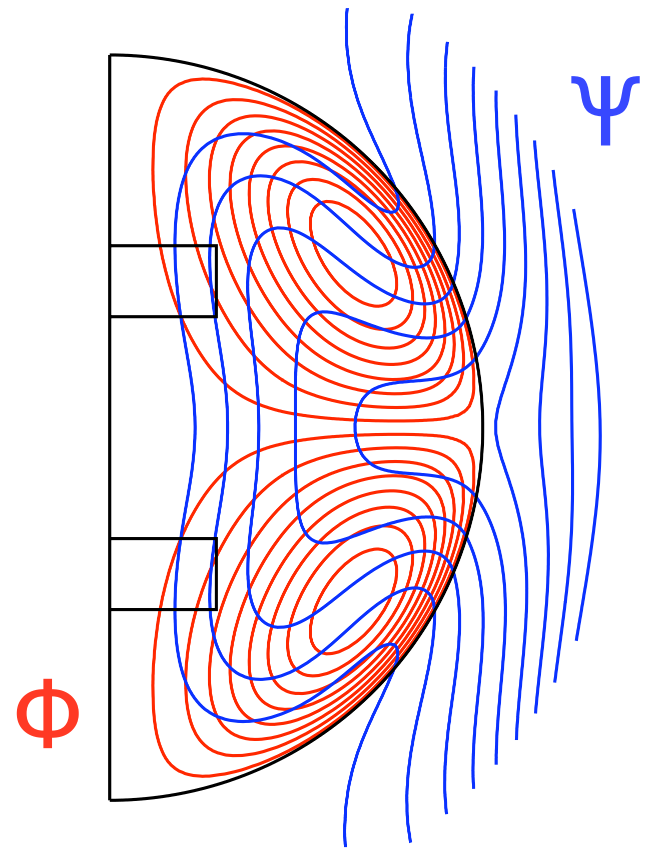
$$= \sigma \frac{|\nabla\Phi \times \nabla\Psi|}{s^2} \quad (4)$$

Switching to flux coordinates (Ψ, ℓ) where $d^3x = \frac{d\ell d\Psi}{B_p}$, the dipole becomes

$$\mu_z = \sigma \int \int |\nabla\Phi \times \nabla\Psi| \frac{d\ell d\Psi}{s B_p} \quad (5)$$

$$= \sigma \int d\Psi \int \frac{\partial \Phi}{\partial \ell} d\ell \equiv 0 \quad (6)$$

Proof continued



Integrating Φ along open poloidal flux contours gives

$$\int_a^b \frac{\partial \Phi}{\partial \ell} d\ell = \Phi(b) - \Phi(a) = 0$$

since vessel boundary had $\Phi = const.$ Closed poloidal flux contours give

$$\oint \frac{\partial \Phi}{\partial \ell} d\ell \equiv 0$$

Therefore, $\mu_z = 0$ for axisymmetric flows. QED

- Conclusion: symmetry breaking fluctuations must be responsible for observed dipole
  - consistent with an α-effect and the self-generated toroidal field: $J_\varphi = \sigma \alpha B_\varphi$

Question: Does a simple Ohm's law make sense?

