Fluctuations of the energy flux in wave turbulence

Stéphan Fauve

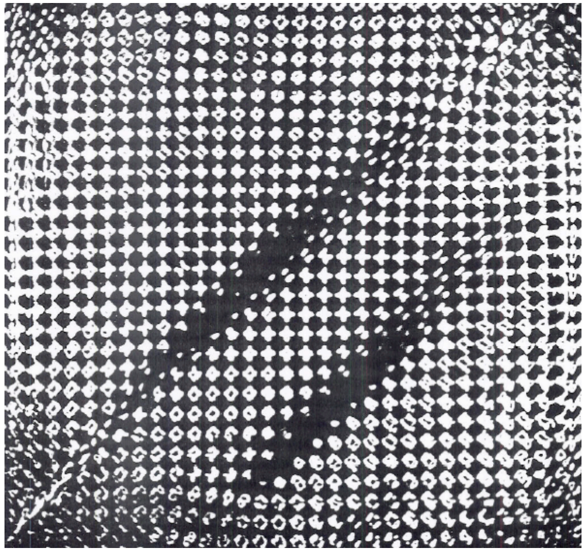
LPS - ENS, Paris

Wolfgang Pauli Institute, Vienna, february 15th 2008

What are the statistical properties of the energy flux needed to maintain a dissipative system in a stationary regime?

- Faraday waves: S. Ciliberto, S. Douady and S. Fauve, EPL (1991).
- Turbulence: R. Labbé, J. F. Pinton and S. Fauve, J. Physique II (1996).
- Turbulent convection: S. Aumaître and S. Fauve, EPL (2003).
- Granular gases: S. Aumaître, S. Fauve, S. McNamara and P. Poggi, EPJB (2001). S. Aumaître, J. Farago, S. Fauve and S. McNamara, EPJB (2004). S. Aumaître, A. Alastuey and S. Fauve, EPJB, (2006).
- Wave turbulence: E. Falcon, S. Aumaître, C. Falcon, C. Laroche and S. Fauve, PRL (2008)

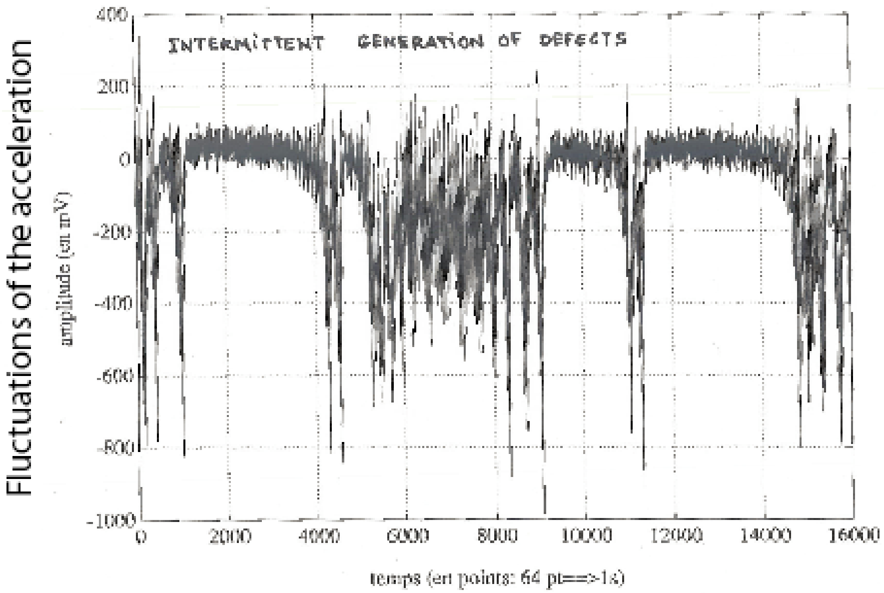
Faraday waves: fluctuations of the acceleration due to intermittent generation of defects in the wave field



$$E = \frac{1}{2}\int_V v^2\,\mathrm{d}V + g\int_V z\,\mathrm{d}V + \gamma S$$

$$\frac{\mathrm{d}E}{\mathrm{d}t} = \frac{\mathrm{d}g}{\mathrm{d}t}\int_V z\,\mathrm{d}V - \nu\int_V \Omega^2\,\mathrm{d}V - \nu\int_S (\boldsymbol{v}\times\boldsymbol{\Omega})\cdot\boldsymbol{n}\,\mathrm{d}S$$

$$g = g_0 + A\cos(2\omega t) + \delta g(t)$$



Fluctuations of the energy flux from injection to dissipation

S. Ciliberto, S. Douady and S. Fauve, Europhys. Lett. 15, 23-28 (1991)

Energy flux: $$\frac{dE}{dt} = I - D$$

Example :
Navier Stoles turbulence

$$E = \frac{1}{2}\rho \int_V v^2 dV,$$

$$\rho\left[\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v}\cdot\nabla)\mathbf{v}\right] = -\nabla p + \rho\nu\Delta\mathbf{v} + \mathbf{f}$$

$$I = \int_V \mathbf{f}\cdot\mathbf{v}\, dV,$$

I > 0 or I < 0 whereas D > 0

$$D = \rho\nu \int_V \omega^2 dV,$$

Dissipative systems

- Fluid turbulence (Navier-Stokes equation): viscosity
- Granular gases: inelastic collisions

No thermostat, no ad-hoc dissipation for energy conservation

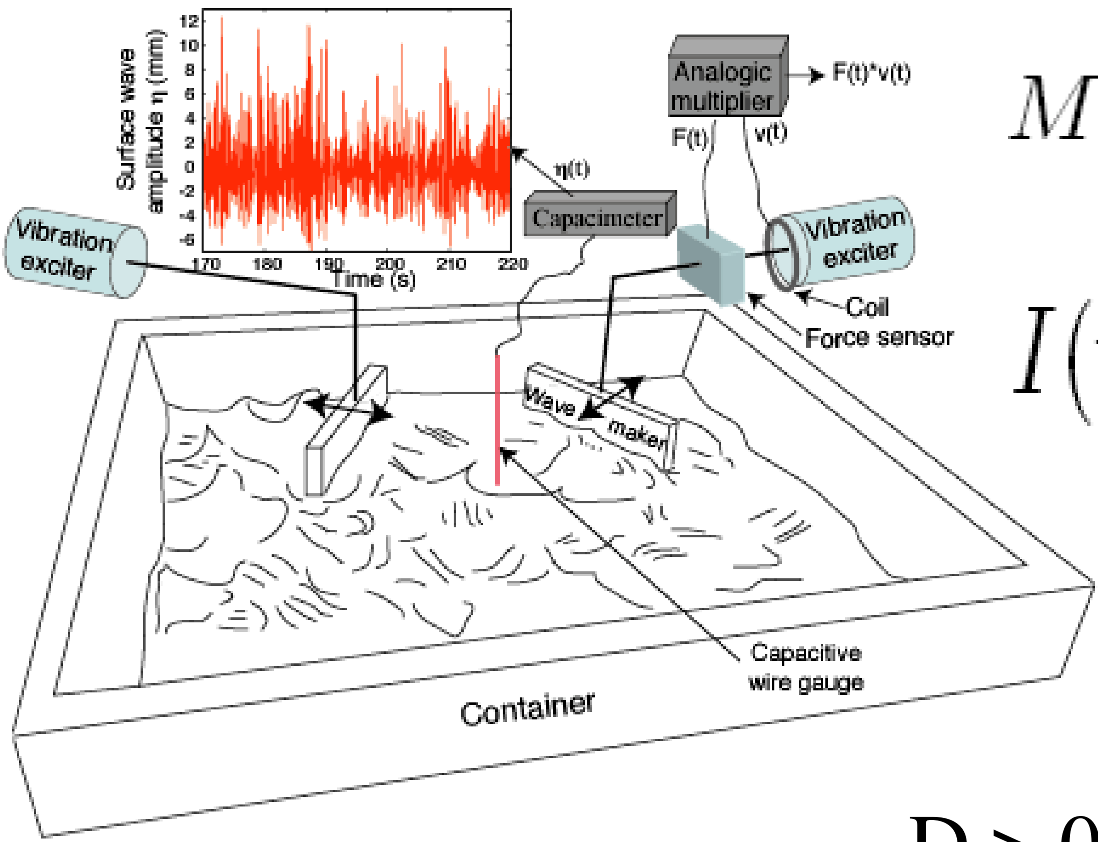
A simple example: a RC circuit driven by a random voltage

$$\frac{dV(t)}{dt} = -\gamma V(t) + \xi(t) + f_0$$

(i) ξ (t) and γ : heat bath $\Rightarrow$ fluctuation dissipation relation
f_0 : external operator $\Rightarrow$ drives the system slightly out of equilibrium

