Turbulence models: Local vs non local Mean vs fluctuations

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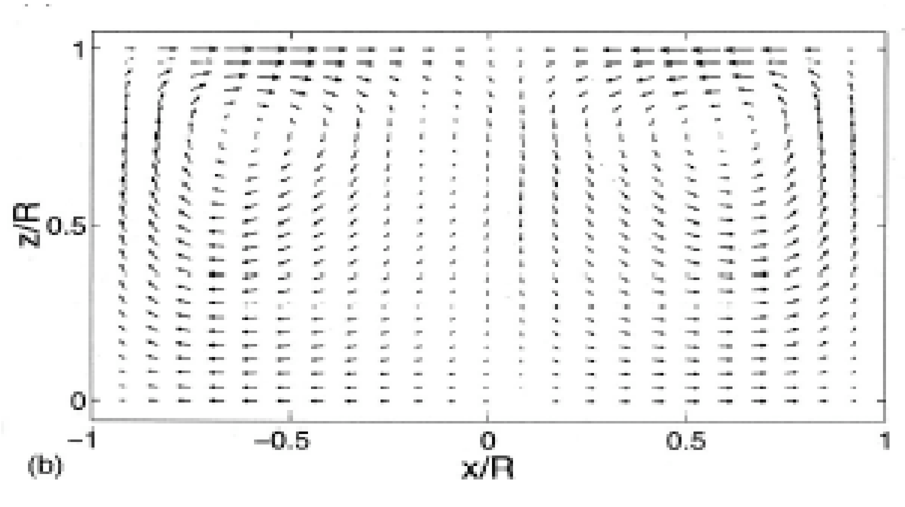
Turbulences

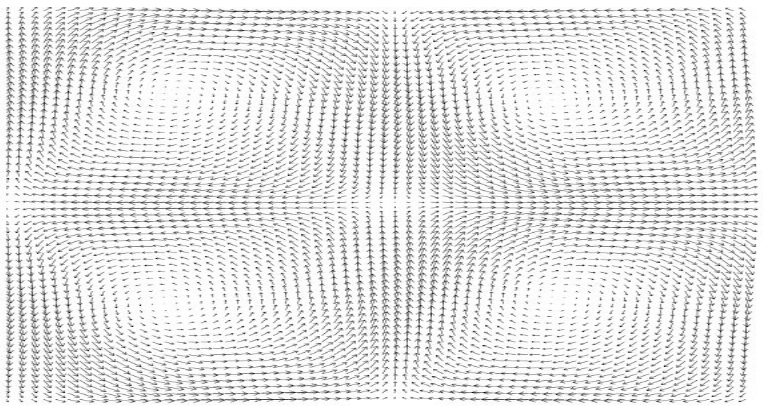
$$\partial_t u + u \bullet \nabla u = -\nabla p + \nu \Delta u + f$$

$$f = (\sin x \cos y \cos z, -\cos x \sin y \cos z, 0)$$

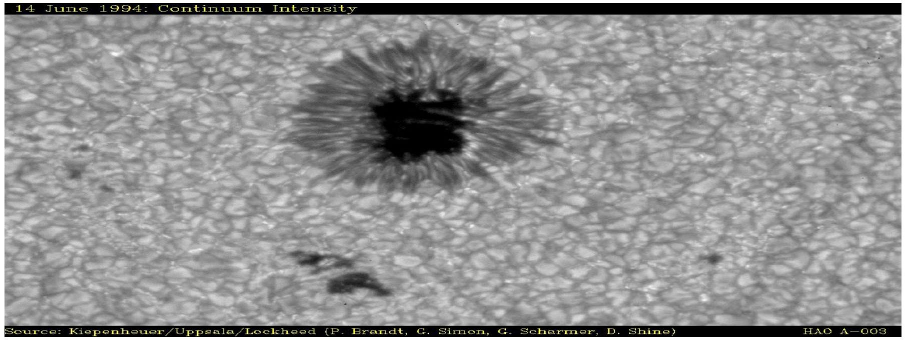
Von Karman $\mathrm{Re} = 10^6$

Taylor-Green flow $\mathrm{Re} = 10^3$





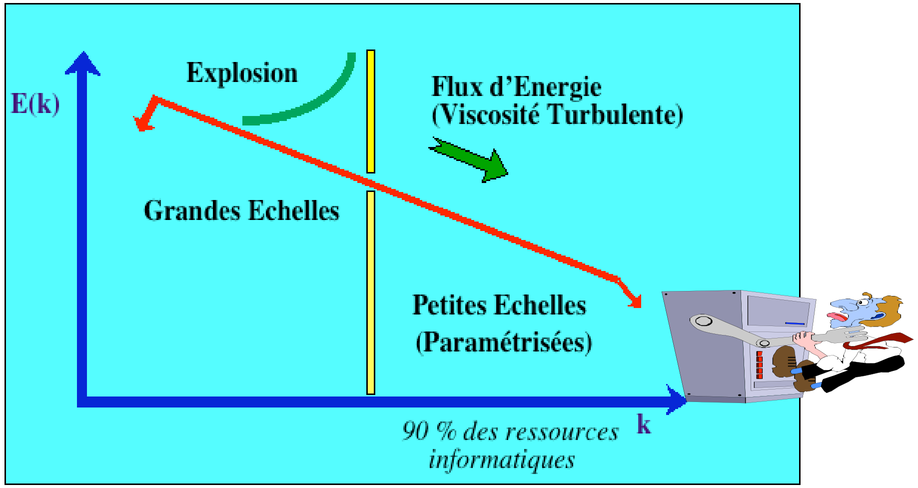
IS IT SUFFICIENT TO KNOW BASIC EQUATIONS?



Dissipation scale	Granule	Solar spot	Giant convection cell
0.1 km	$10^3\,km$	$3\cdot 10^4\,km$	$2\cdot 10^5\,km$