Measured in Sodium Experiment

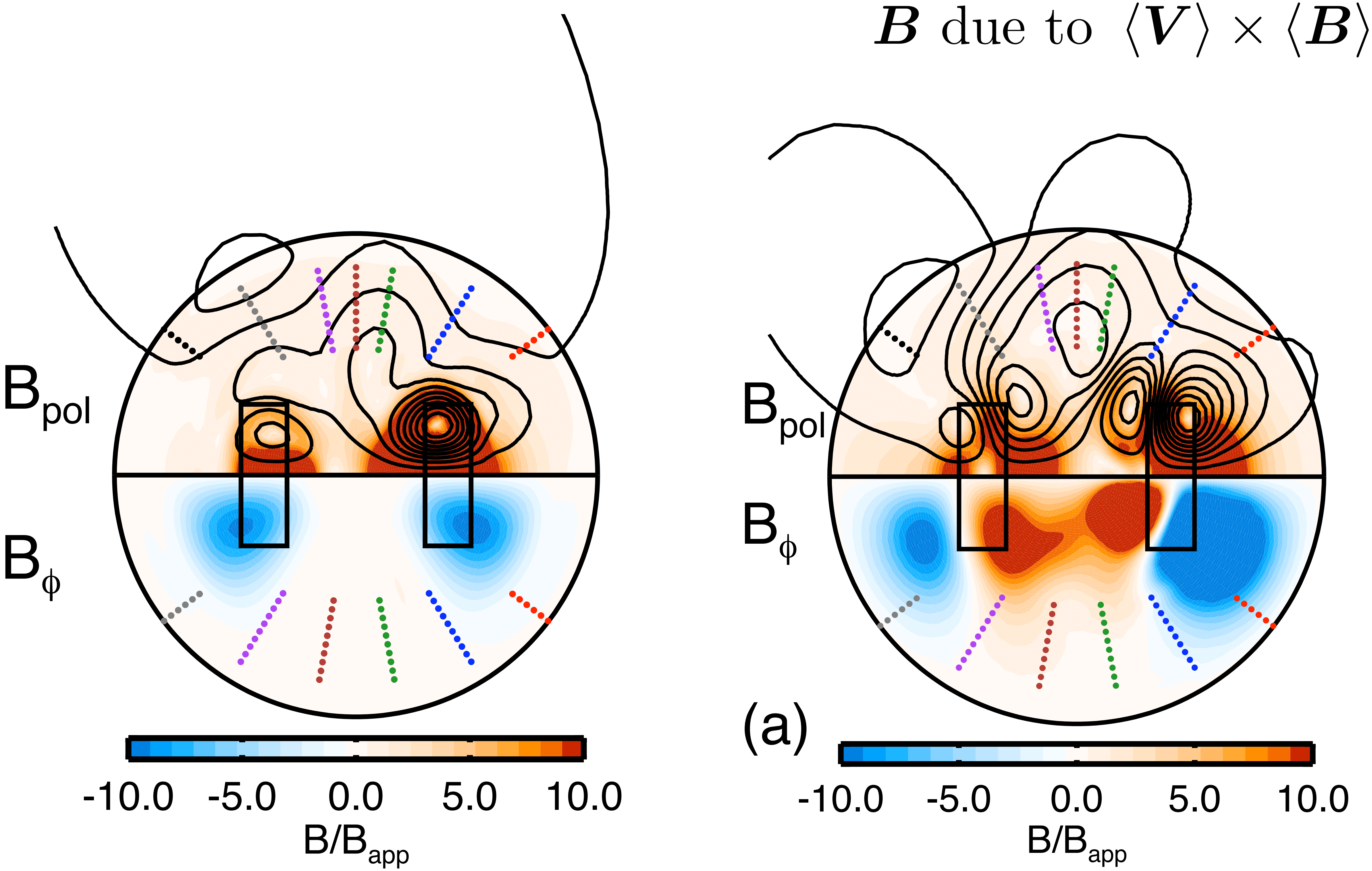
$$\langle \boldsymbol{J} \rangle = \sigma \left(\langle \boldsymbol{E} \rangle + \langle \boldsymbol{V} \rangle \times \langle \boldsymbol{B} \rangle + \left\langle \widetilde{\boldsymbol{v}} \times \widetilde{\boldsymbol{b}} \right\rangle \right)$$