(ii) γ macroscopic damping (fluctuations can be neglected)
f_0 =0, ξ (t) : external operator $\Rightarrow$ drives the system strongly out of equilibrium

Gravity-capillary waves



$$M\dot{V} = F(t) + F_R(t)$$

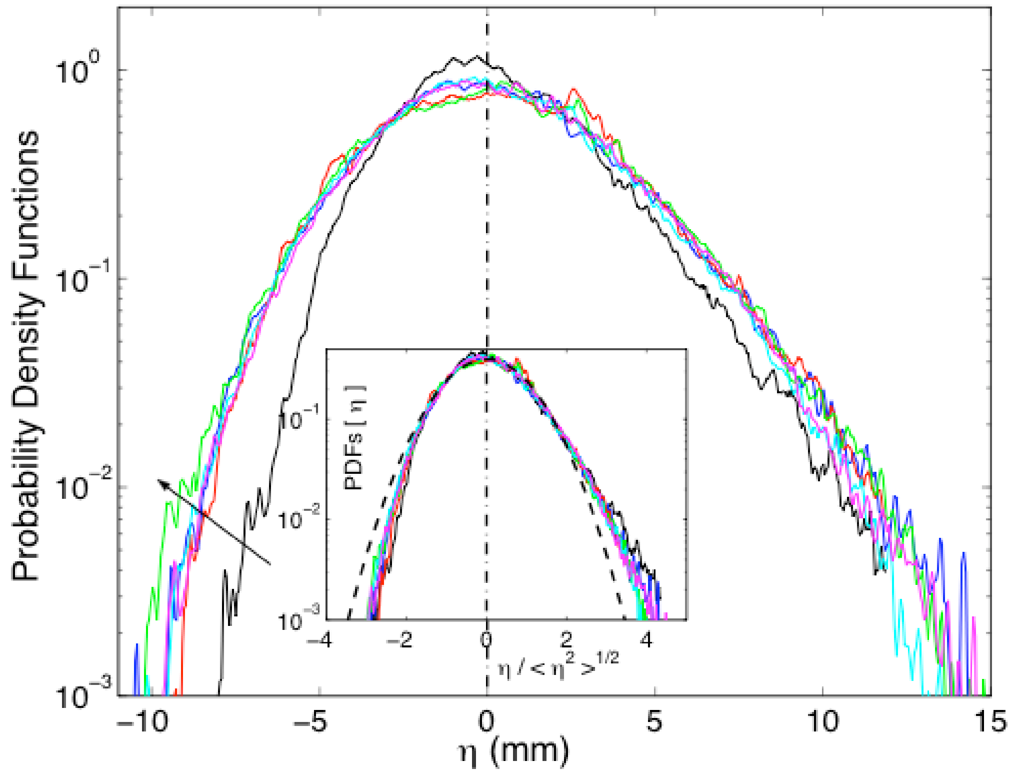
$$I(t) = -F_R(t)V(t)$$

I > 0 or < 0

D > 0

Viscous dissipation

PDF of the waves amplitude

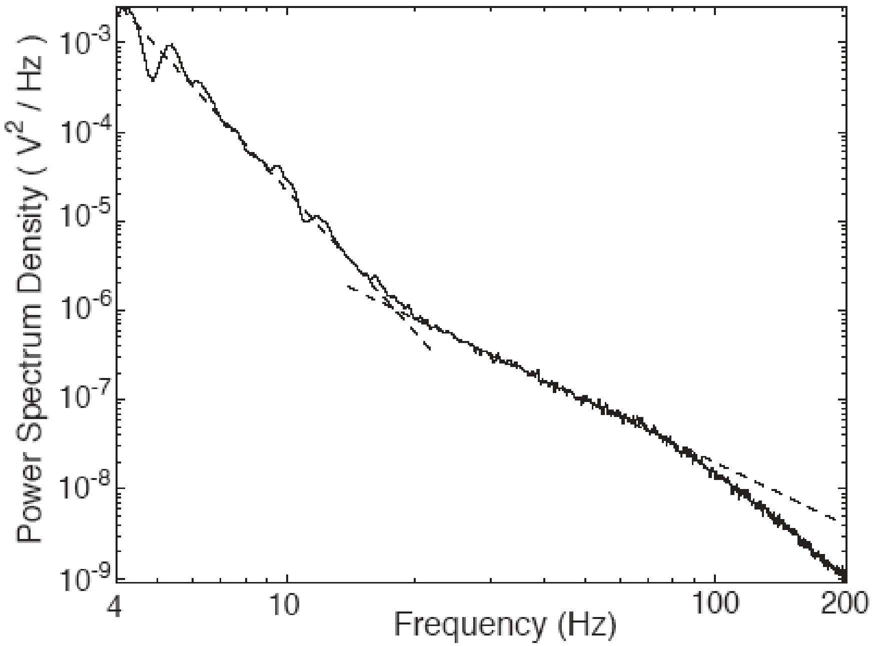


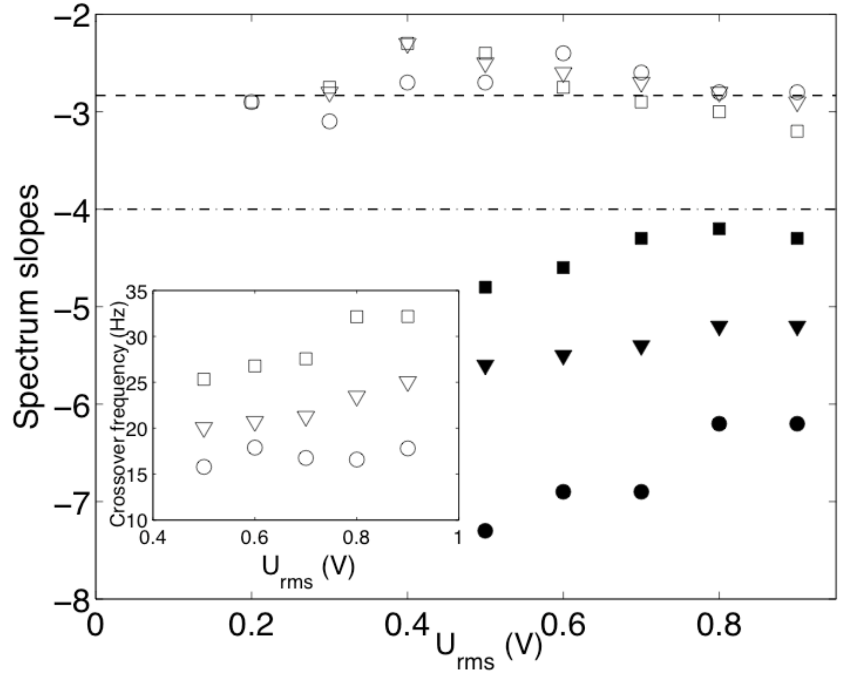
Non Gaussian PDF of the amplitude of the waves

Subsists when the height of the layer is large compared to the largest wavelength

Effect of the nonlinearities of the waves

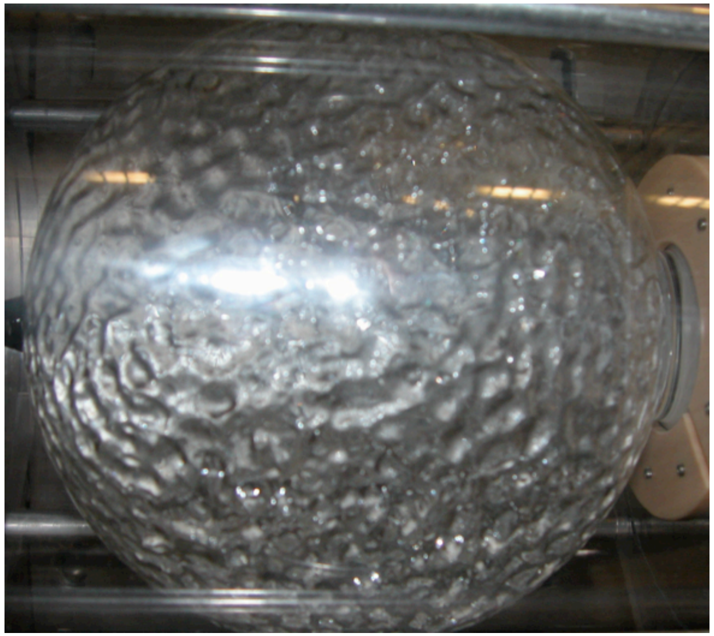
Spectra in the gravity and capillary ranges