Waste of computational resources
Time-scale problem

Necessity of parametrization



Mathematics

$$\partial_t u_i + u_j \nabla_j u_i = -\nabla_i p + \frac{1}{\mathrm{Re}} \Delta u_i + f_i$$

$$u = \overline{u} + u'$$

- Spatial filter: LES
- Statistical average: RANS

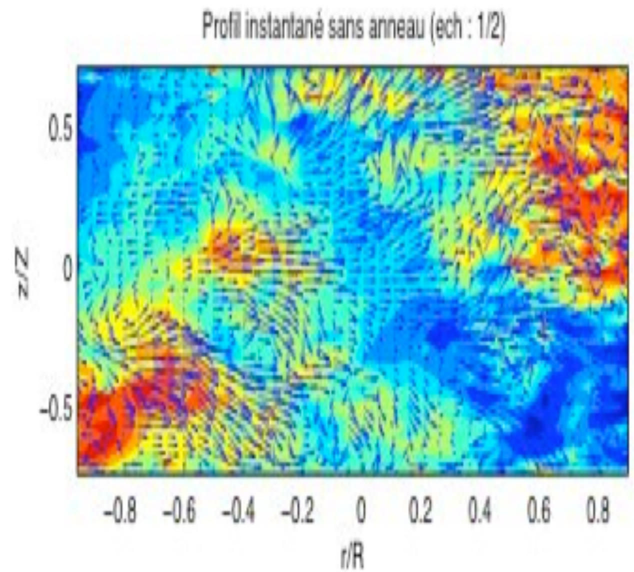
$$\partial_t \overline{u}_i + \overline{u}_j \nabla_j \overline{u}_i = -\nabla_i \overline{p} + \frac{1}{\mathrm{Re}} \Delta \overline{u}_i + \bar{f}_i - \nabla_j \tau_{ij}$$

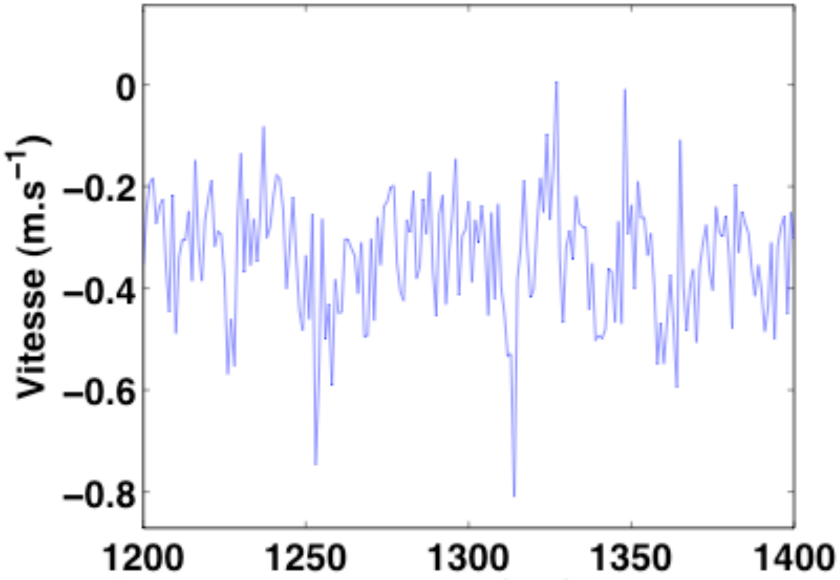
Reynolds stress

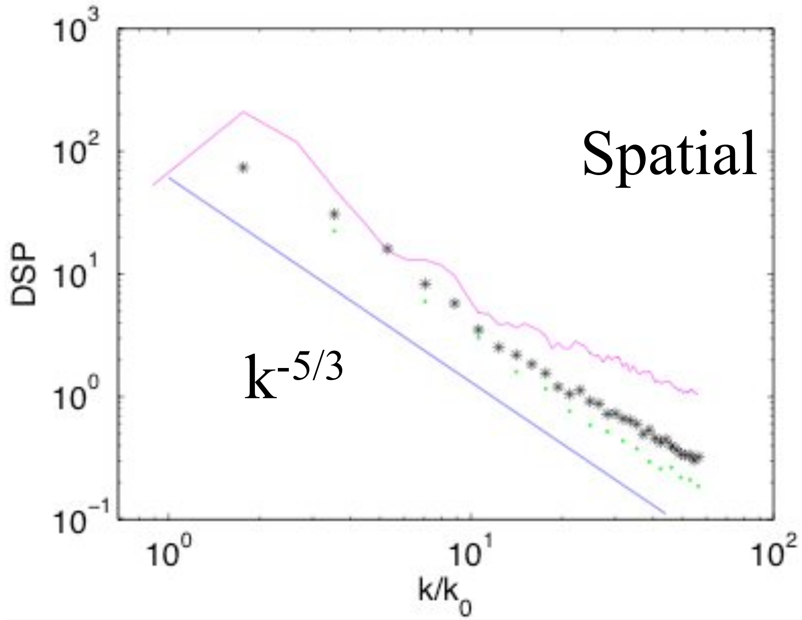
$$\tau_{ij} = \overline{\overline{u}_i \overline{u}_j} - \overline{u}_i \overline{u}_j + \overline{\overline{u}_i u'_j} + \overline{u'_i \overline{u}_j} + \overline{u'_i u'_j}$$ LES

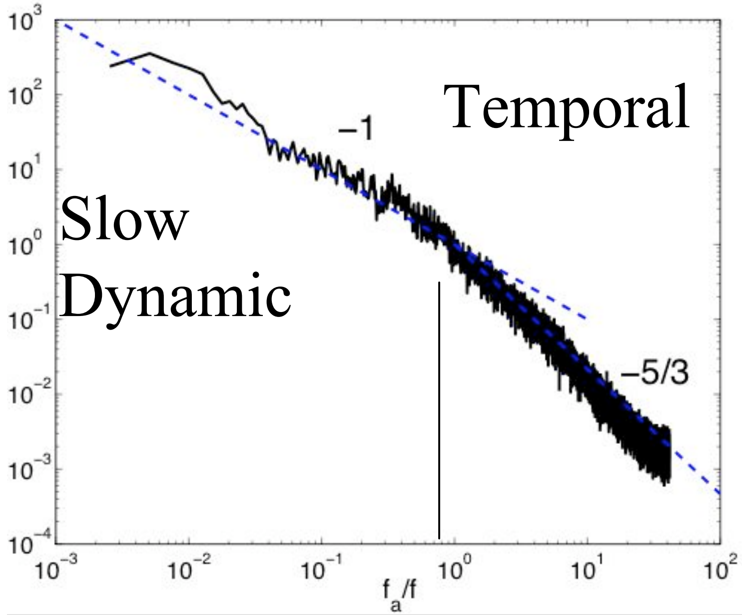
$$\tau_{ij} = +\overline{u'_i u'_j}$$ RANS