Measured by LDV

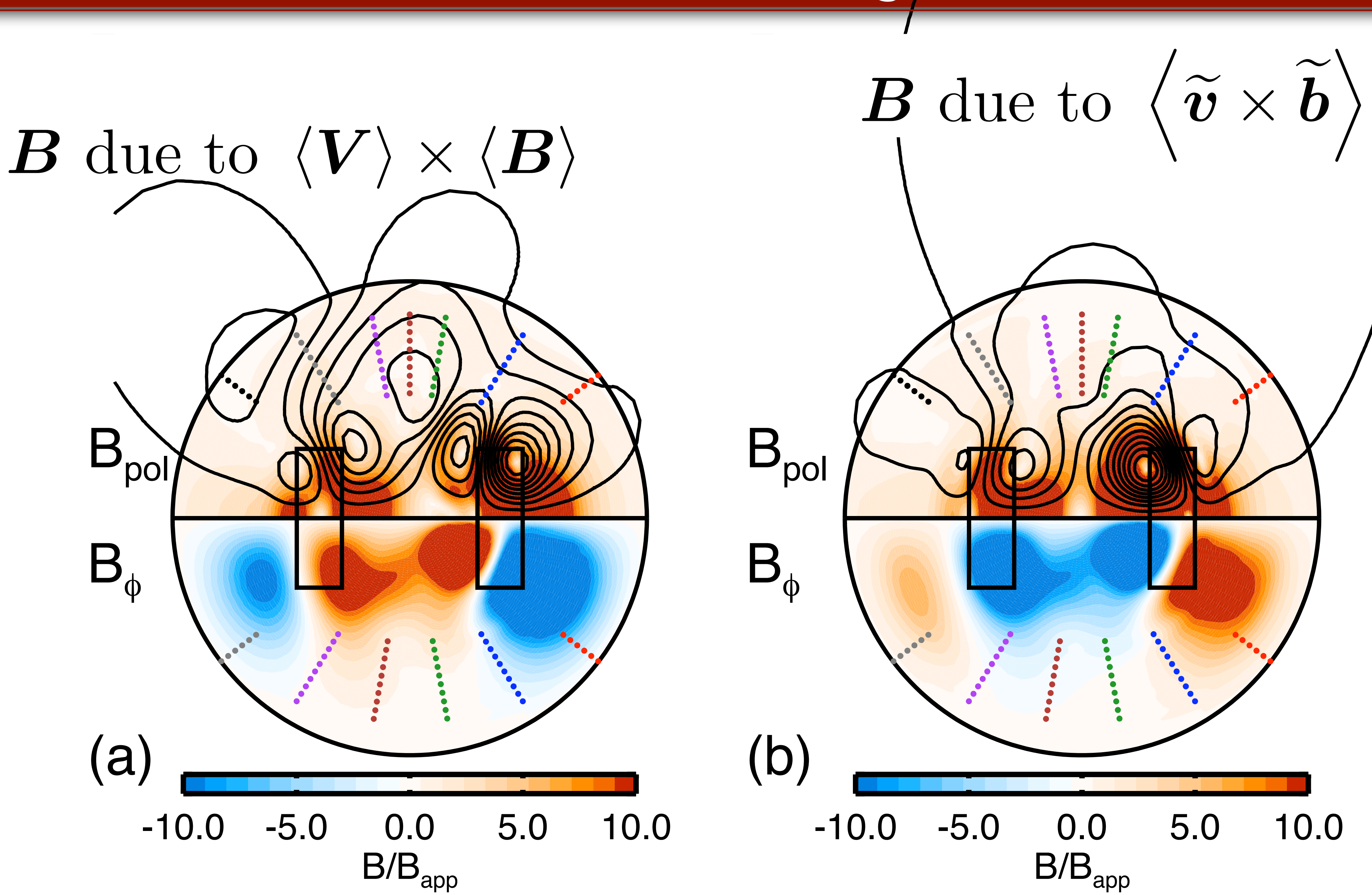
Fluctuation Driven Currents

$$\langle \ldots \rangle \equiv \frac{1}{2\pi T} \int^{T} \int_{0}^{2\pi} \ldots \, d\phi dt$$

<V>x<B> does not account for measured field: turbulence must be generating current

