Chaotic dynamics of the magnetic field generated by a turbulent flow in the VKS experiment

Stéphan Fauve

LPS - ENS-Paris

Wolfgang Pauli Institute, Vienna, february 14th 2008

MHD equations and dimensionless parameters

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{V} \times \mathbf{B}) + \frac{1}{\mu_0 \sigma} \nabla^2 \mathbf{B}$$

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v}.\nabla)\mathbf{v} = -\nabla \left(\frac{\mathbf{p}}{\rho} + \frac{\mathbf{B^2}}{\mathbf{2}\mu_\mathbf{0}} \right) + \nu \nabla^\mathbf{2} \mathbf{v} + \frac{\mathbf{1}}{\mu_\mathbf{0} \rho} (\mathbf{B} \cdot \nabla)\mathbf{B}.$$

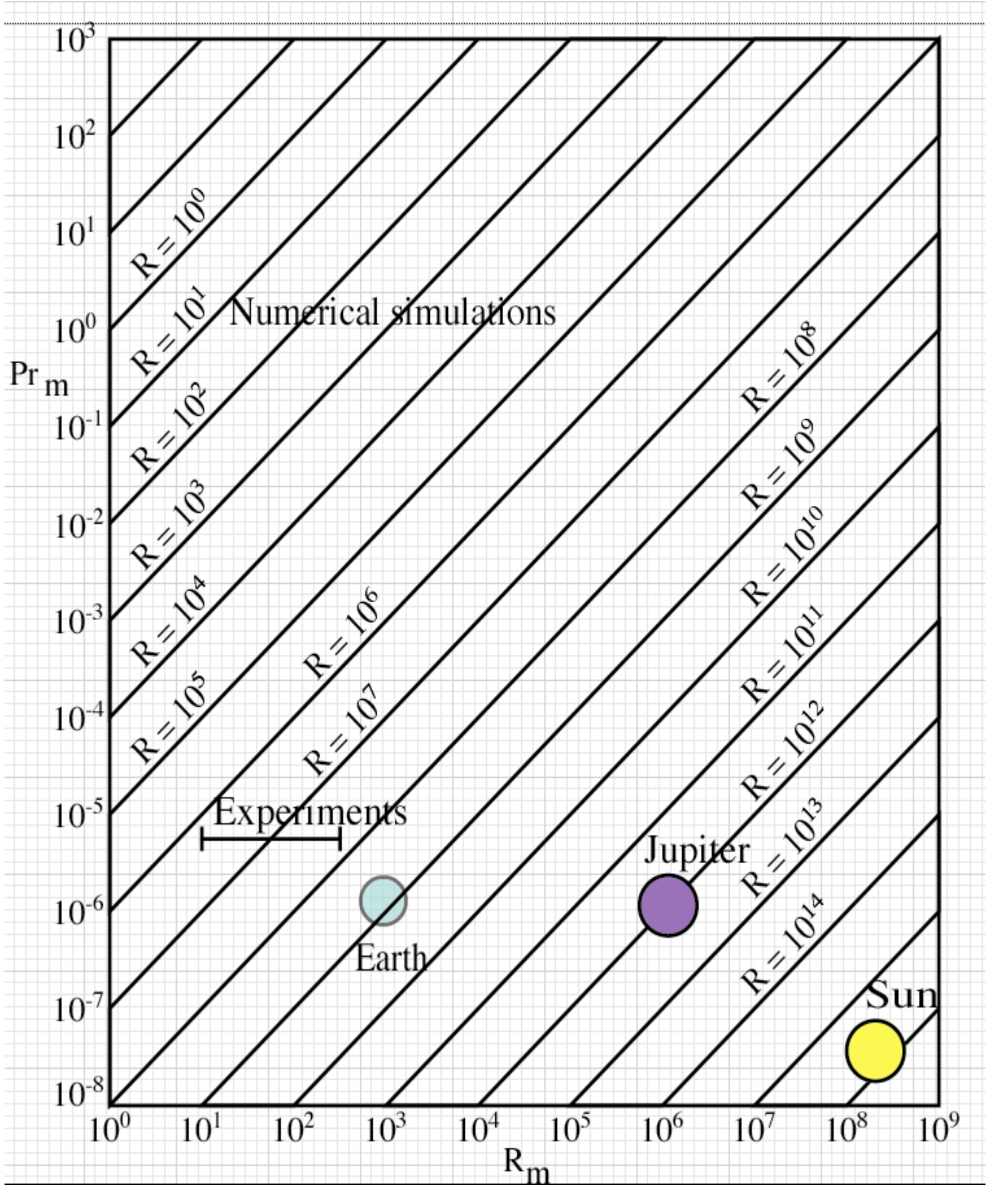
- fluid density: ρ
- kin. viscosity: ν
- velocity : V
- domain size: L
- mag. permeability: μ_0
- elec. conductivity : σ

$Re = VL / \nu$

$R_m = \mu_0 \sigma VL$

$P_m = \mu_0 \sigma \nu$

Numerical simulations, experiments and the universe



$$Rm = \mu_0 \sigma L V$$

$$Pm = \mu_0 \sigma \nu$$

Power $P \propto \rho L^2 V^3$
to drive a turbulent flow

$$\Rightarrow \; Rm \propto \mu_0 \sigma \left(PL/\rho\right)^{1/3}$$

Using liquid sodium,
100 kW to reach Rm = 50
with L = 1m

Dynamos in the case of small Pm

$Pm = \mu_0 \sigma \nu < 10^{-5}$ for liquid metals
$\Rightarrow Re = Rm/Pm > 10^6$ for Rm above Rm^c

$\Rightarrow$ a bifurcation from a strongly turbulent regime

- Dynamo threshold: $R_m^c = f\,(Re)$

  Effect of turbulence on the dynamo threshold ?

- Magnetic energy density: $B^2 = \mu_0\, \rho V^2\ g\,(R_m, Re)$

  How much magnetic energy can be generated ?