Spectra and scales

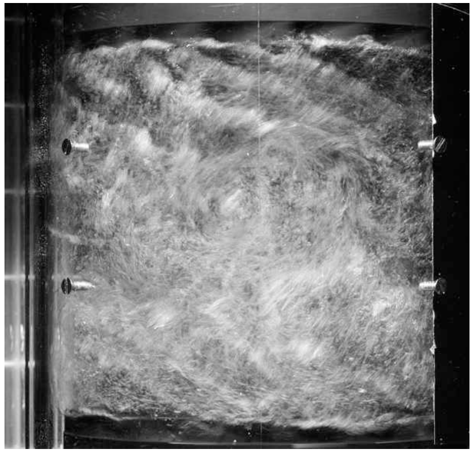


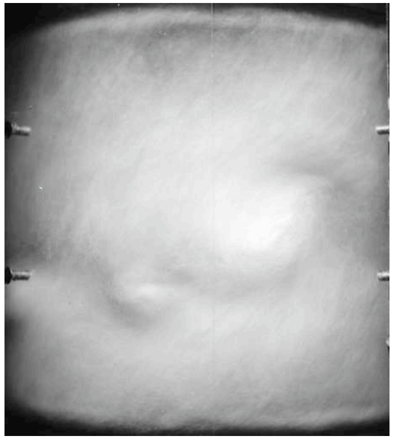


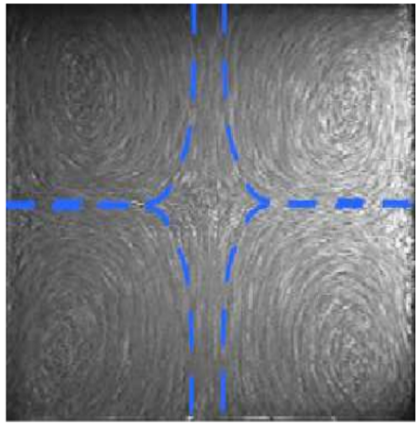




Components of turbulence





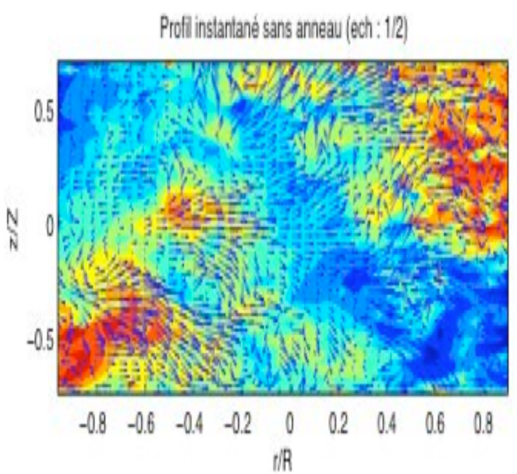


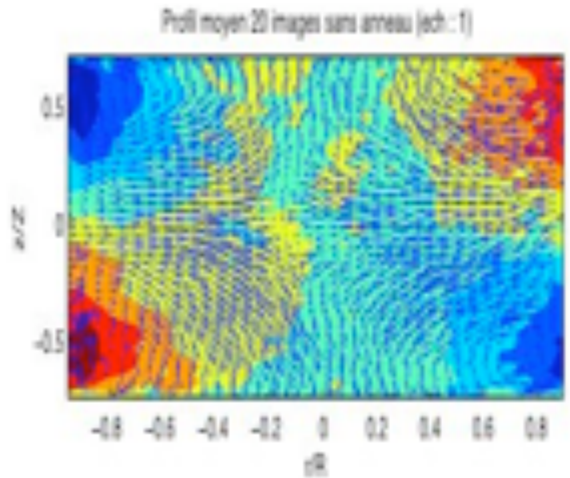
(a)