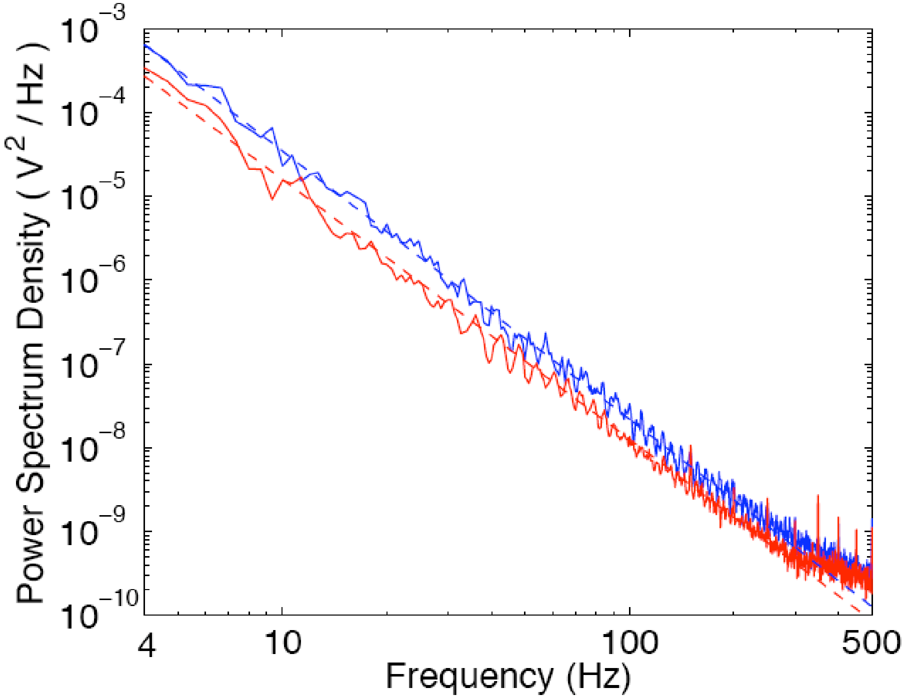




The slope depends on the forcing in the gravity range
It is roughly constant in the capillary range

Capillary waves in zero gravity





C. Falcon, E. Falcon, U. Bortolozzo, S. Fauve (2007)

Wave turbulence

-Weak nonlinear interactions between a continuum of waves:

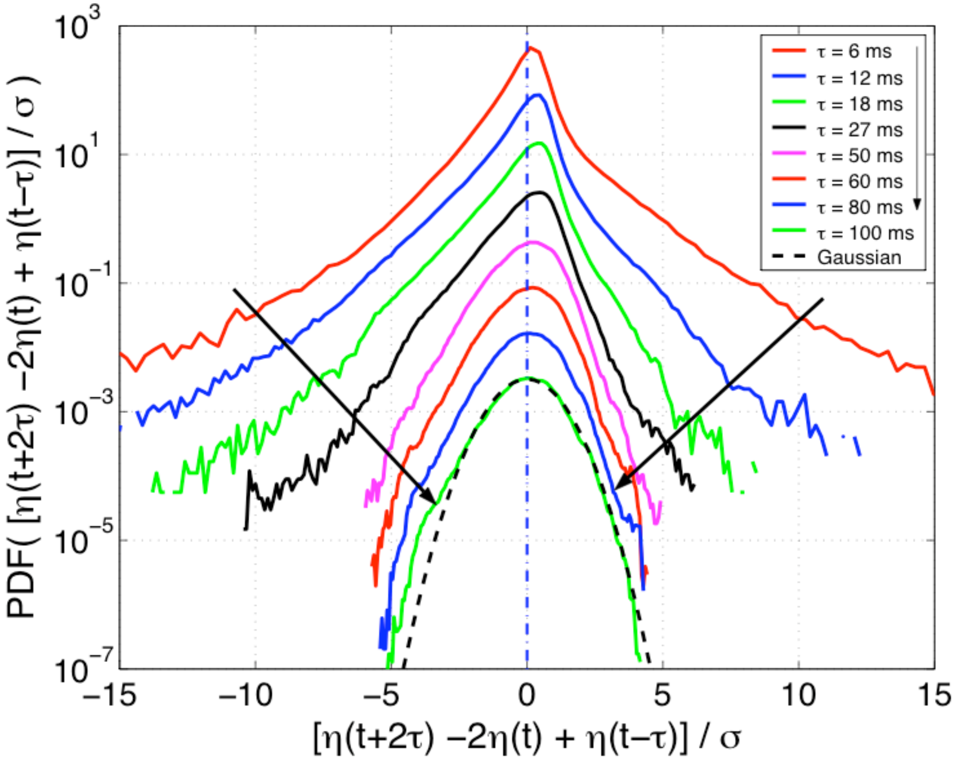
- Kinetic-like equations for the wave intensity at wavenumber k or frequency f

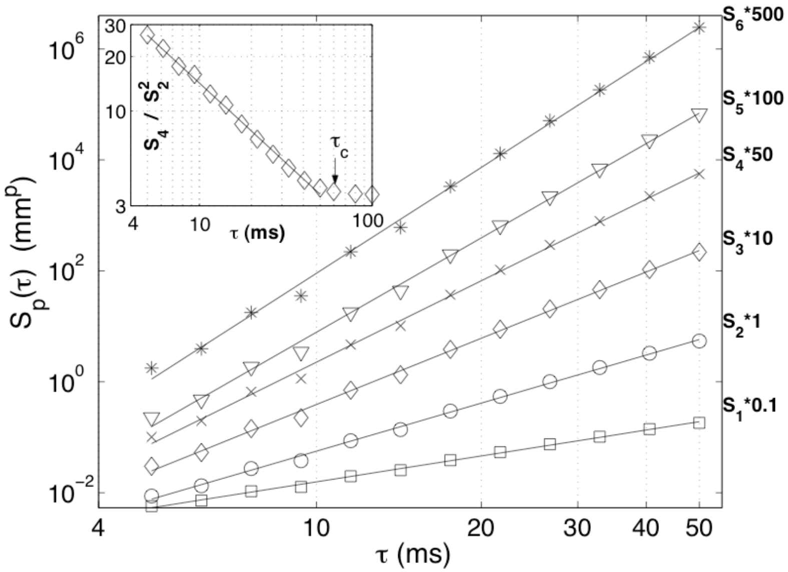
-In addition to equilibrium spectra, there are solutions that are related to a constant energy flux in wavenumber or frequency space (Zakharov et al., 1967)

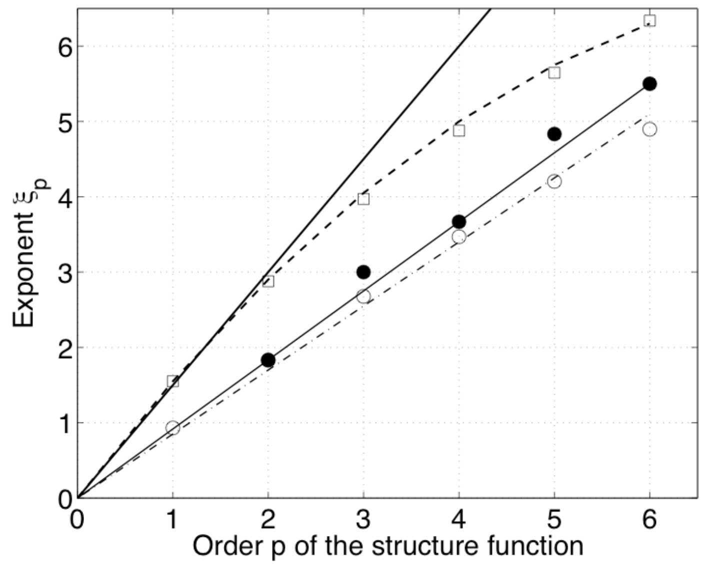
$$S_\eta(f) \propto \epsilon^{1/2}\left(\frac{\gamma}{\rho}\right)^{1/6} f^{-17/6} \quad \text{for capillary waves,}$$