« Laminar or turbulent » dynamos

$$\mathbf{V}(\mathbf{r},\mathbf{t}) = \langle \mathbf{V} \rangle (\mathbf{r}) + \tilde{\mathbf{v}}(\mathbf{r},\mathbf{t})$$

$$\frac{\partial \mathrm{B}}{\partial t} = \nabla \times [\langle \mathbf{V} \rangle (\mathbf{r}) \times \mathbf{B}] + \nabla \times [\tilde{\mathbf{v}}(\mathbf{r},\mathbf{t}) \times \mathbf{B}] + \frac{1}{\mu_0 \sigma} \nabla^2 \mathbf{B}$$

Two different approaches:

1) Try to avoid (large scale) fluctuations by using flows with strong geometrical constraints (Karlsruhe, Riga)

2) Study the effect of velocity fluctuations on the dynamo instability (VKS)

VKS experiment :

CEA, ENS-Lyon, ENS-Paris

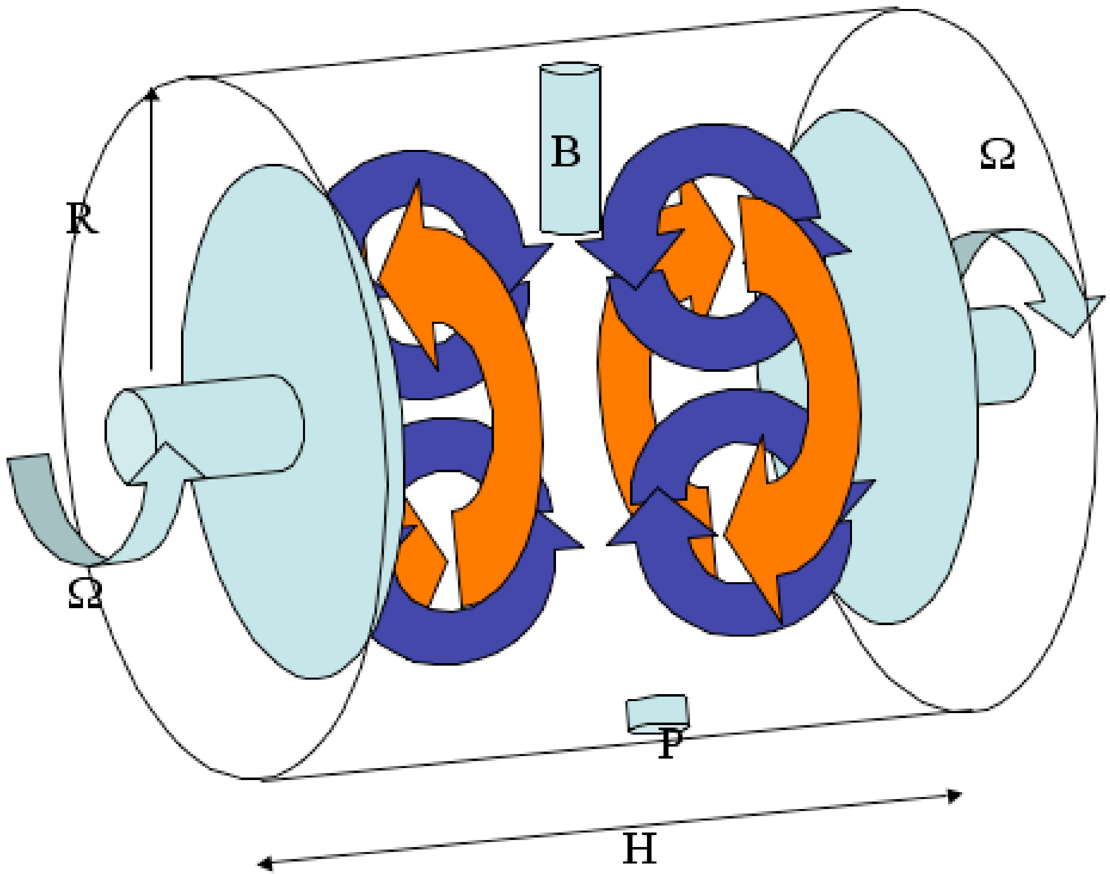
S. Aumaître, A. Chiffaudel, F. Daviaud, B. Dubrulle, R. Monchaux, F. Ravelet

M. Bourgoin, P. Odier, N. Plihon,, J. F. Pinton, R. Volk

M. Berhanu, S. Fauve, N. Mordant, F. Pétrélis

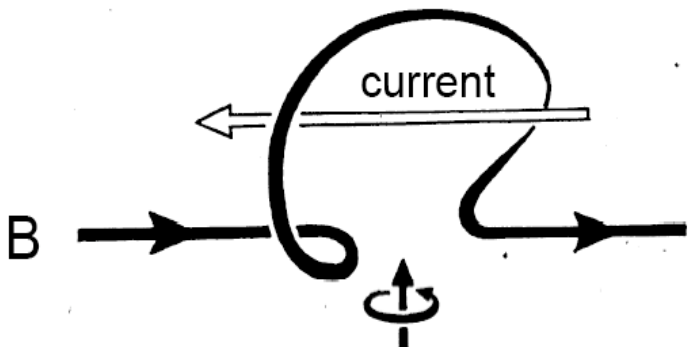
A « turbulent » dynamo ?
Motivations for the von Karman flow

- Strongly turbulent flow
- Differential rotation
- Helicity
- « Analogy » B - Ω
- Global rotation