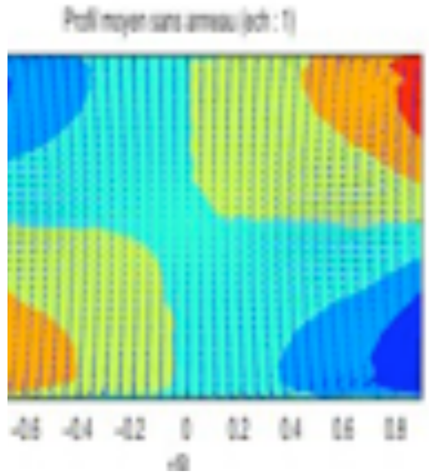
instantaneous

Average over 20

Average over 5000



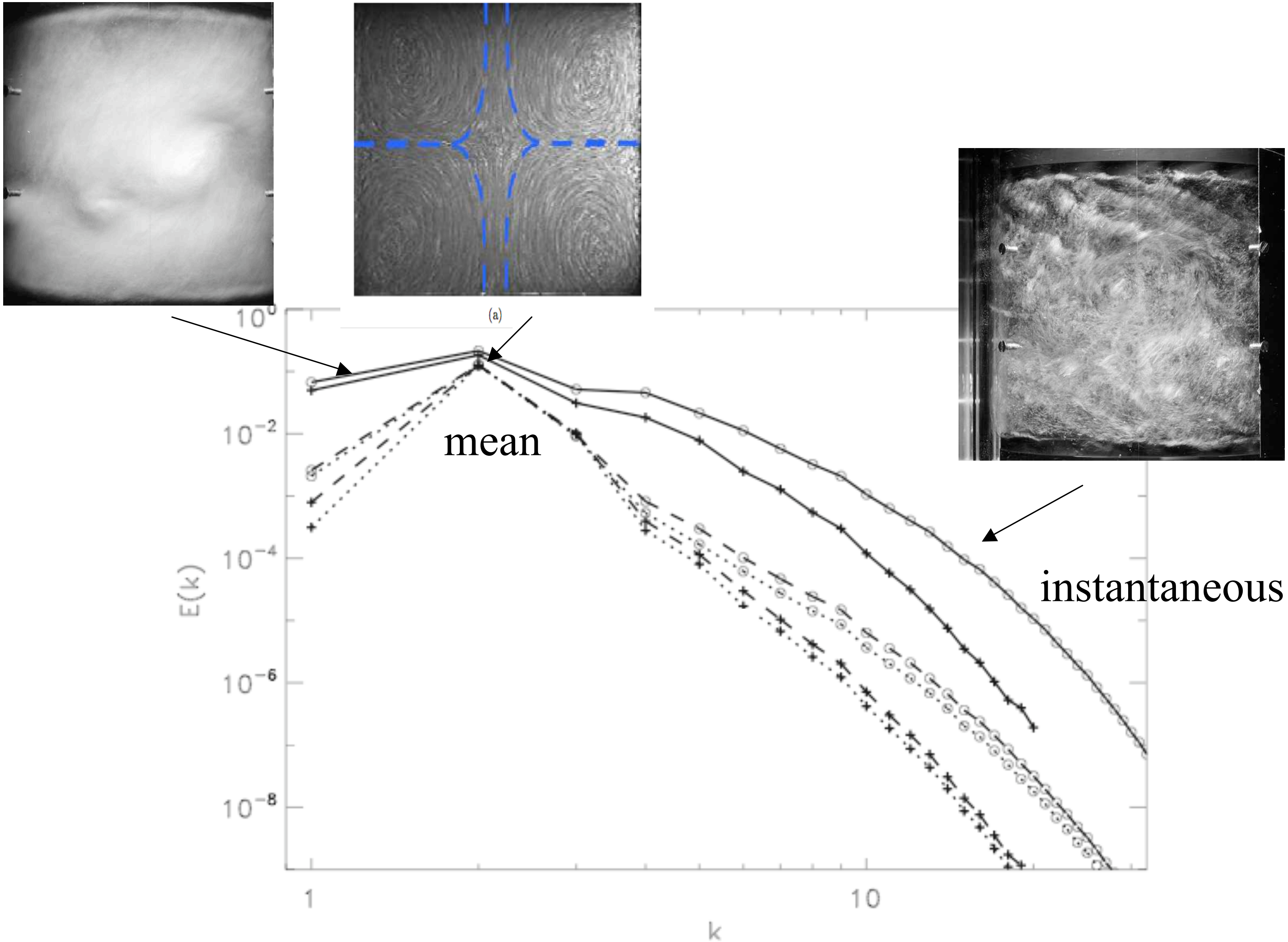




Small scale fluctuations

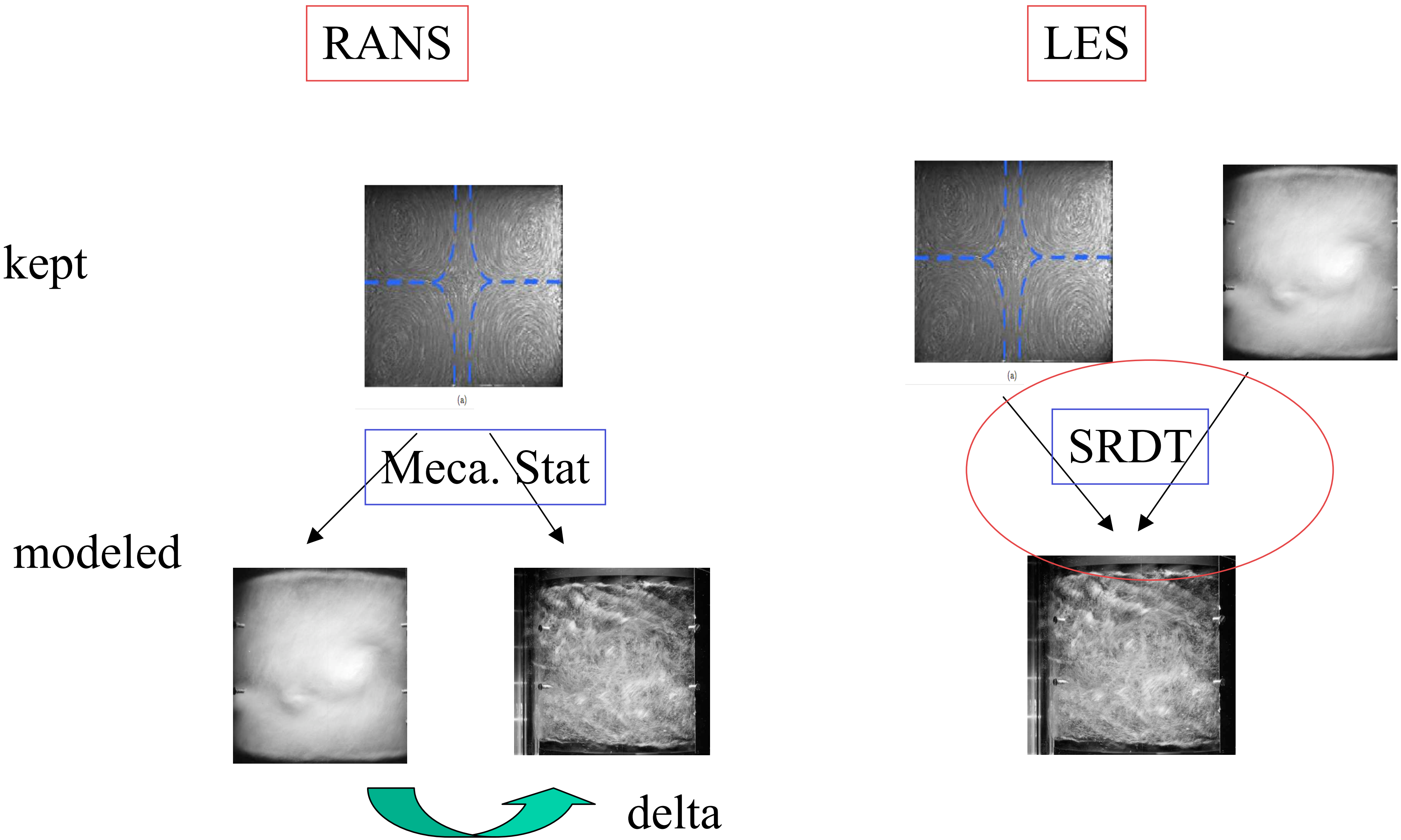
Large scale fluctuations

Mean flow



(a)
mean
instantaneous
E(k)
k
10^0
10^{-2}
10^{-4}
10^{-6}
10^{-8}
1
10

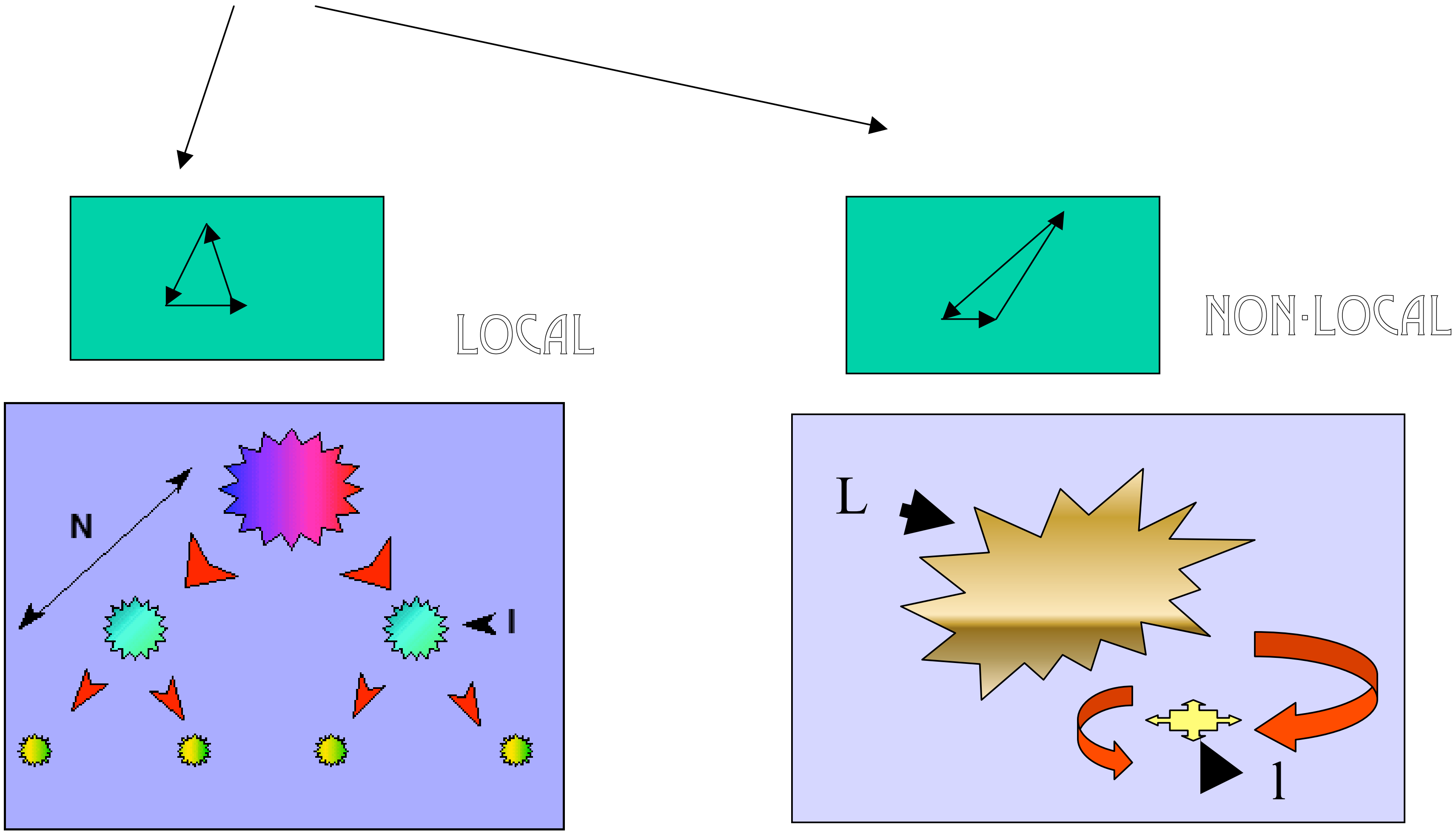
Summary



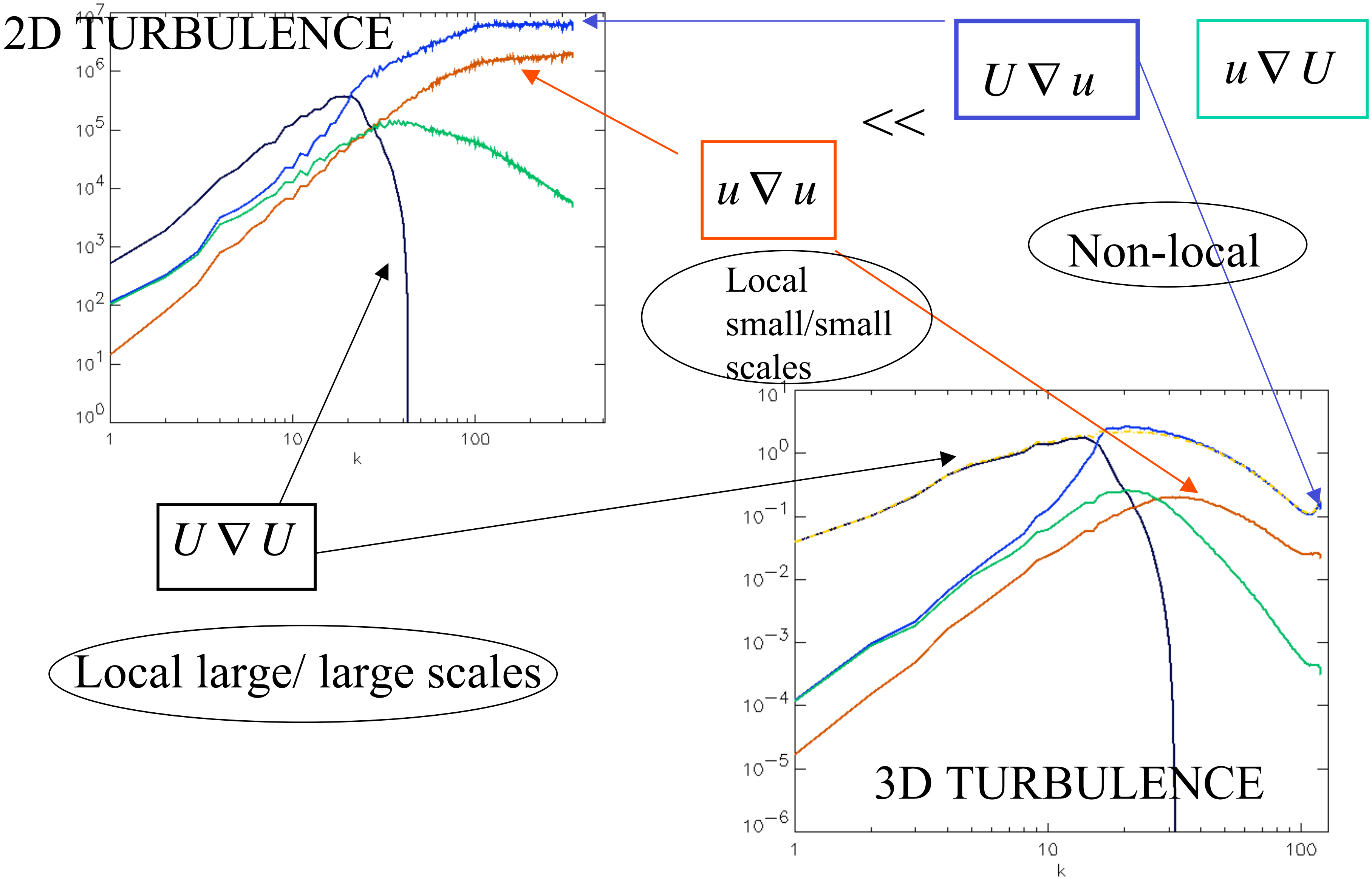
LOCAL VS NON-LOCAL INTERACTIONS

- Navier-Stokes equations : two types of triades

$$\partial_t u + u \bullet \nabla u = - \nabla p + \nu \Delta u + f$$