$$S_\eta(f) \propto \epsilon^{1/3} g f^{-4} \quad \text{for gravity waves,}$$

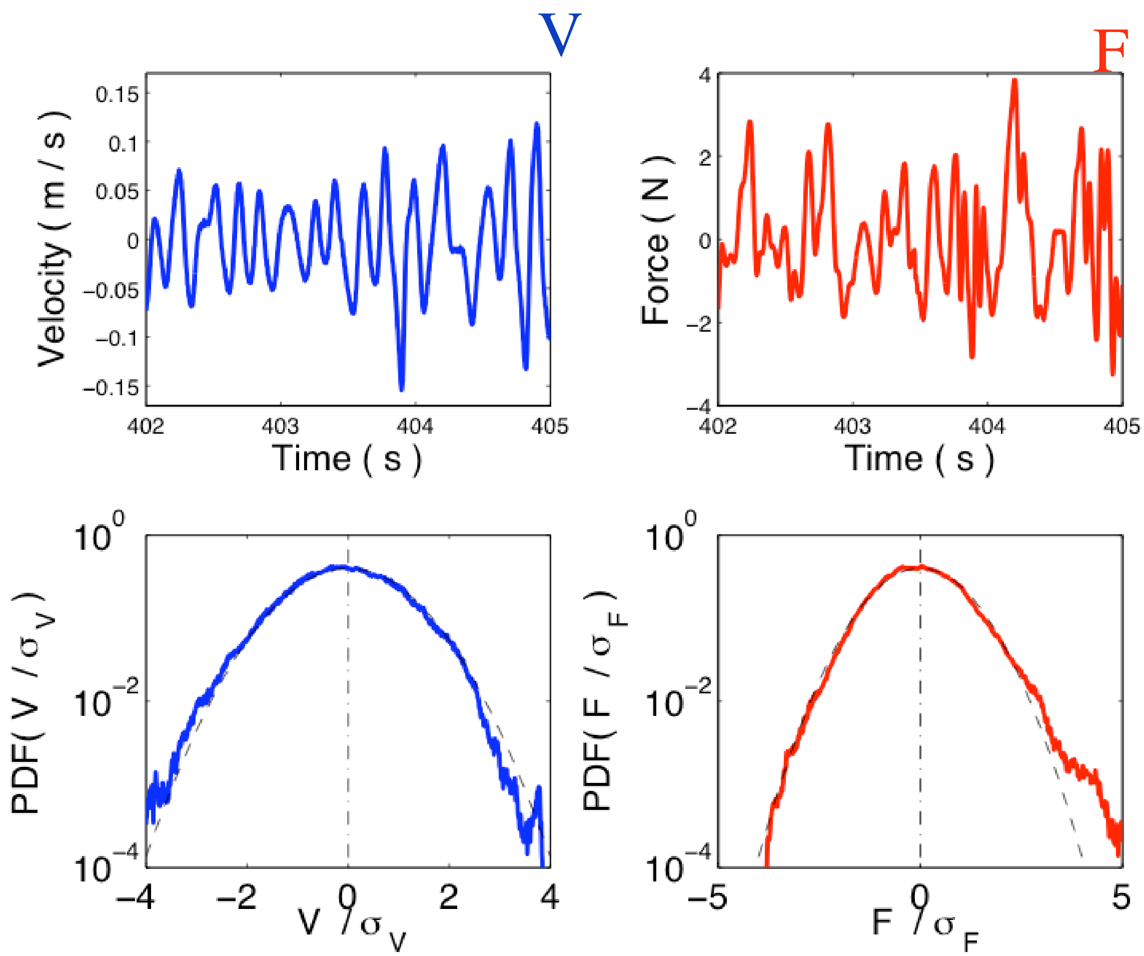
Intermittency of the surface velocity







Measurement of the force F applied to the wave maker and of its velocity V with random driving voltage



$\sigma_F = K\ \rho\ S_P\ \sigma_V$

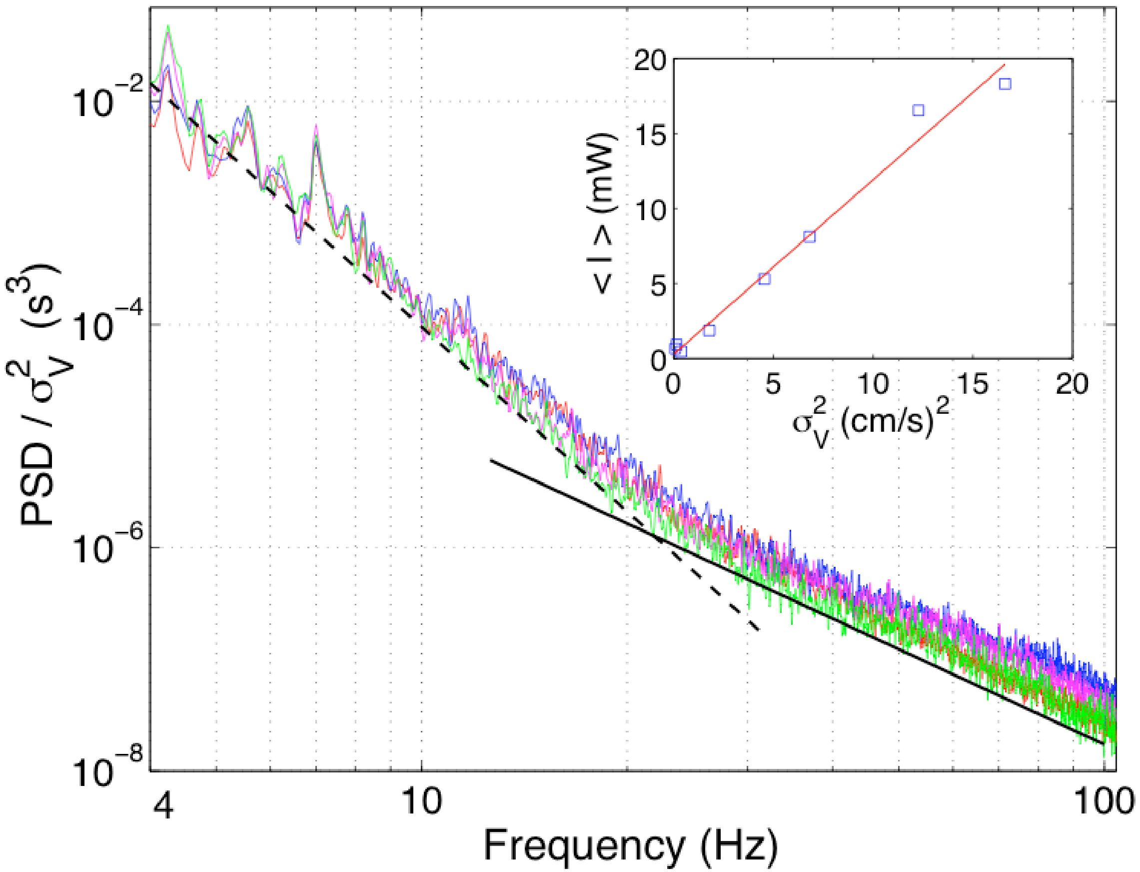
For mercury:

$\sigma_V \sim 5$ cm/s

$\sigma_F \sim 1$ N

Scaling versus the mean energy flux

$<I> = c\, \rho\, S_P\, \sigma_V^2$ $\varepsilon = <I> / \rho\, S_P$



For mercury:

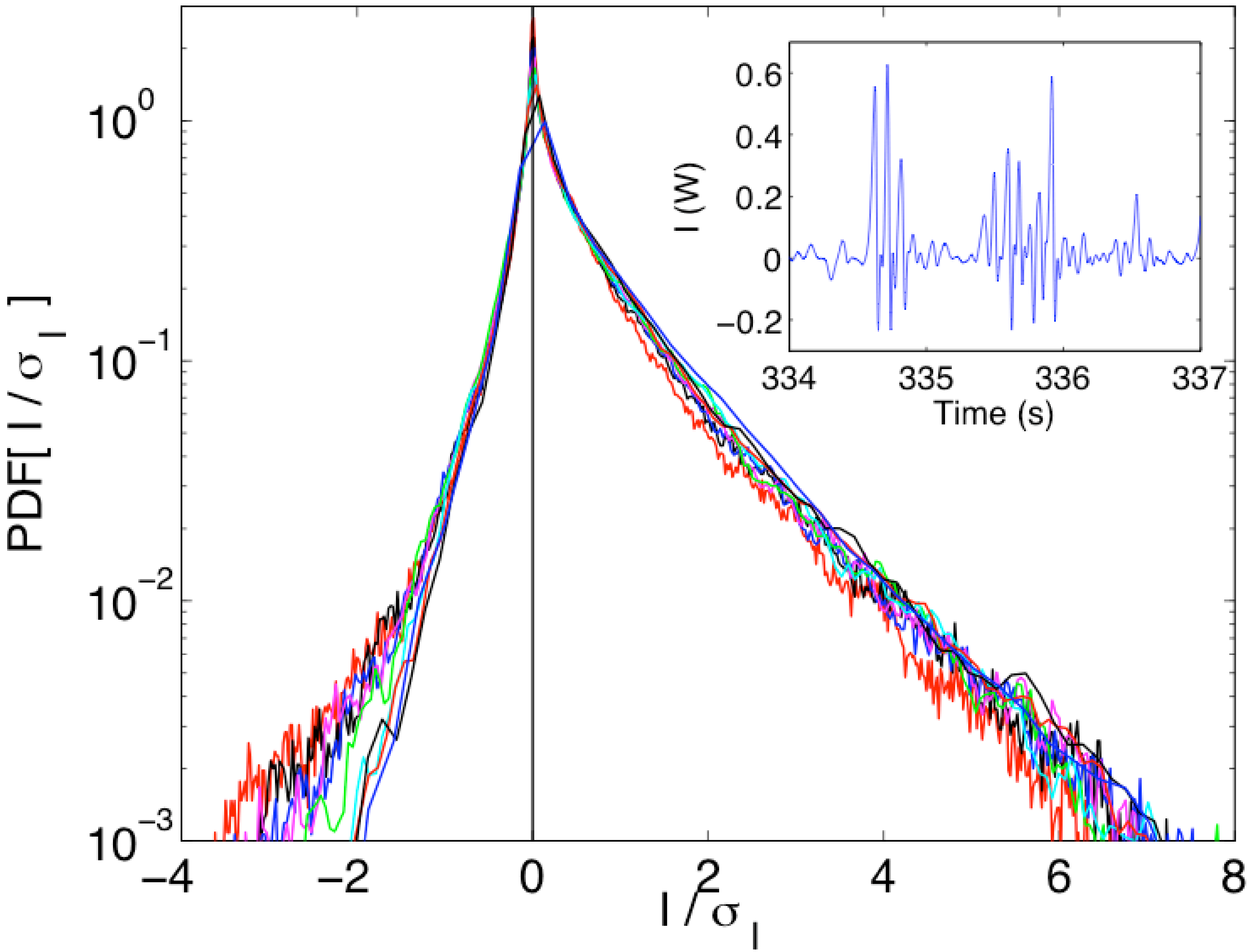
$\sigma_V \sim 5$ cm/s

$\sigma_F \sim 1$ N

$<I> \sim 30$ mW

$\sigma_I \sim 100$ mW

Fluctuations of the injected power I = F V



Effect of inertia

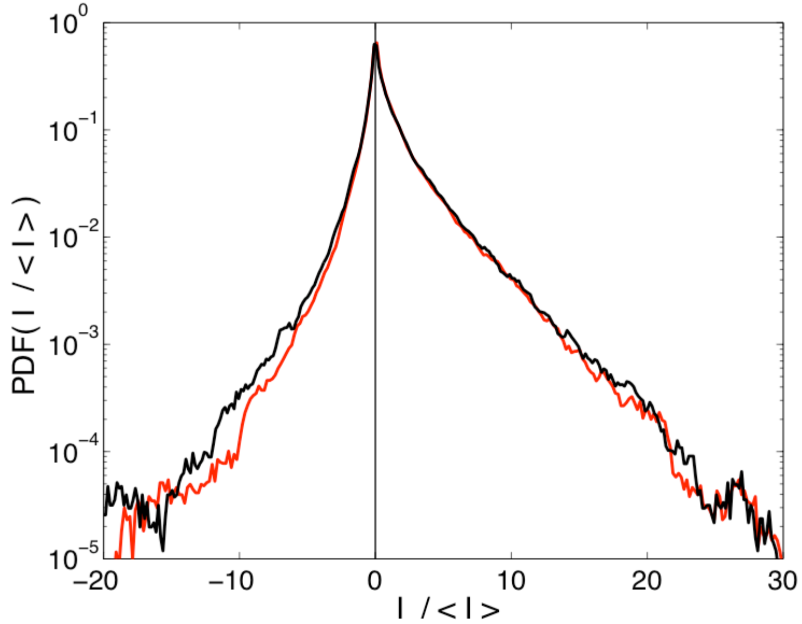
- F applied by the shaker to the wave maker
- F_R force applied by the wave maker to the fluid