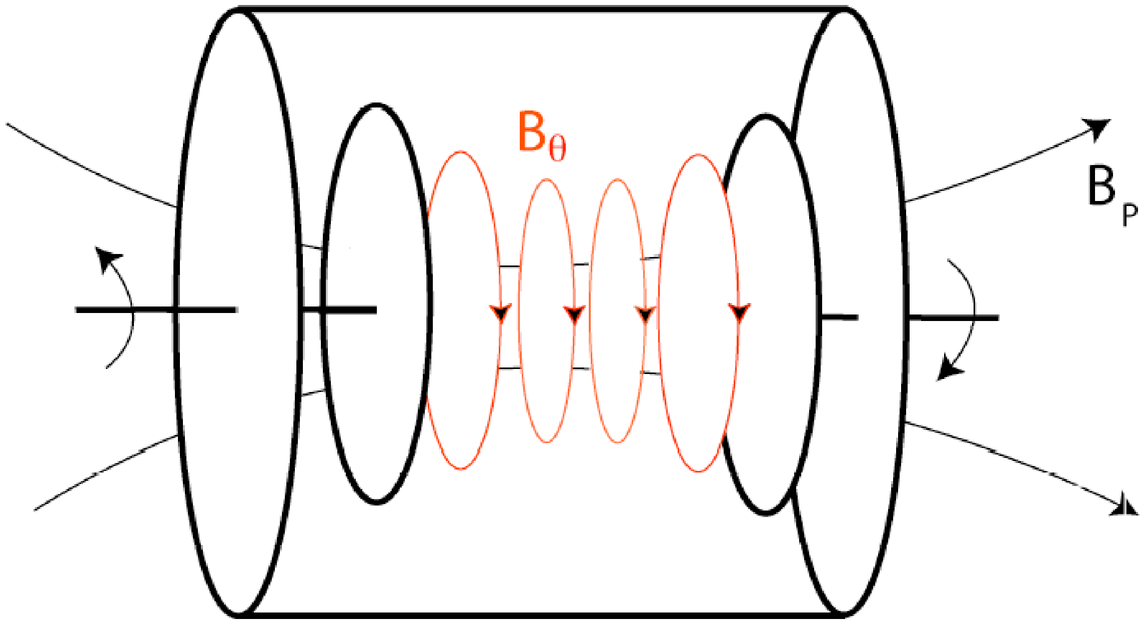


An instability from a strongly turbulent flow regime

Geometry of the mean magnetic field :
rôle of non axisymmetric velocity fluctuations



Magnetic induction by helical flows
« alpha effect » (Parker 1955)

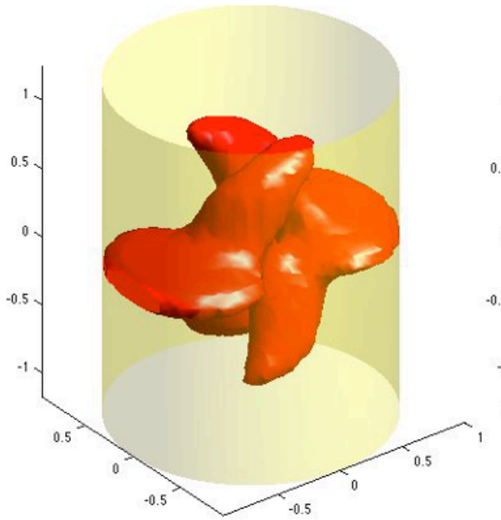


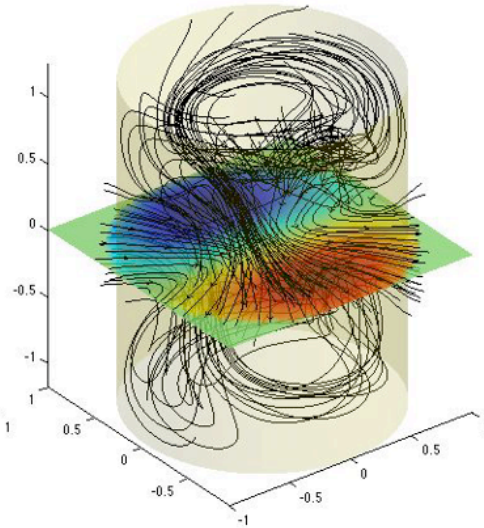
$Bp \rightarrow B\theta$
helicity or
differential rotation

$B\theta \rightarrow Bp$
helicity

Pétrélis et al., GAFD (2007)

Geometry of the magnetic field generated by the mean flow alone





Non axisymmetric fluctuations are averaged out in the mean flow

The magnetic field should break axisymmetry (Cowling, 1934)

Equatorial dipole (C. Gissinger, AGU 2007)
Marié et al. (2003), Bourgoin et al. (2004)
Similar to the predictions for Madison dynamo experiment

The VKS dynamo is not generated as if the mean flow were acting alone

Effect of turbulent fluctuations on the threshold of the dynamo generated by <V>

- No threshold shift to first order in the fluctuations
- Strong increase of Ohmic losses with large scale fluctuations

$$
\begin{aligned}
|\hat{B}|^2 &= A(k/k_\sigma)^\alpha && \text{si } k/k_\sigma < 1 \\
&= A(k/k_\sigma)^{-11/3} && \text{si } k/k_\sigma > 1,
\end{aligned}
$$

$$
\begin{aligned}
\frac{1}{\sigma}\langle j^2\rangle &\propto \frac{\langle B\rangle^2}{\mu_0^2\sigma L^2} Rm^{(9+3\alpha)/4} && \text{si } \alpha > -3, \\
&\propto \frac{\langle B\rangle^2}{\mu_0^2\sigma L^2} && \text{si } \alpha < -3,
\end{aligned}
$$