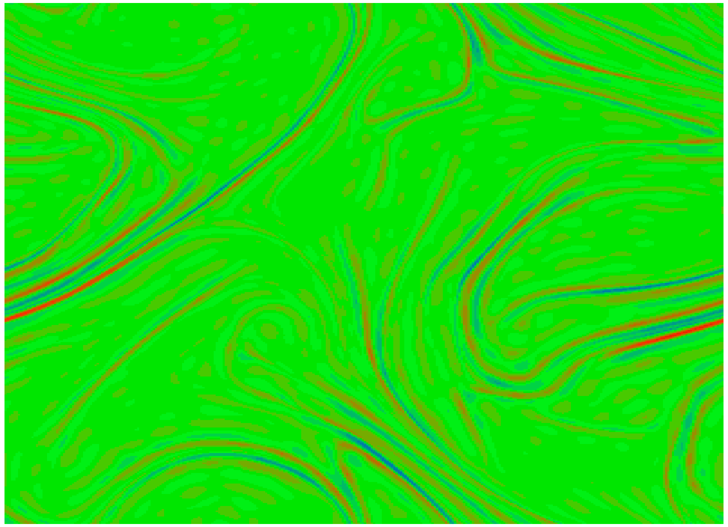


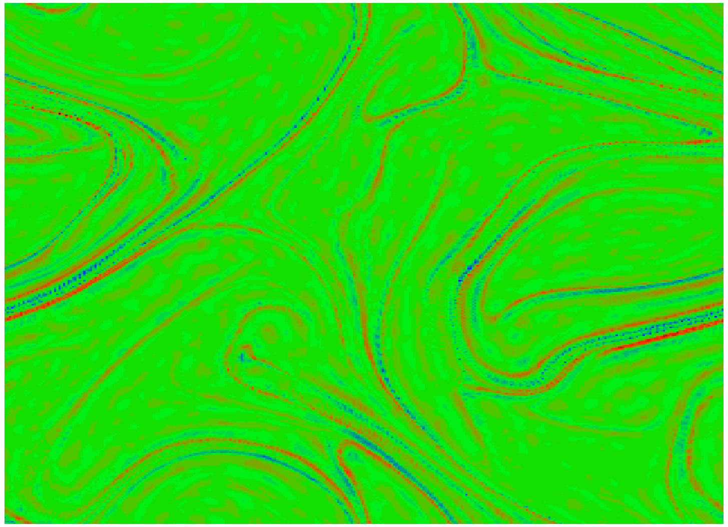
A PRIORI TESTS IN NUMERICAL SIMULATIONS



DYNAMICAL TESTS IN NUMERICAL SIMULATIONS

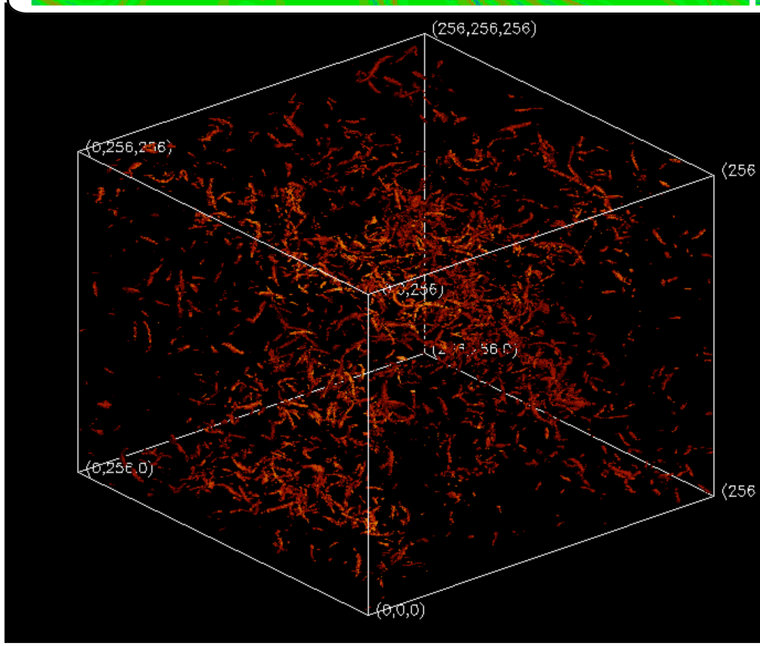
2D
DNS

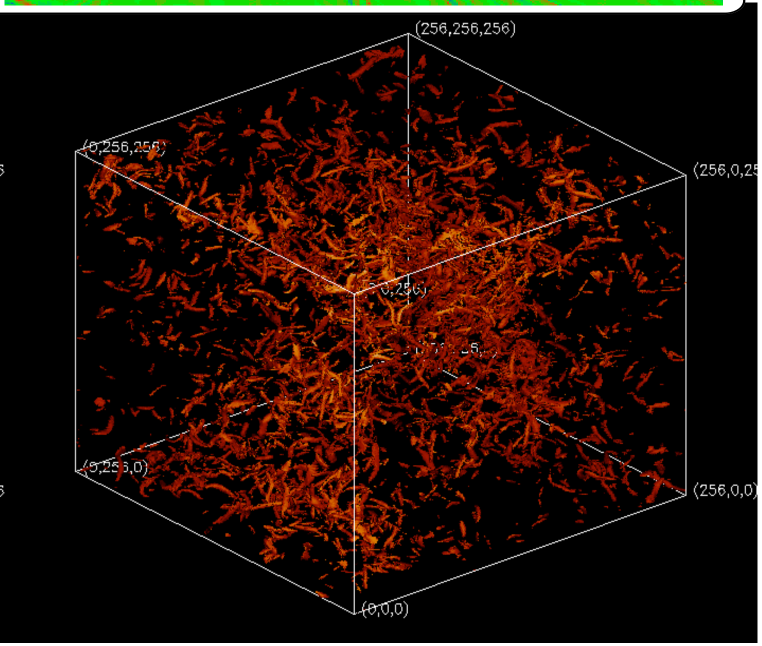




2D
RDT

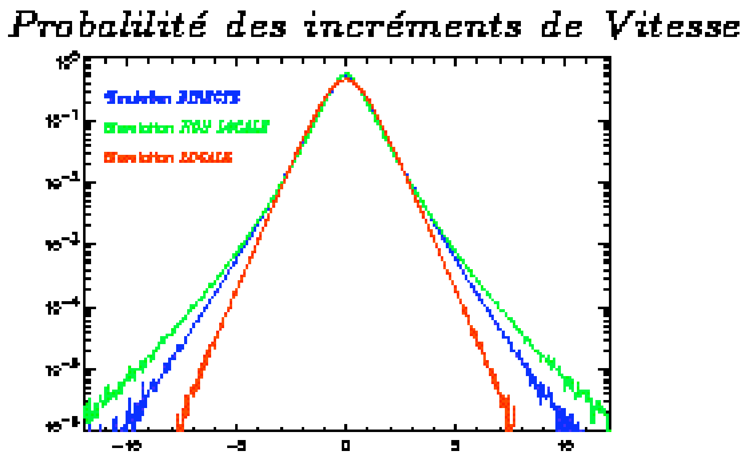
3D
DNS

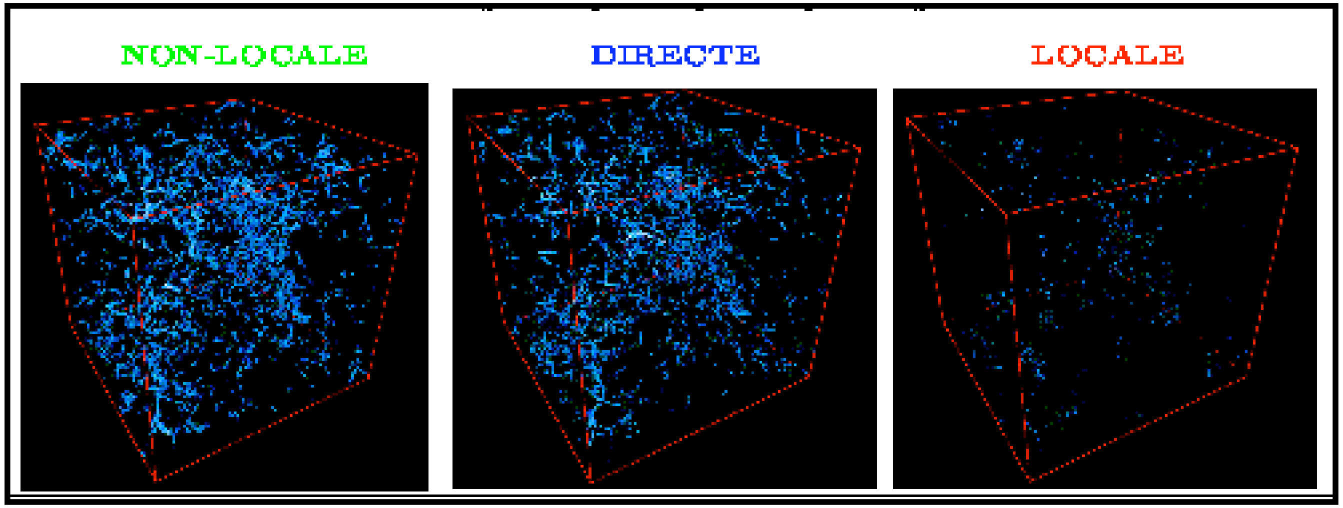




3D
RDT

LOCAL VS NON-LOCAL TURBULENCE





THE RDT MODEL

Equation for large-scale velocity

$$\partial_t U_i + U_j \nabla_j U_i = -\nabla_i P + \nu \Delta U_i - \nabla_j \left(\overline{u_i U_j + u_j U_i + u_i u_j} \right)$$