$$M\dot{V} = F(t) + F_R(t)$$

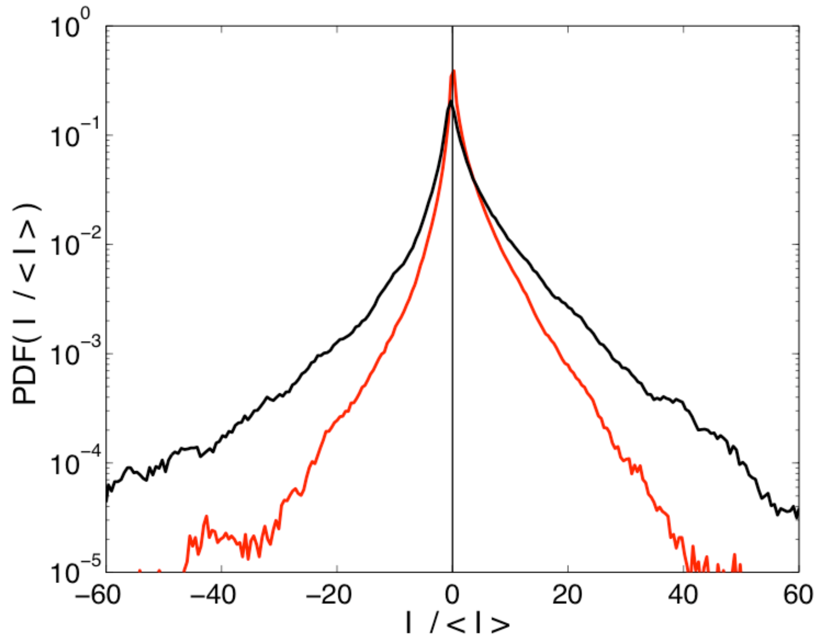
Negligible correction for Hg

Important correction for H_2O

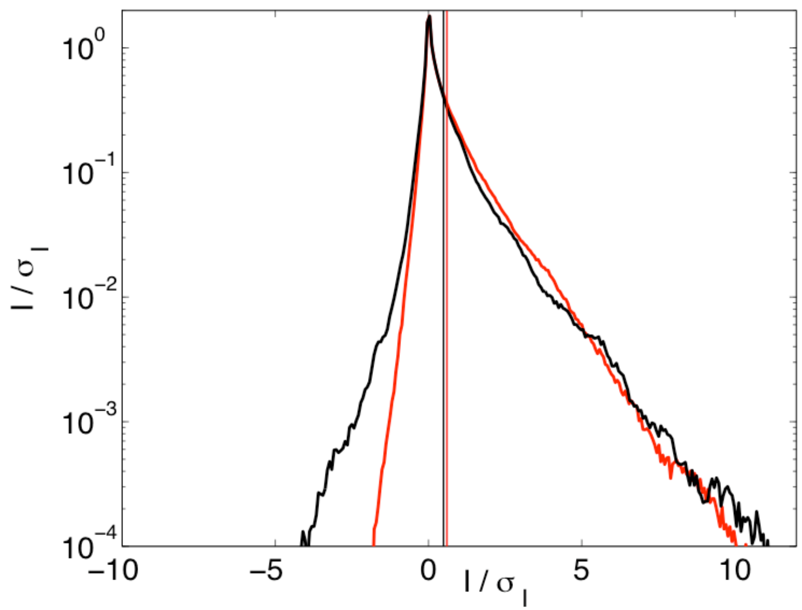
Hg

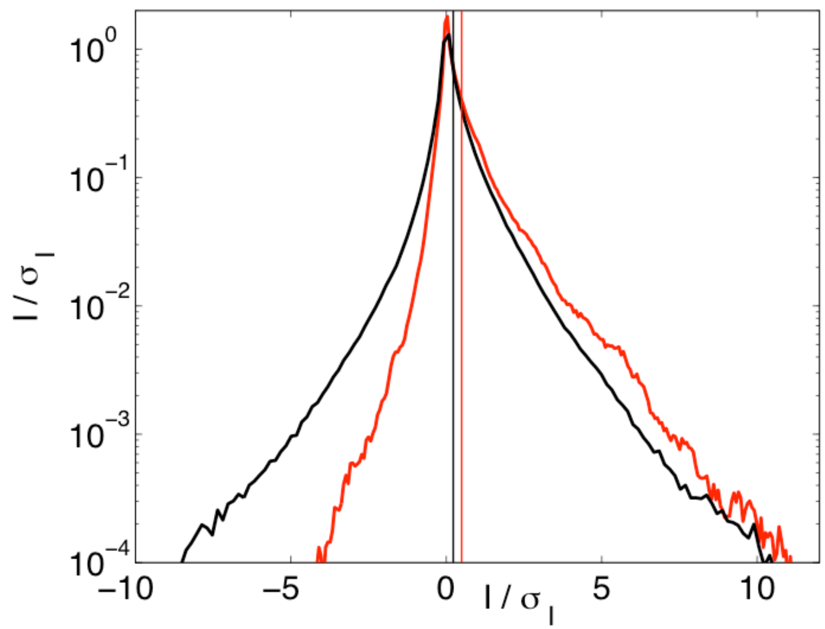


H_2O



Effect of the size of the container and of the fluid parameters



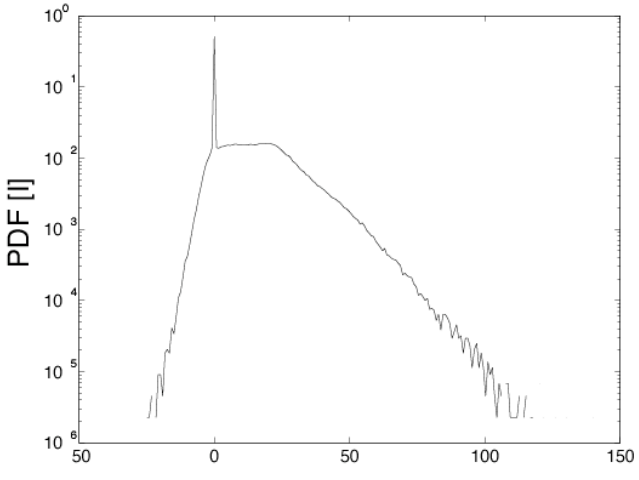


Mercury
Water

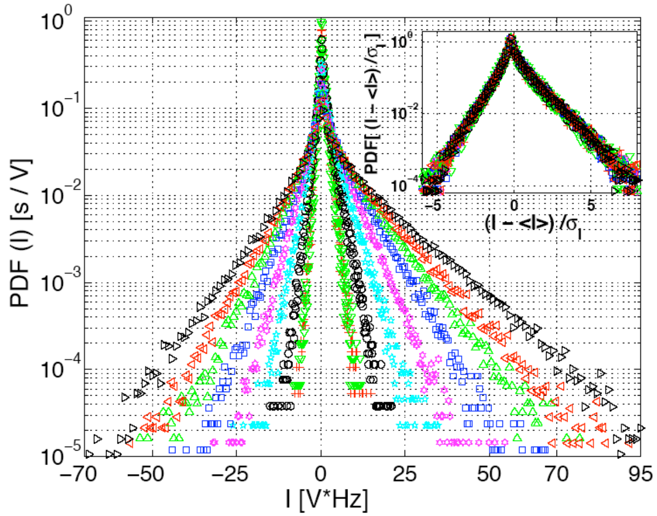
Small container
Large container

$$\sigma_I \propto \langle I \rangle = c\rho S_P \sigma_V^2$$

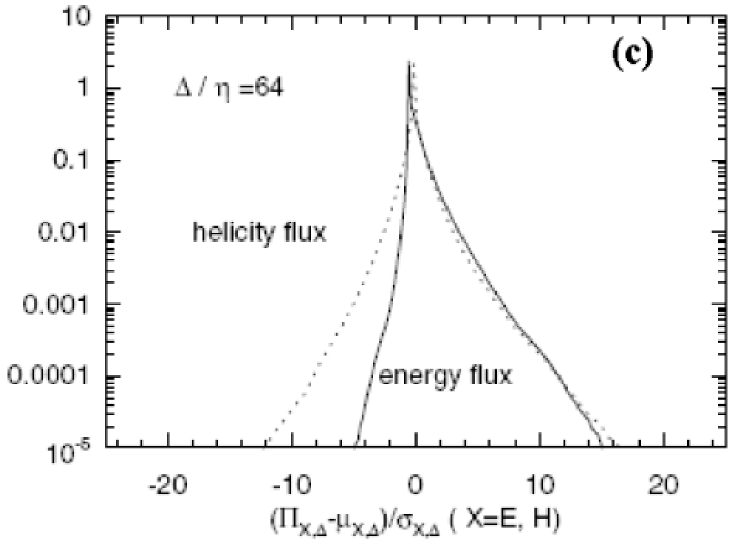
PDF of the injected power in different systems



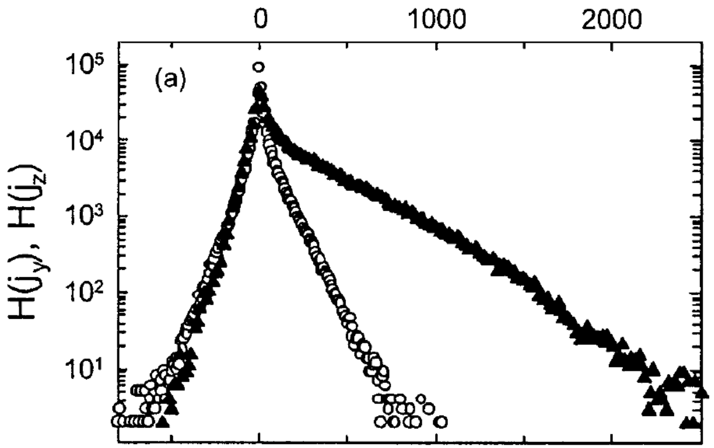
Aumaitre et al., granular gas



Falcon et al., RC circuit



Chen et al., turbulence



Shang et al., convection

A simple model for the pdf of the injected power with random forcing (Phys. Rev. Lett. 2008)

$$\frac{dv(t)}{dt} = -\gamma v(t) + f(t) \qquad \frac{df(t)}{dt} = -\beta f(t) + \xi(t)$$

Bivariate normal distribution for P(v, f)