Dissipation joule augmentée par un facteur

-$R_m^{5/2}$ si $\alpha = 1/3$

-$R_m^{3/2}$ si $\alpha = -1$

By-passing the Cowling theorem

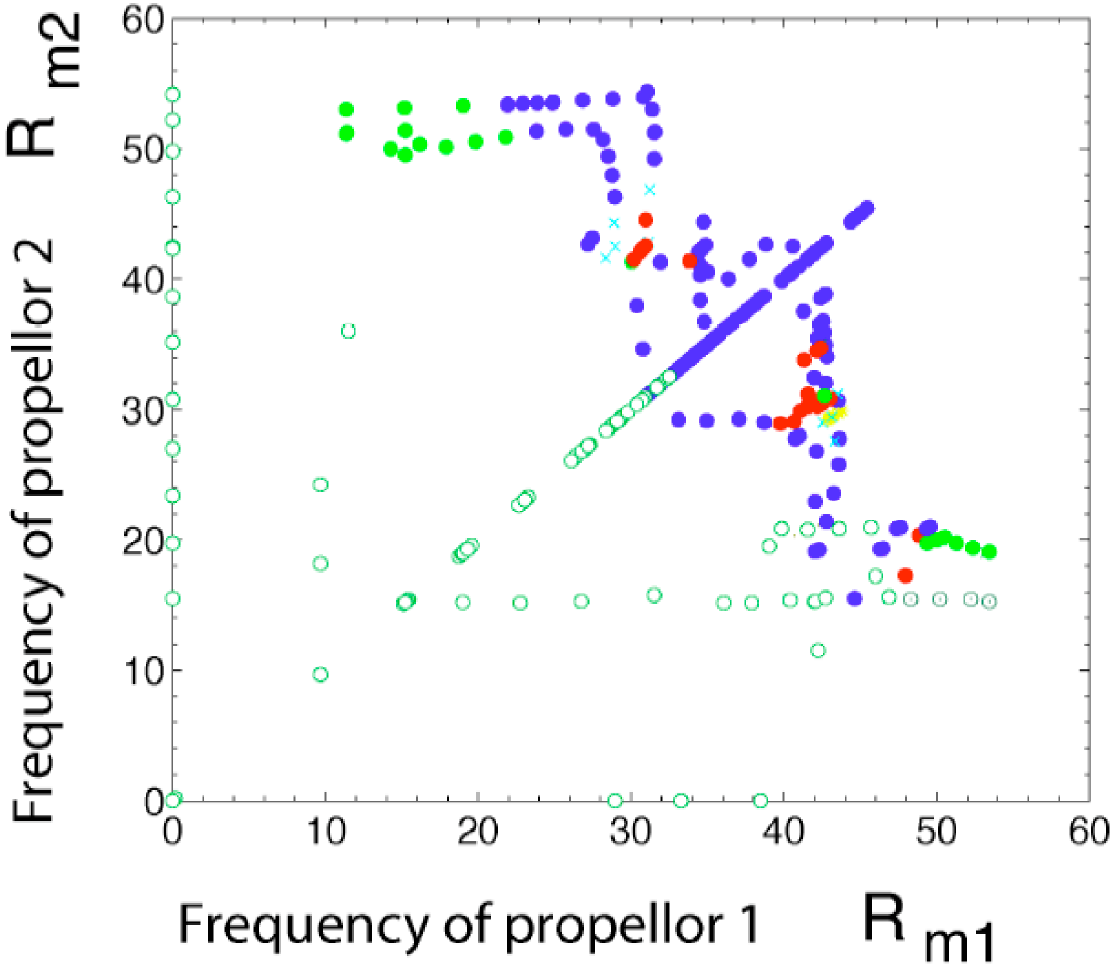
- Very often, fluctations delay the bifurcation threshold because they favor the disordered phase
- This is much more efficient when there exists a neutral mode due to broken axisymmetry
- Small turbulent fluctuations are enough to kill the equatorial dipole.

Even without velocity fluctuations, the equatorial dipole bifurcates to an axisymmetric dipole via a secondary bifurcation

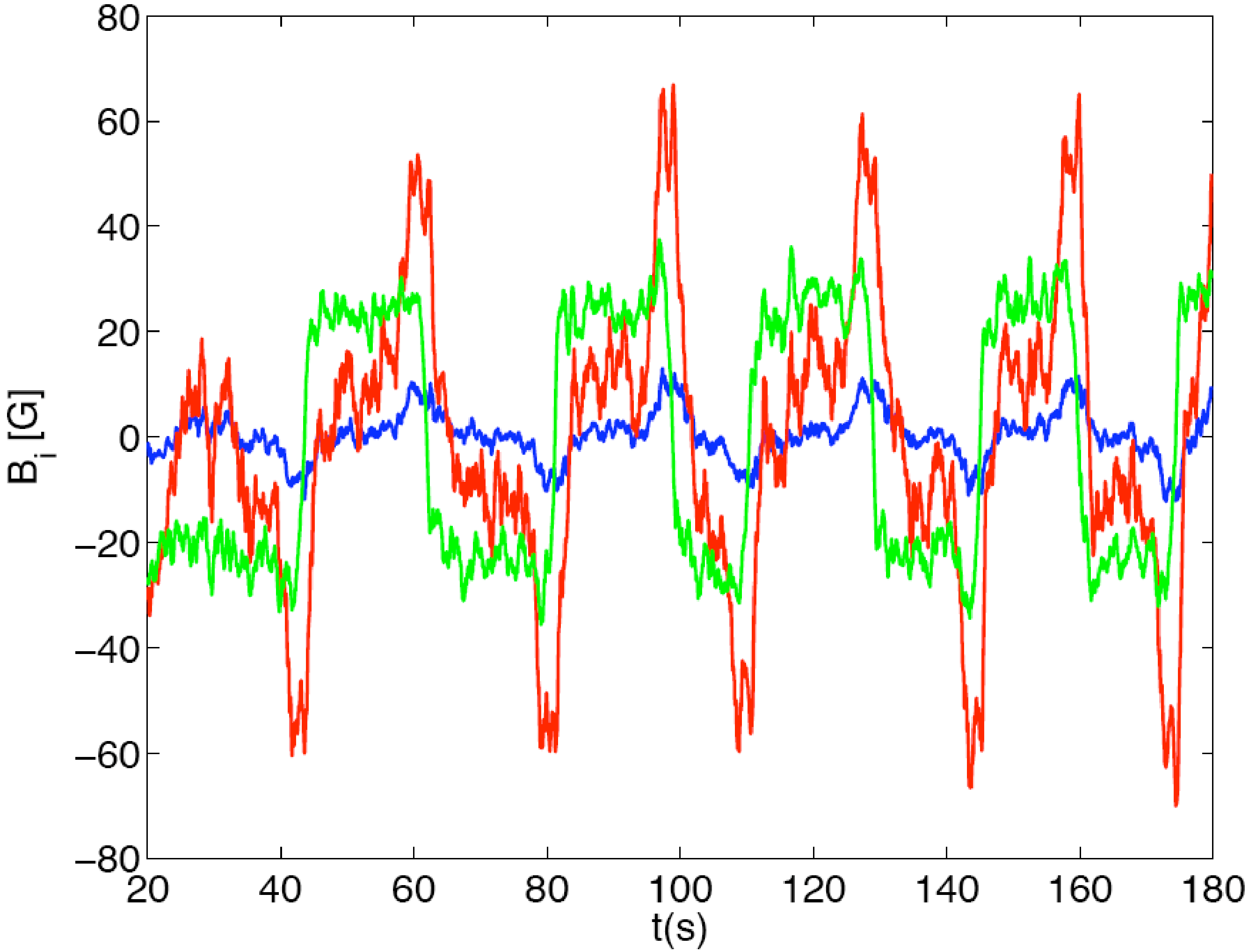
(Gissinger et al. AGU 2007)

Dynamics in parameter space

- Stationary dynamos
- Oscillatory dynamos
- Intermittent dynamos and field reversals