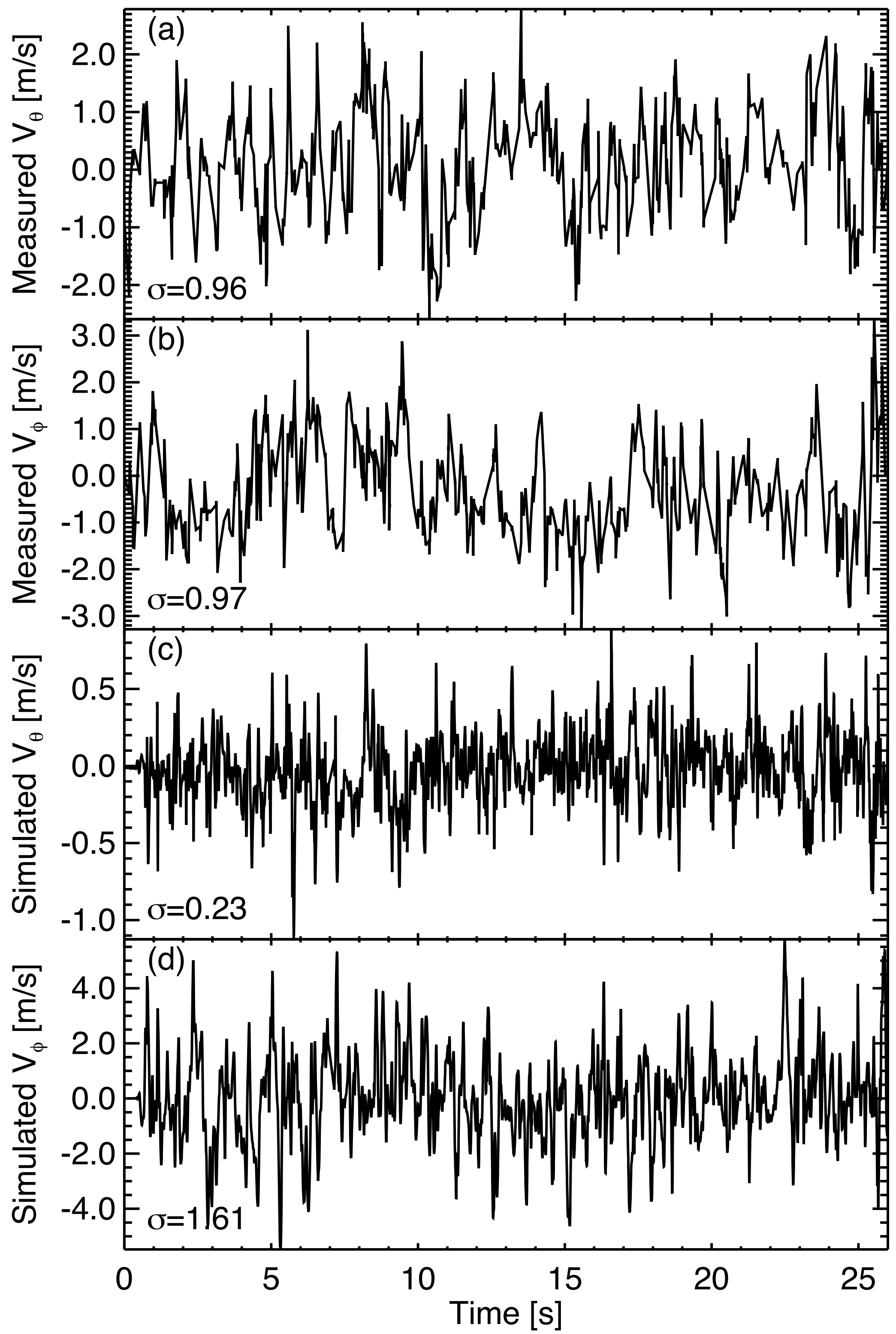


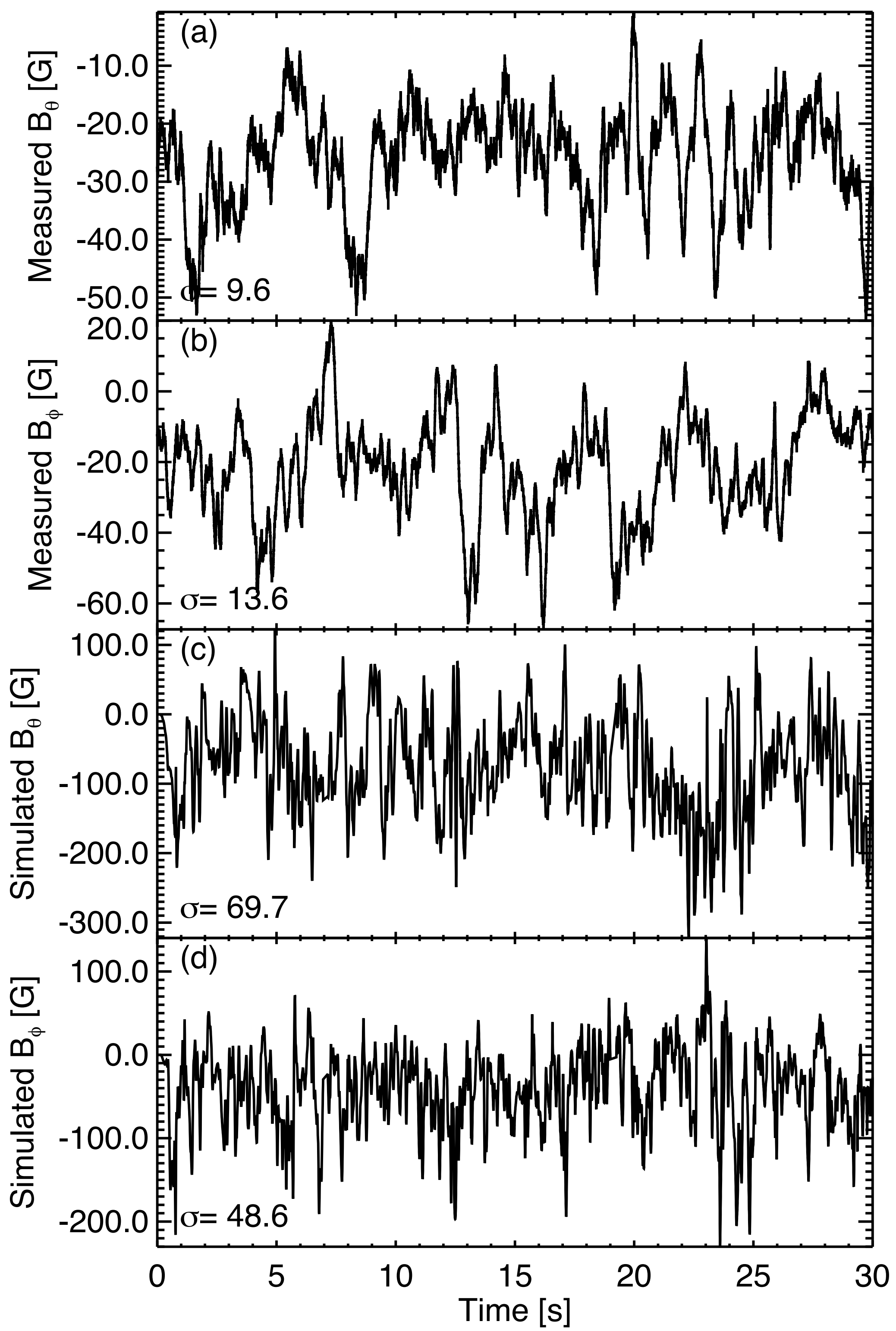
Field can be separated into mean-flow, mean-field driven currents and fluctuation generated currents