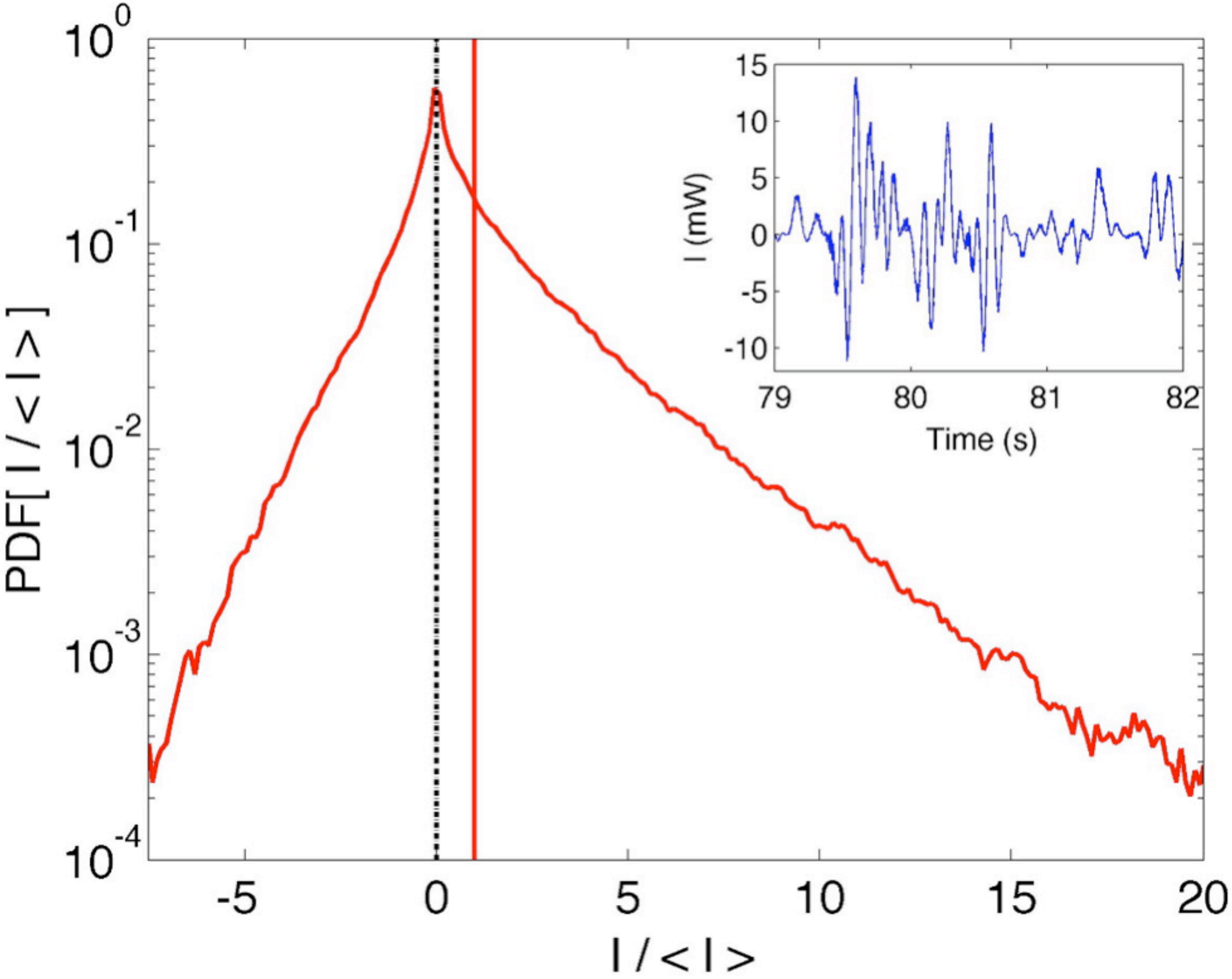
$\sigma_f^2 = D/2\beta$; $\sigma_v^2 = D/2\beta\gamma(\beta+\gamma)$; $\rho = \langle I\rangle/\sigma_f\sigma_v$; $\langle I\rangle = D/\,2\beta(\beta+\gamma)$

$$\mathcal{P}(v,f) = \frac{1}{2\pi\sigma_v.\sigma_f(1-\rho^2)^{1/2}} \exp\left[-\frac{1}{2(1-\rho^2)}\left(v^2/\sigma_v^2 - 2\rho v f/(\sigma_v\sigma_f) + f^2/\sigma_f^2\right)\right.$$

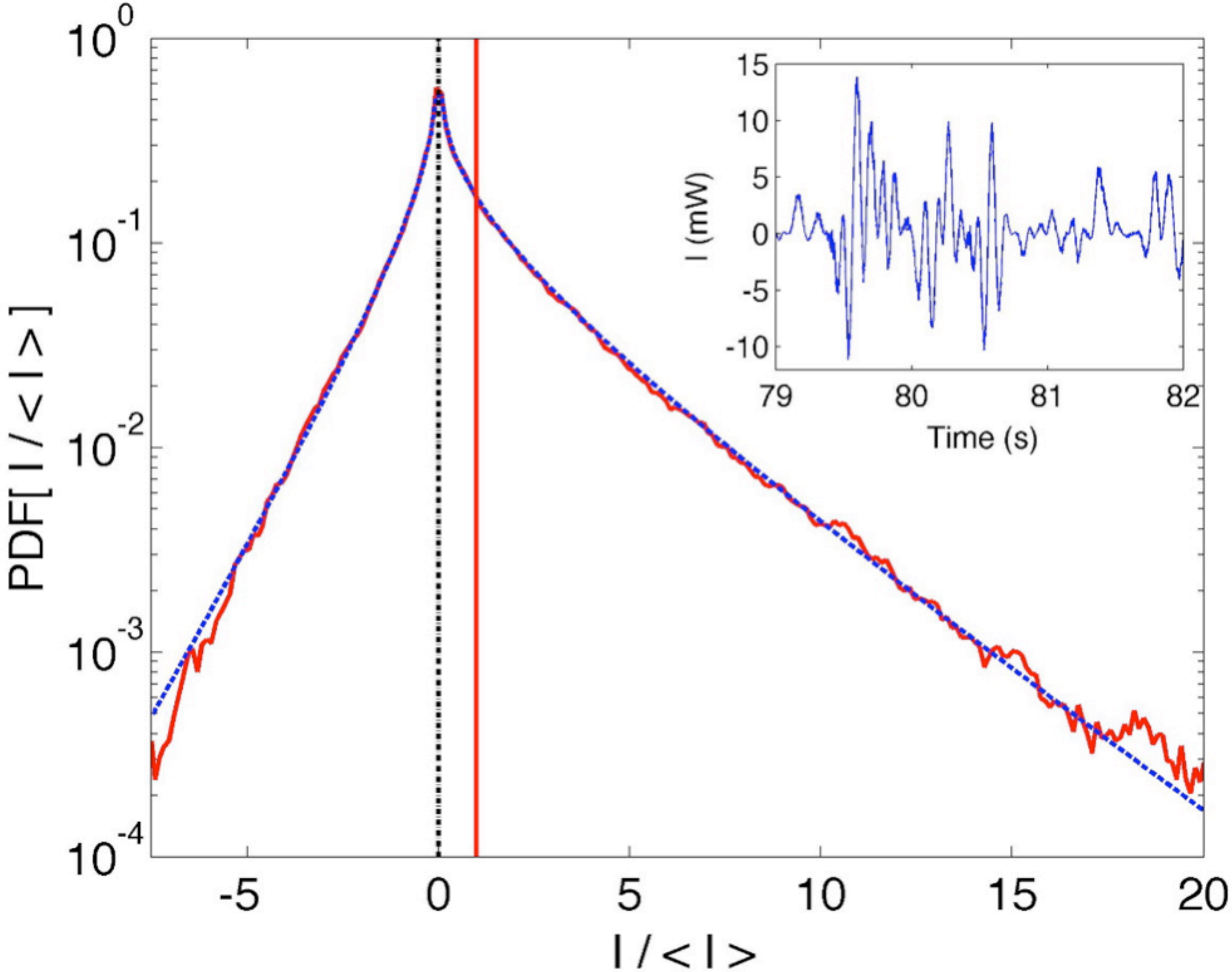
$$P(\tilde{I}) \propto \frac{1}{\sqrt{\tilde{I}}} \exp(\rho\tilde{I} - |\tilde{I}|), \quad \text{with} \quad \tilde{I} = \frac{I}{(1-\rho^2)\sigma_f\sigma_v},$$

Exponential tails, cusp for I=0, asymmetry related to <I>

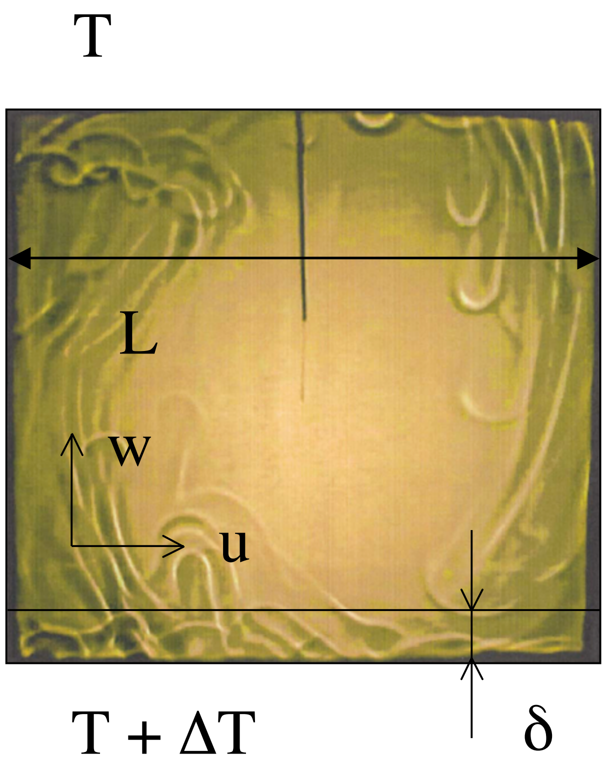
PDF of the injected power in wave turbulence