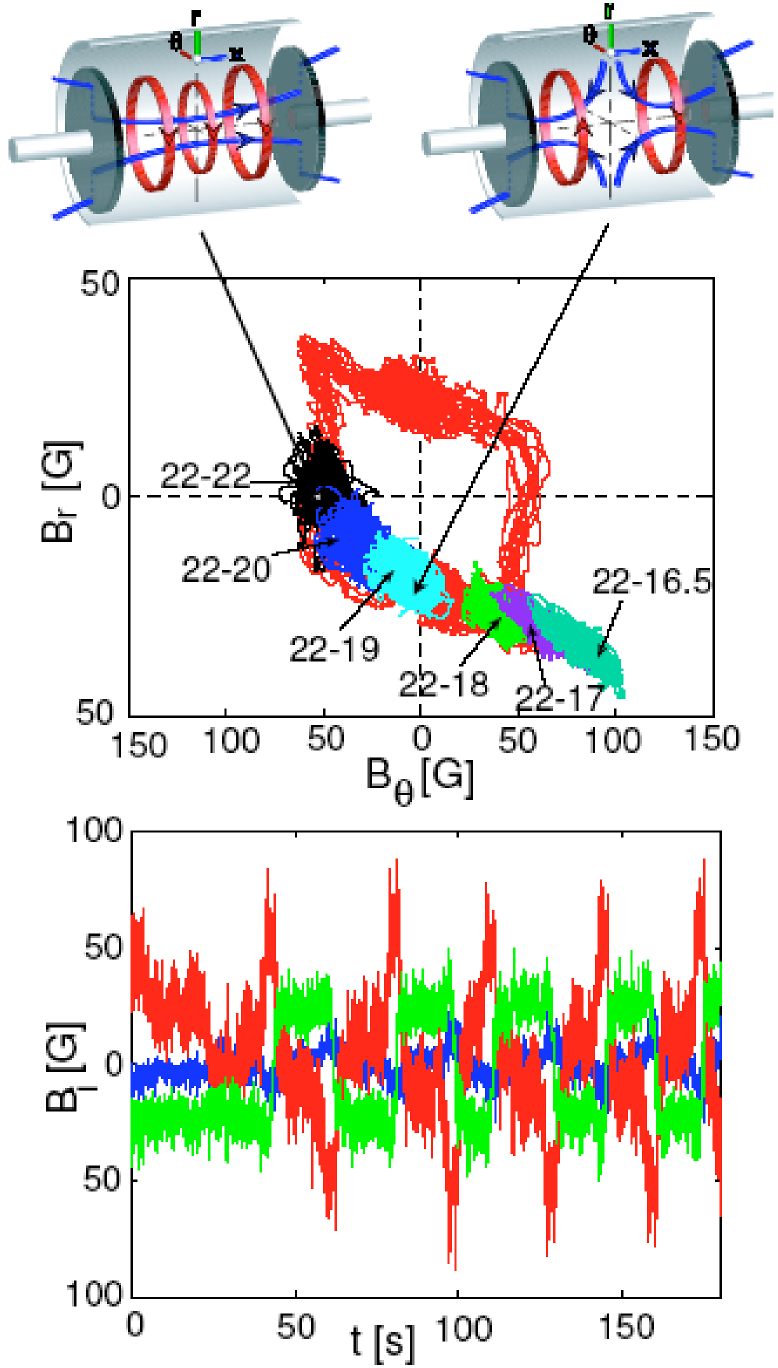


From stationary to time dependent dynamos: A relaxation oscillator

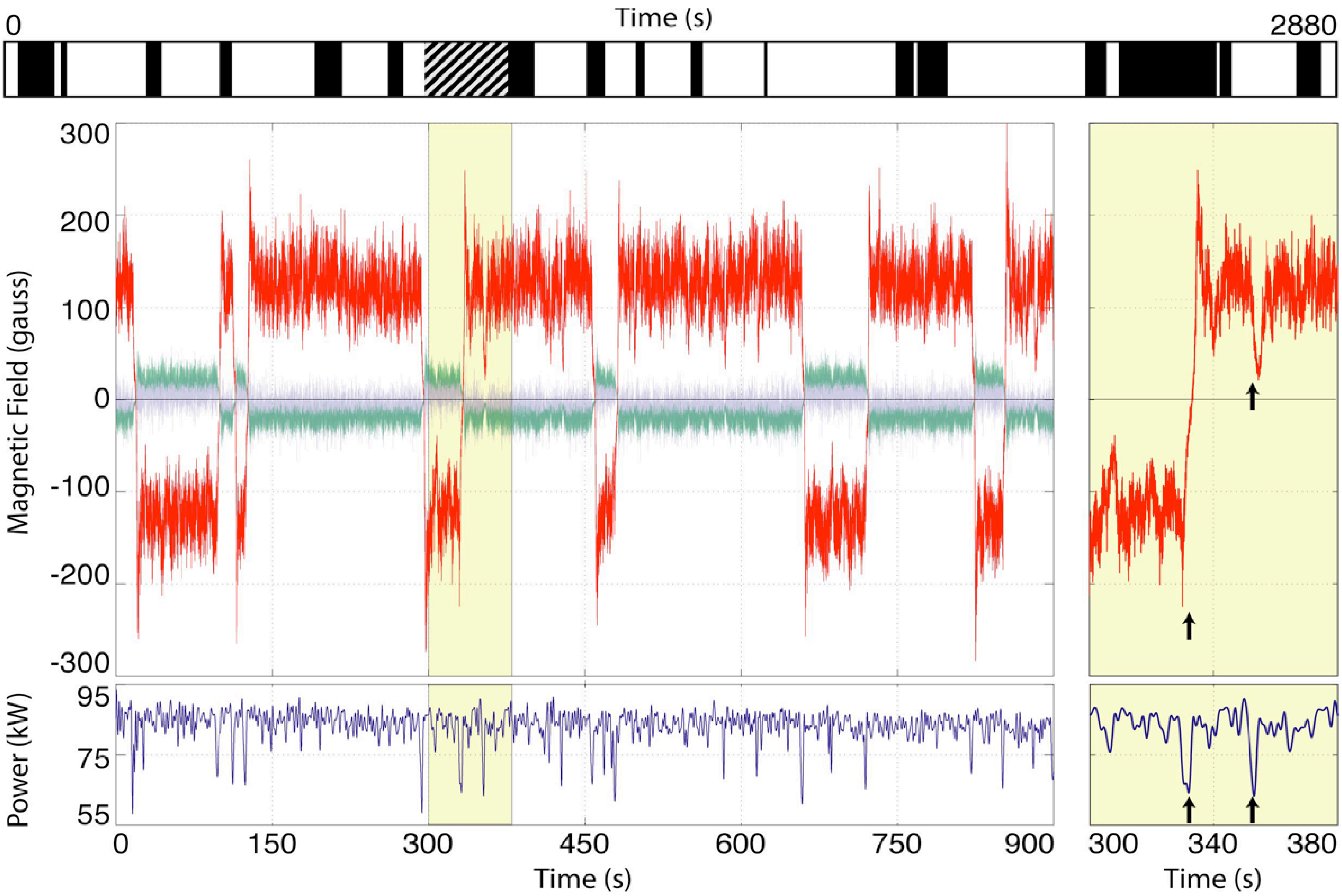


Excitability

The relaxation oscillation for f1=22 Hz, f2=18.5 Hz bifurcates from fixed points located on the limit cycle as in the