Linear stochastic inhomogeneous equation (RDT)

Reynolds stresses

Equation for small scale velocity

$$\partial_t u_i + U_j \nabla_j u_i = -u_j \nabla_j U_i - \nabla_i p + \nu_t \Delta u_i + f_i$$

Turbulent viscosity

Forcing (energy cascade)

Multiplicative Noise+friction

Friction

Additive noise

Qualitative explanation of (small scale) intermittency

Approximation: Gaussians, delta-correlated noises

$$\partial_t P_k = \partial_u\left(\nu_T k^2 u P_k\right) + D\partial_u u \partial_u u P_k - \lambda \partial_u u \partial_u P_k - \lambda \partial_u^2 u P_k + \alpha \partial_u^2 P_k$$

- friction — regularization (large/small scale)
- multiplicative noise — Log-normal
- coupling — skewness
- additive noise — Gaussian

$$P_k = C_k \exp \int_0^u \frac{-\nu_T k^2 y - Dy + \lambda}{Dy^2 - 2\lambda y + \alpha} dy$$

$u << k\nu$

$$P = \frac{C}{\left(Du^2 - 2\lambda u + \alpha\right)^{1/2 + \nu k^2 / 2D}}$$

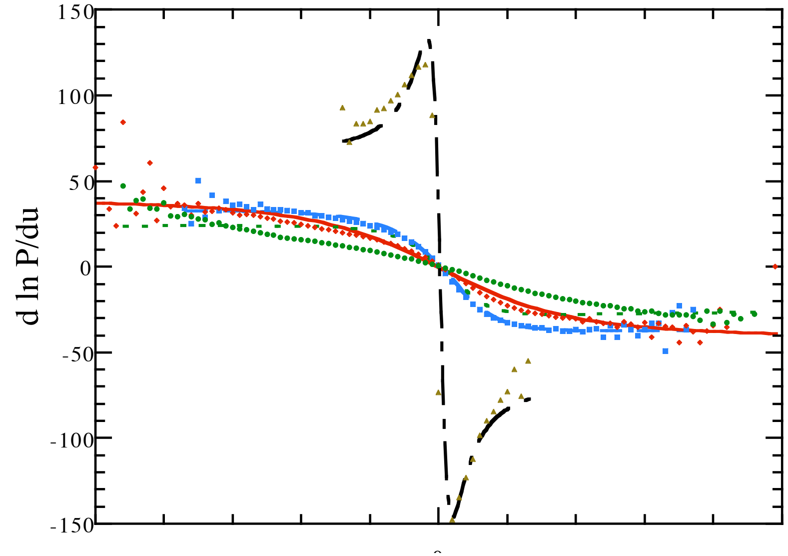
$u >> k\nu$

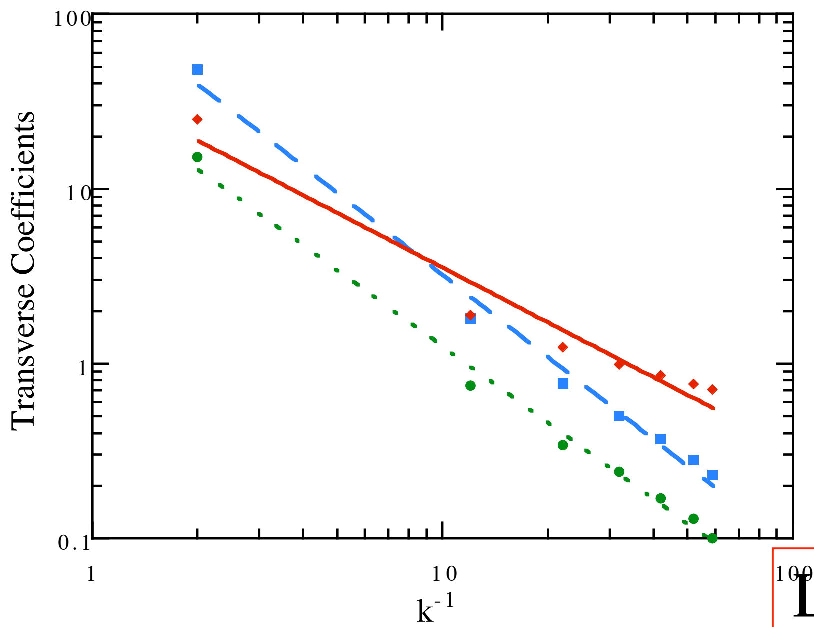
$$\frac{d \ln P}{du} = \frac{-B|u|u}{Du^2 - 2\lambda u + \alpha}$$