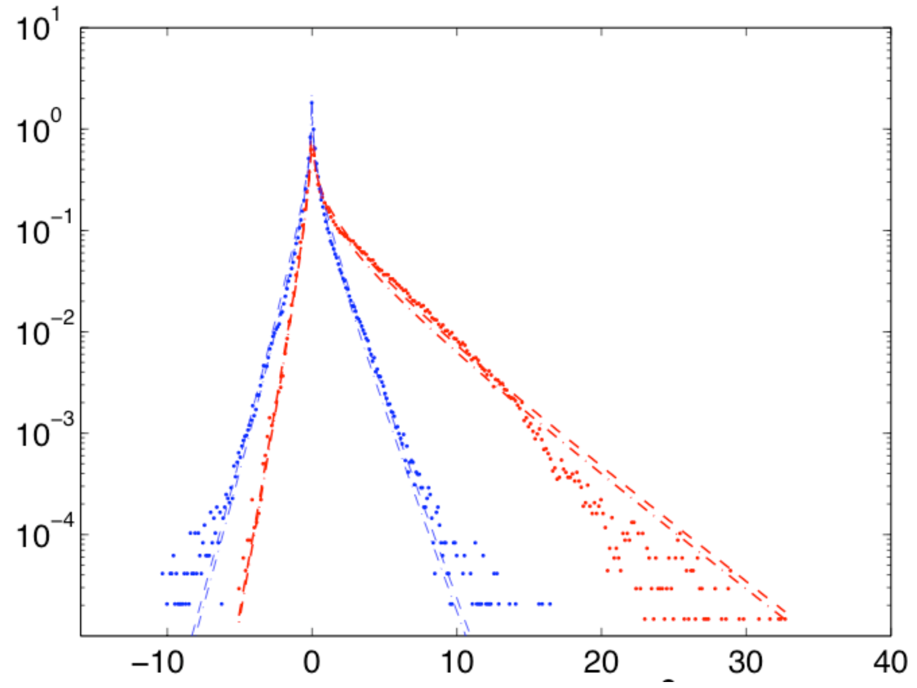


Langevin fit of the PDF of the injected power



« Local » heat flux in turbulent convection





local convective heat flux: $u\theta$ or $w\theta$

Shang, Qiu, Tong and Xia, PRL 90, 074501 (2003)

A relation for the variance of the energy flux

$$\frac{dE}{dt} = I - D \qquad \text{obviously implies} \qquad \langle \mathrm{I} \rangle = \langle \mathrm{D} \rangle$$

$$\text{but also} \qquad |\hat{\mathrm{I}}|^2(0) = |\hat{\mathrm{D}}|^2(0)$$

and from the Wiener Khintchin theorem

$$\int_0^\infty \left[\langle I(t')I(0) \rangle - \langle I \rangle^2 \right] dt' = \int_0^\infty \left[\langle D(t')D(0) \rangle - \langle D \rangle^2 \right] dt'$$

$$\sigma_I^2 \, \tau_I \propto \sigma_D^2 \, \tau_D$$

Implication for Kolmogorov type cascade

$$\frac{dE_{1,2}}{dt} \approx F(k_1, t) - F(k_2, t) \qquad \text{in the inertial range}$$

$$\sigma_F^2 \, \tau_k \propto \sigma_F^2 \, \epsilon^{-1/3} \, k^{-2/3} = \text{constant}$$

In this picture, the standard deviation of the fluctuations of the energy flux should increase proportionally to $k^{1/3}$ within the inertial range

An experimental or at least a numerical check can provide a quantitative test of the cascade picture and show some connection with intermittency

Large deviation theorem

$$I_\tau(t) = \frac{1}{\tau}\int_t^{t+\tau} I(t')dt'$$

$$\Pi(I_\tau(t) = U) \approx C \cdot \exp(-\tau F_I(U))$$

F_I is the large deviation function. It is minimum for U = <I>

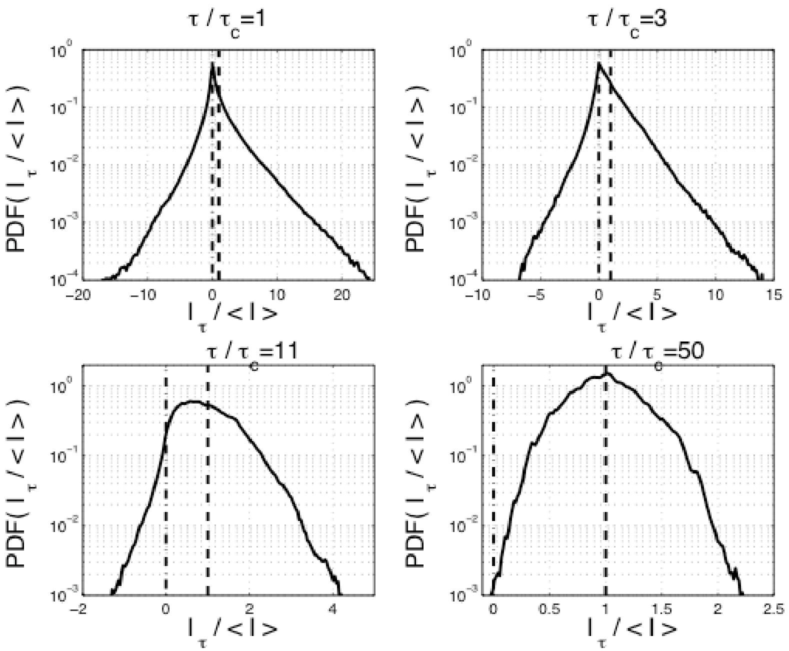
The large deviation function of I and D have the same Taylor expansion in the vicinity of <I> = <D> (but their tails can be different)

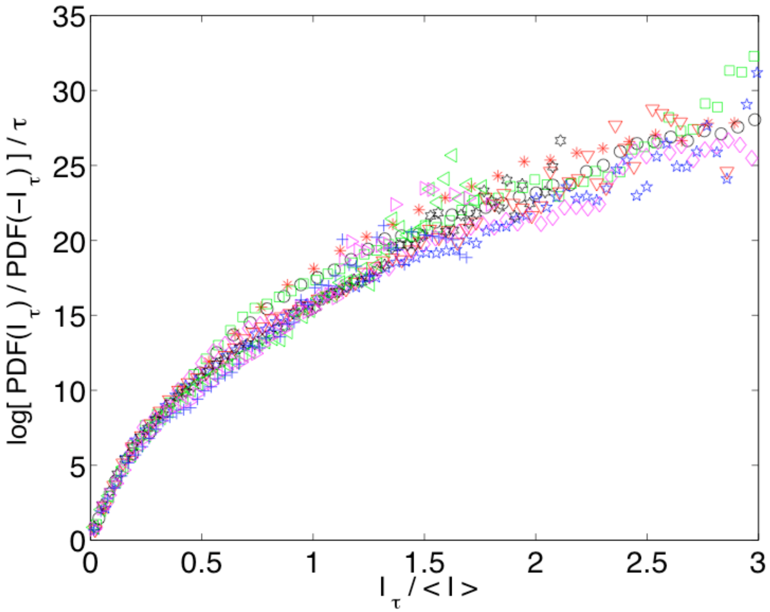
Farago, J. Stat. Phys. (2002)
Visco et al., Europhys. Lett. (2005)

PDF of the averaged injected power

$$I_\tau(t) = \frac{1}{\tau}\int_t^{t+\tau} I(t')dt'$$

$$\frac{1}{\tau}\, Log\, \frac{P(I_\tau = \epsilon)}{P(I_\tau = -\epsilon)}$$





Similar to the analytic result of Farago, J. Stat. Phys. (2002)