case of anexcitable system (for instance, a pendulum submitted to a constant torque)

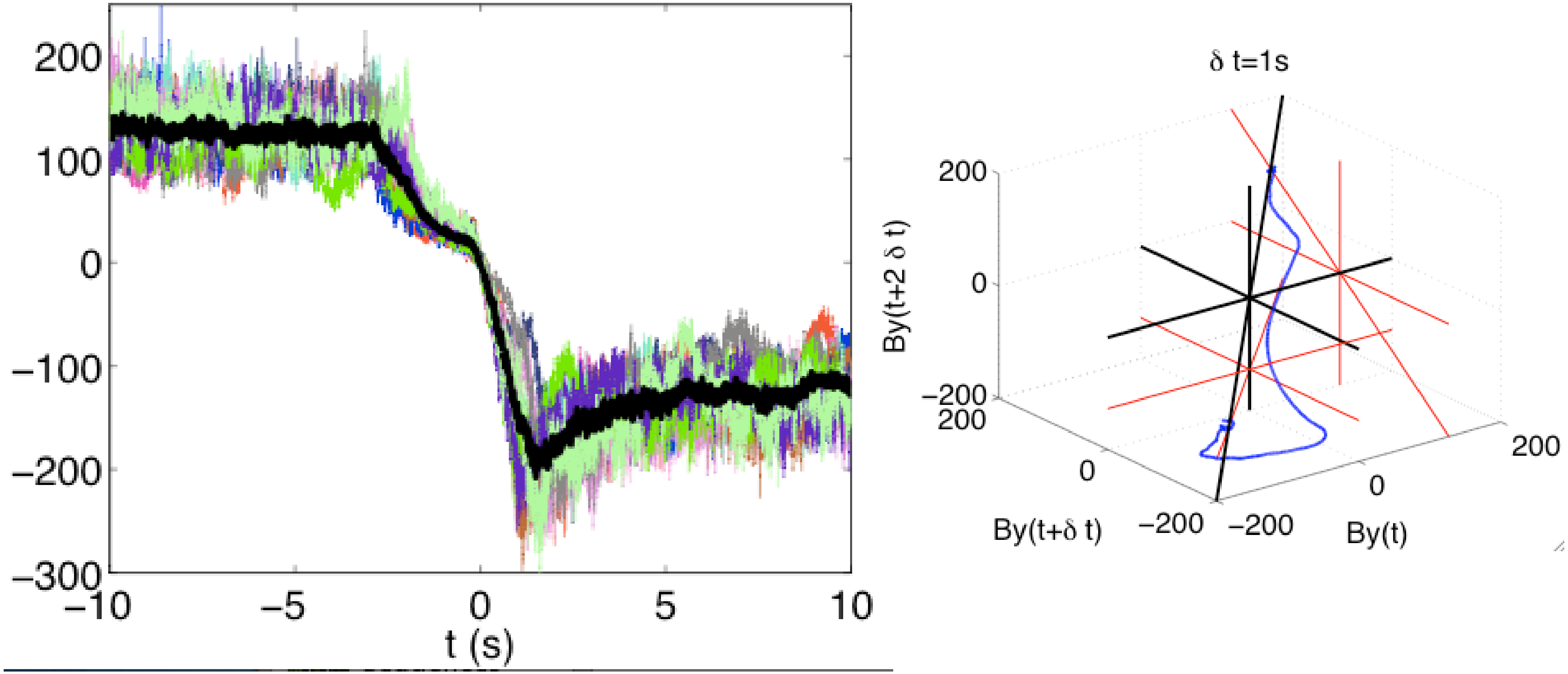


Reversals of the magnetic field



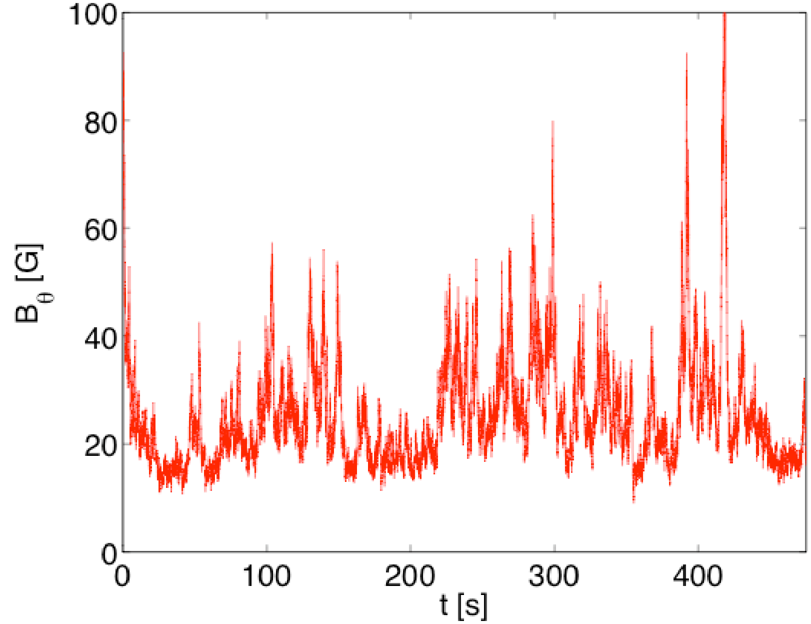
Robustness of reversals of the magnetic field with respect to turbulent fluctuations