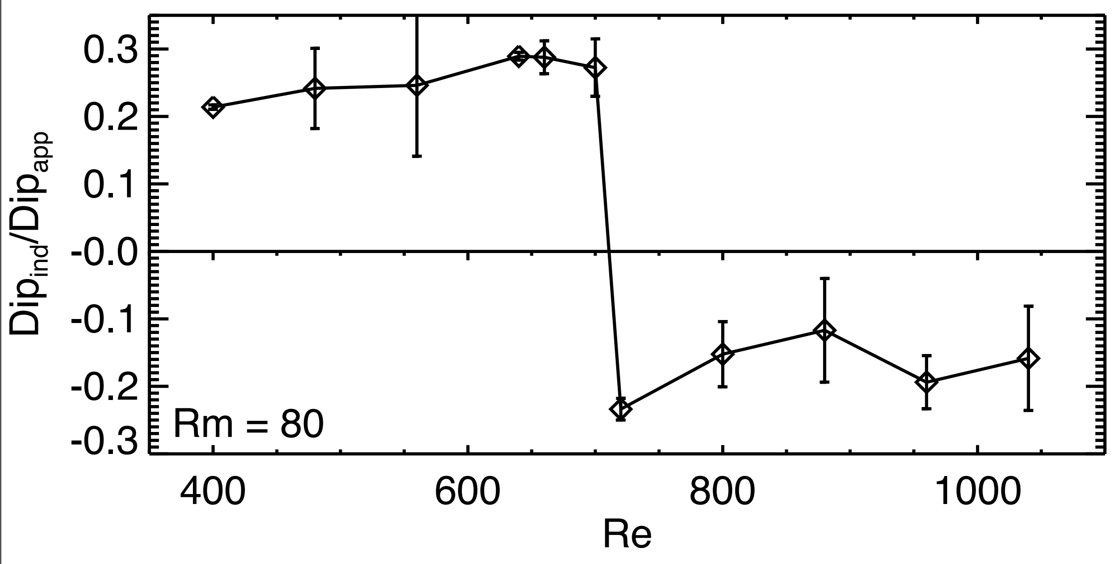
Spence, Nornberg, Jacobson, Parada, Kendrick, and Forest, *Turbulent Diamagnetism in Flowing Liquid Sodium*, Phys. Rev. Lett. **98** 164503 (2007).

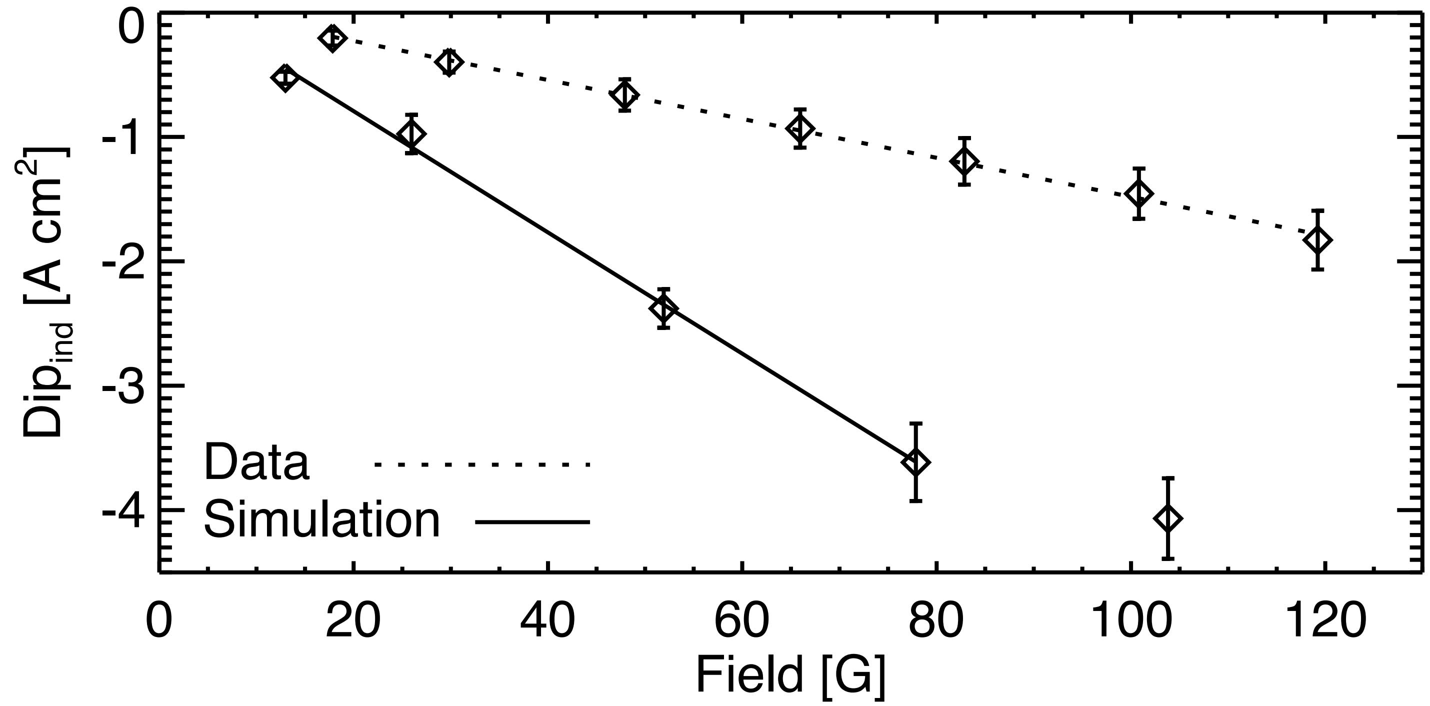
Comparisons between Simulation and Experiment