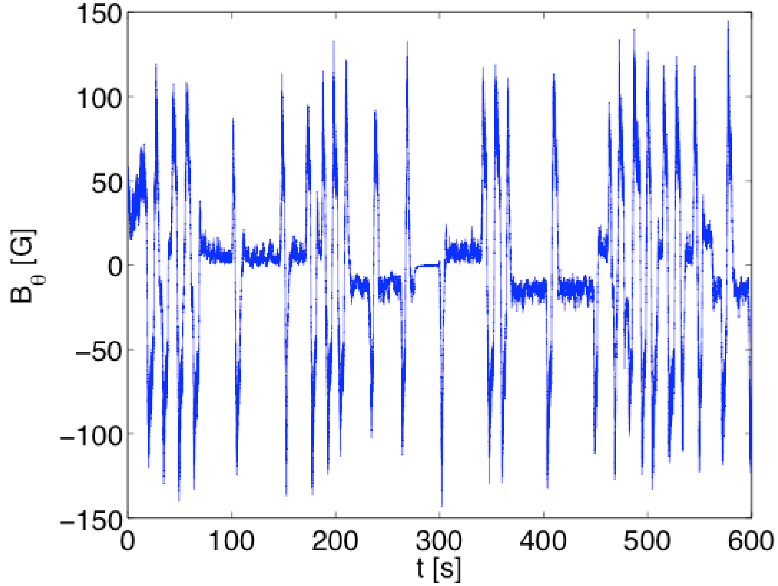
12 superimposed reversals (slow decay, fast recovery, overshoots)

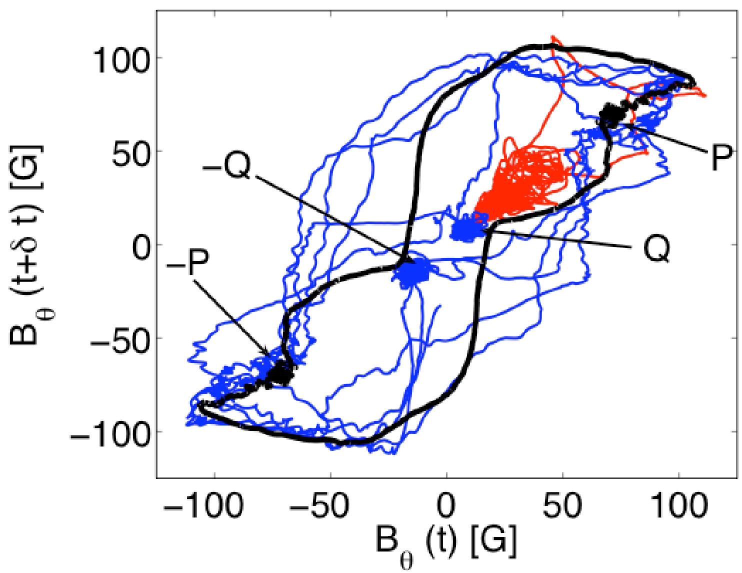


Berhanu et al., Europhysics Letters 77, 59001 (2007)