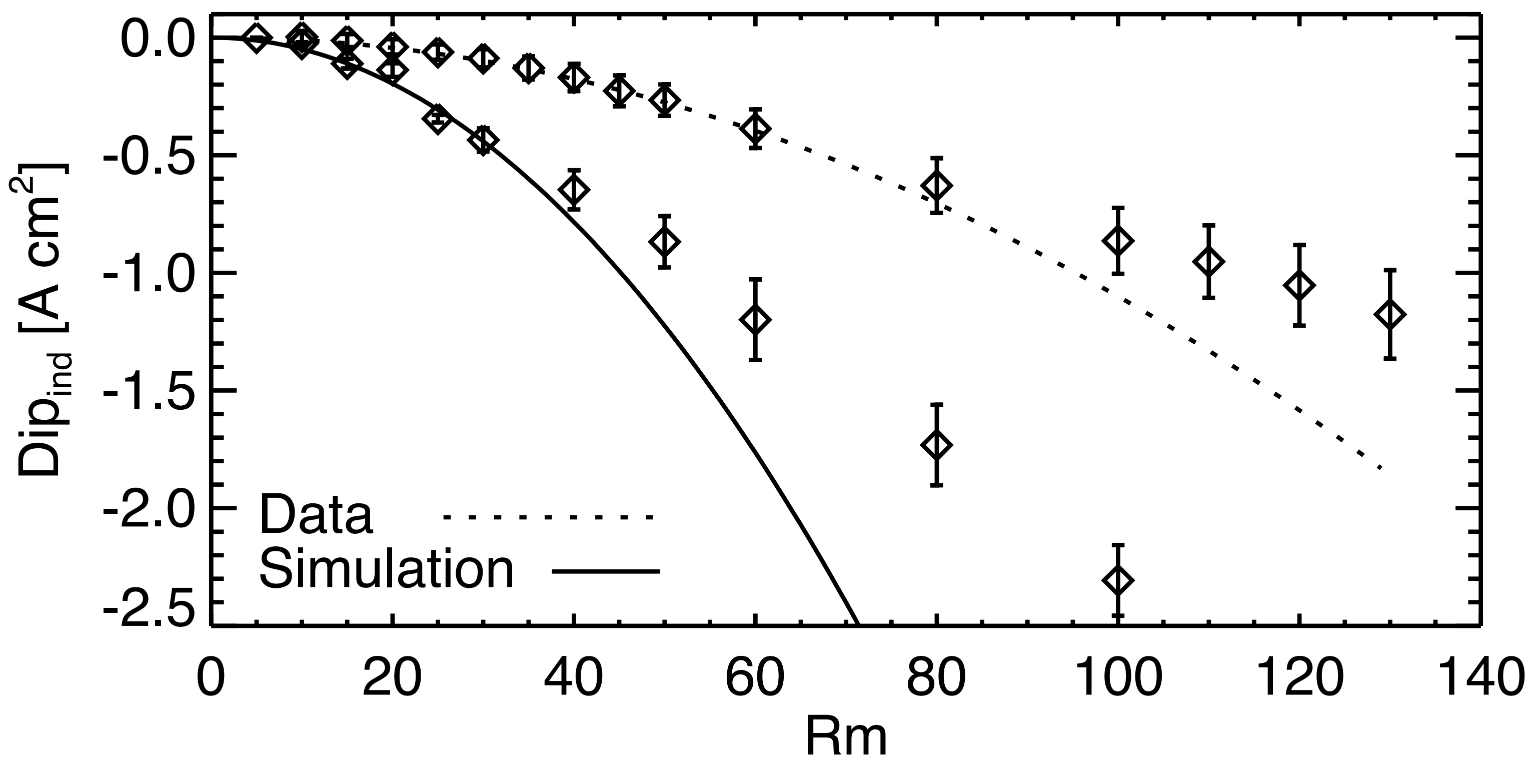




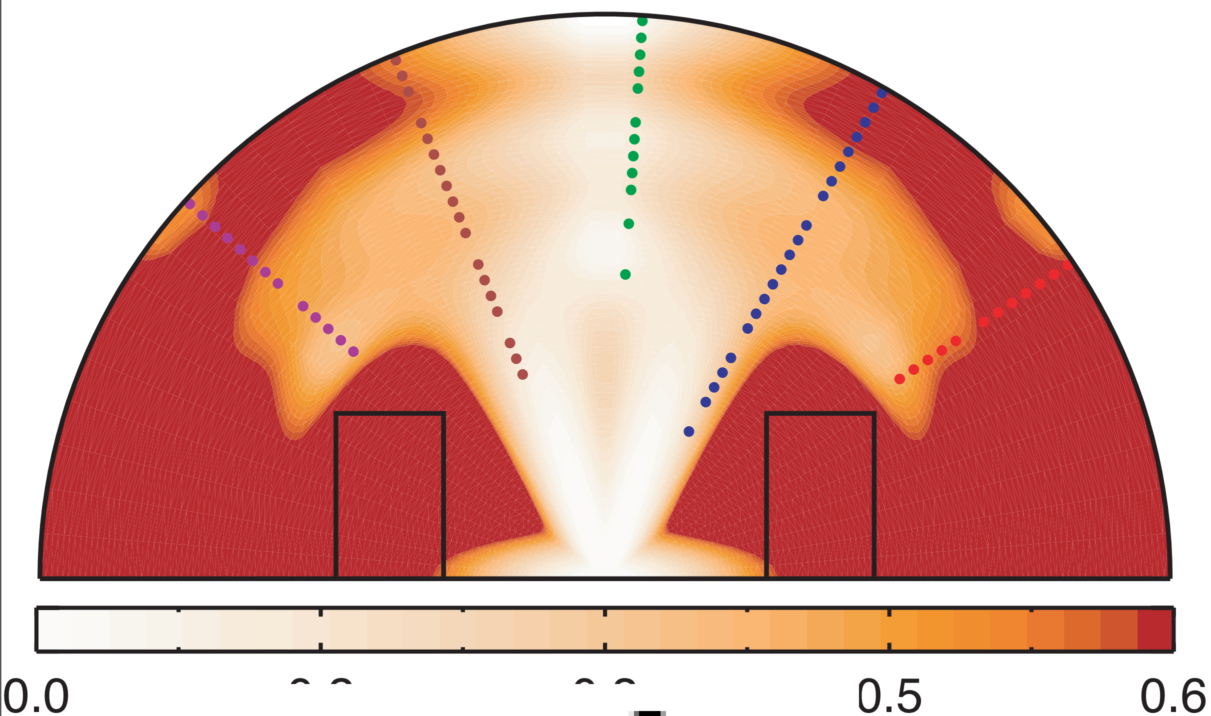
Scalings of Mean-Field Currents





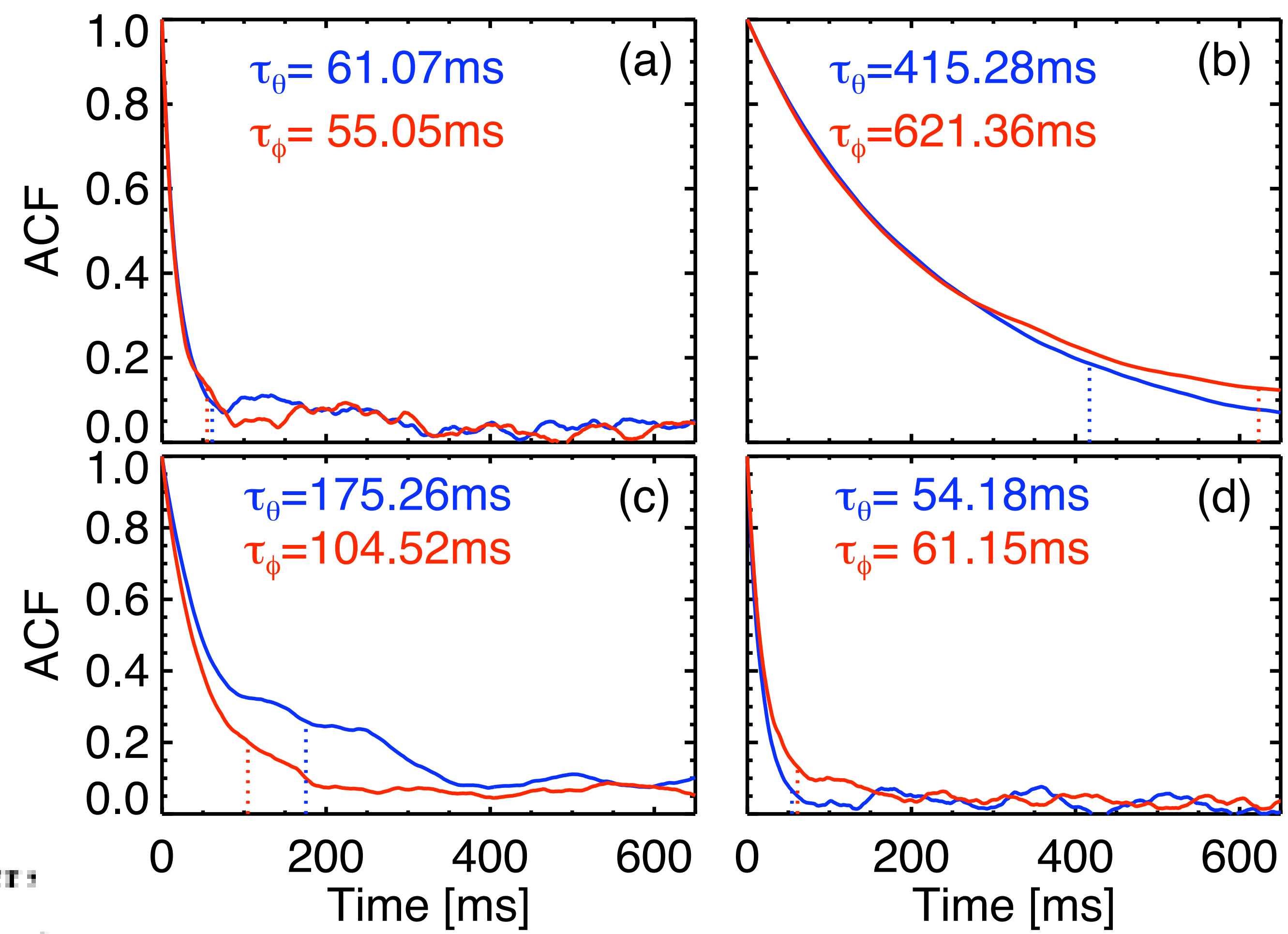


Turbulent resistivity can be large



$$\sigma_T = \frac{\sigma}{(1+\mu_0\sigma\beta)}.$$

$$\beta(\mathbf{r}) = \frac{1}{3}\int_0^\infty \langle \tilde{\mathbf{v}}(\mathbf{r},t)\cdot\tilde{\mathbf{v}}(\mathbf{r},t-\tau)\rangle \, d\tau = \frac{1}{3} v_{\mathrm{rms}}^2 \tau_{\mathrm{corr}},$$



Autocorrelation functions of LDV velocity measurements for Rmtip = 100. The locations are (a) in the bulk flow above and behind the impeller (r = 45 cm,theta = 0.596) (b) deep in the flow at the equator (r = 26 cm, theta= 1.50) (c) near the wall (r = 53 cm, theta = 0.596) and (d) near an impeller (r = 34 cm, theta= 0.596).