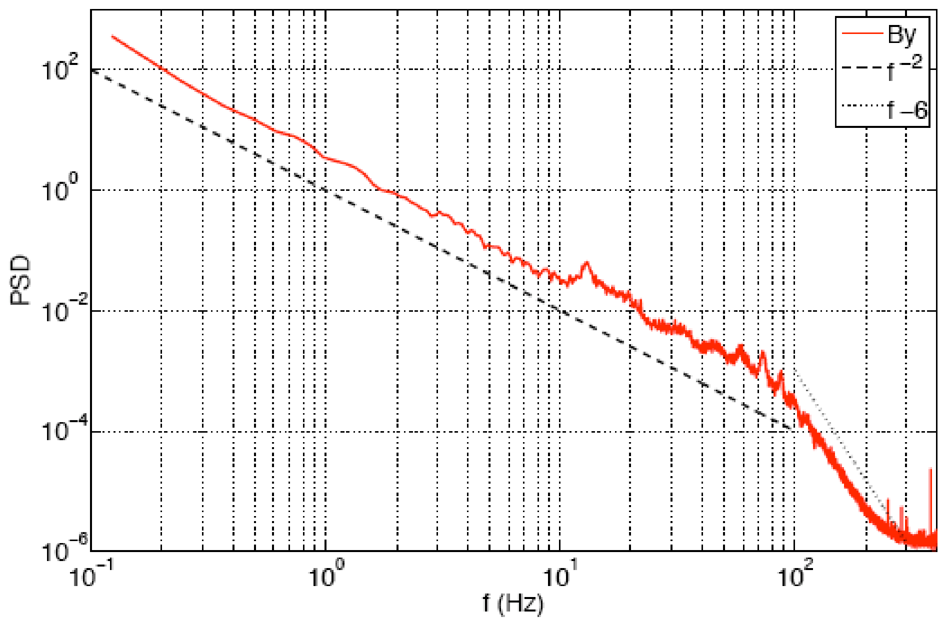
Bursts and reversals



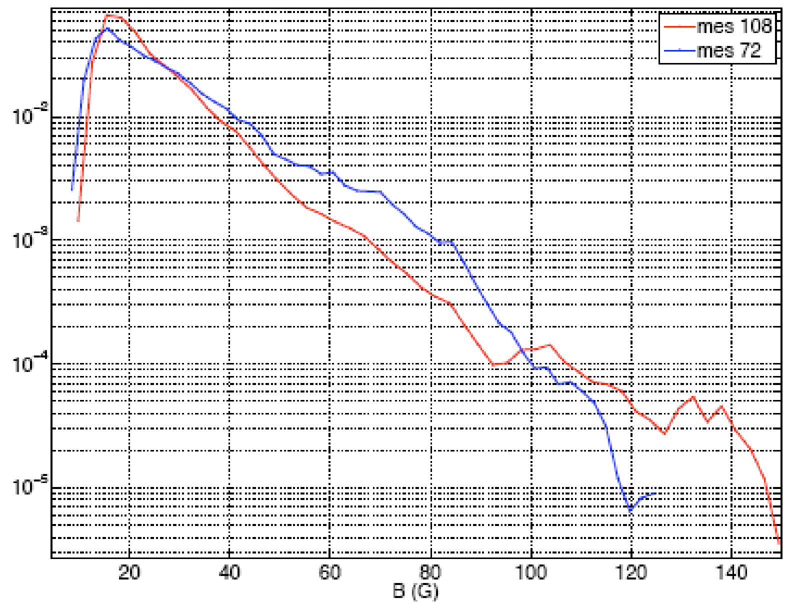




Statistical properties of the bursts

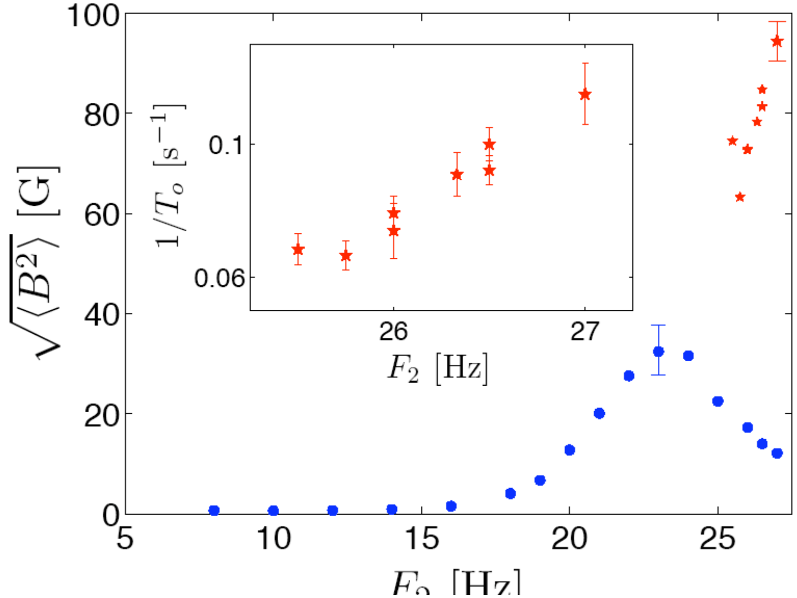


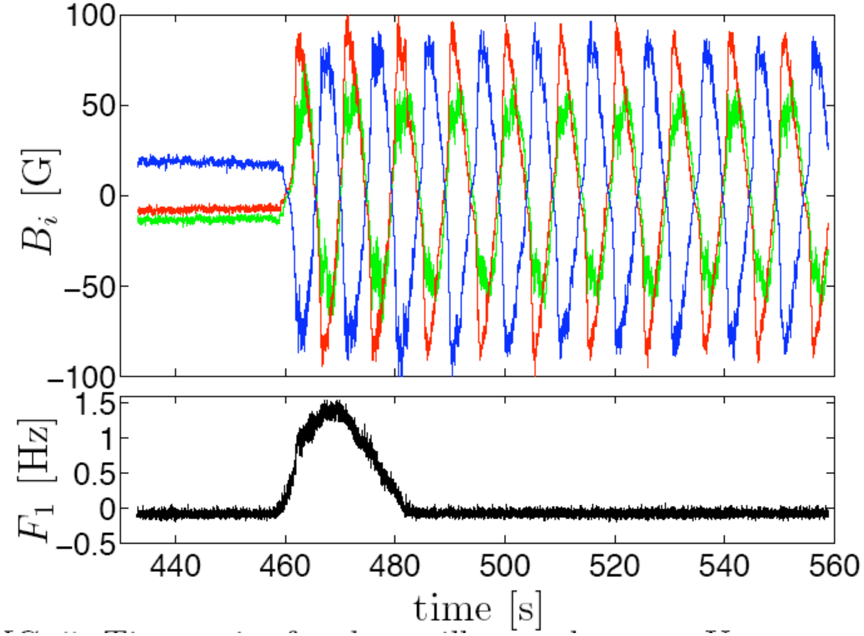
An f^{-2} power spectrum

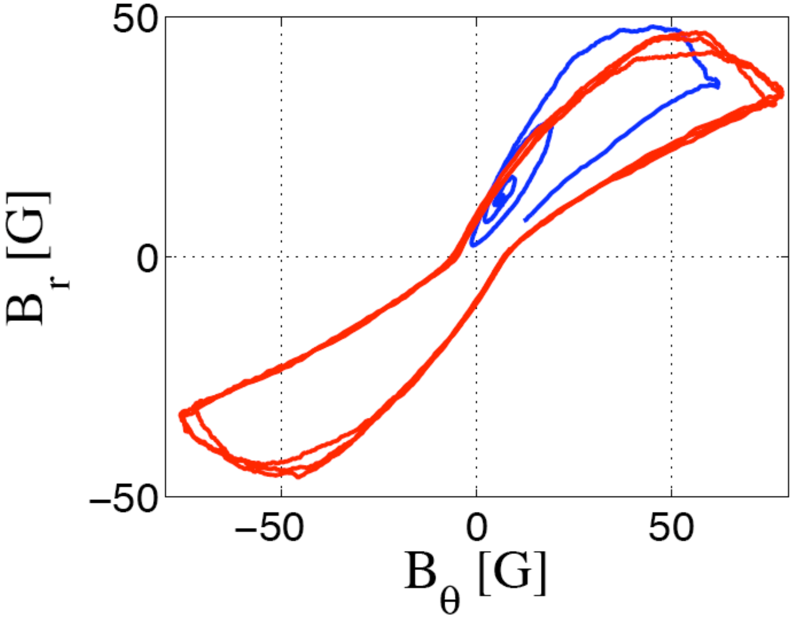


An exponential probability density function

Subcritical oscillatory dynamos and bistability







No transitions between the two metastable regimes generated by turbulent fluctuations on observable time scales

Conclusions

- VKS dynamo not generated by the mean flow alone
- Good agreement for the scaling of the magnetic field
- Many different regimes in a small parameter range
- Large scale dynamics of the field
  - governed by a few modes
  - not smeared out by turbulent fluctuations
- Reversals result from the competition between different modes (no need any external triggering mechanism)