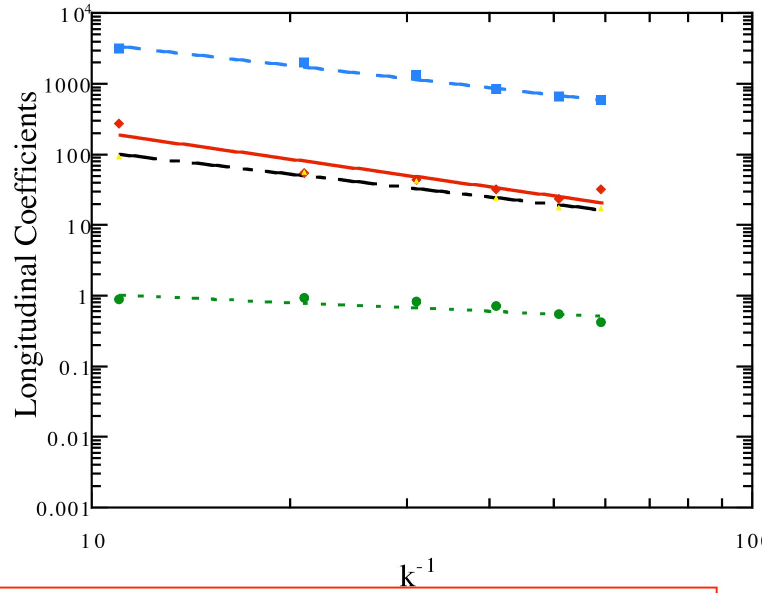
CHECK IN(NUMERICAL) HOMOGENEOUS TURBULENCE

$$\frac{d\ln P}{du} = \frac{-u\sqrt{(m_1 u)^2 + m_2^2} - m_4 u + m_3}{m_4 u^2 - 2m_3 u + 1}$$

4 parameter fit

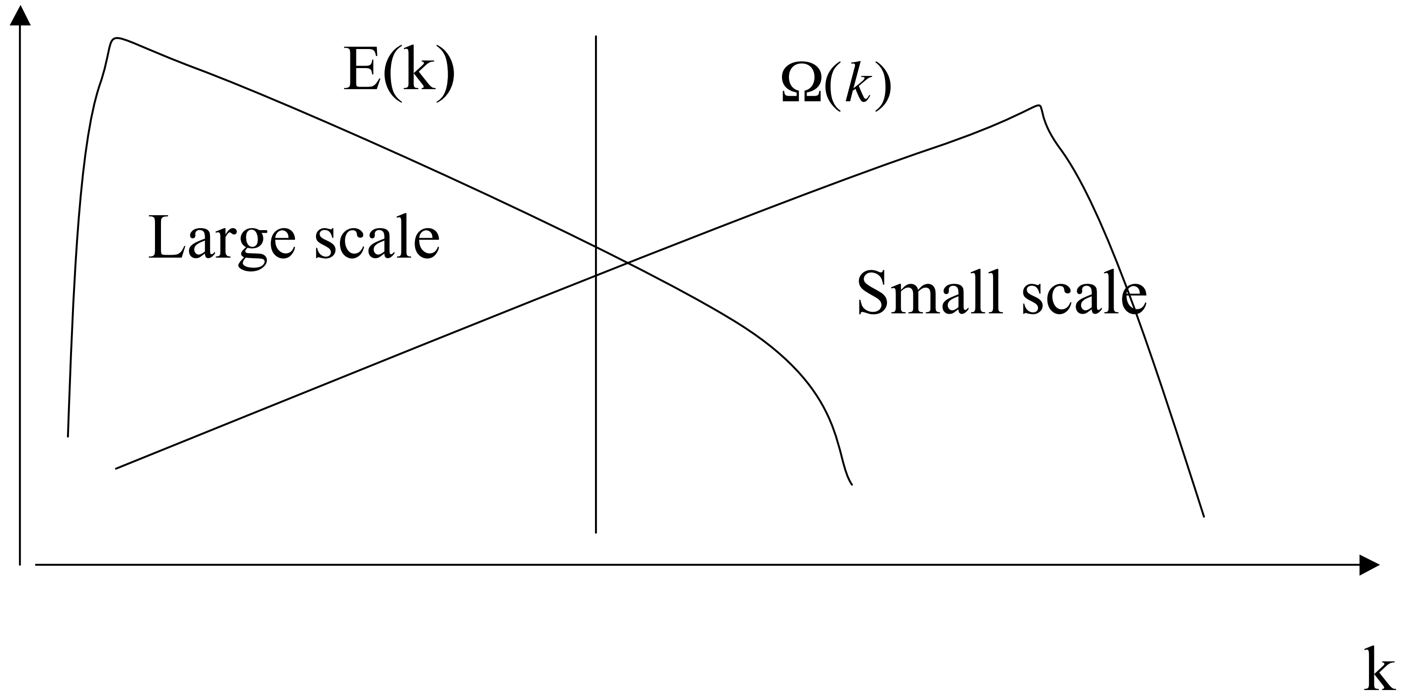






Laval, Dubrulle, Nazarenko 2001

Physical meaning of the model



3 D turbulence: Velocity concentrated at large scale
Vorticity concentrated at small scale

Cf small scale dynamo

Conservation laws of the model

Keeping only leading order terms:

$$\partial_t U = U \times \Omega + u \times \omega - \nabla P + \nu \Delta U$$

$$\partial_t \omega = \nabla \times \left(U \times \omega \right) + \nu_t \Delta \omega + f$$

Analogie with MHD!

Conserved quantities	Analog in MHD
$E = \int (U^2 + u^2) dx$	$E = \int (U^2 + B^2) dx$
$H_k = \int U \bullet \Omega dx$	
$h = \int u \bullet \omega dx$	$h = \int A \bullet B \, dx$
$H_c = \int (U \bullet \omega + u \bullet \Omega) dx$	$H_c = \int U \bullet B \, dx$

LES: what we learned

$$\partial_t \overline{u}_i + \overline{u}_j \nabla_j \overline{u}_i = -\nabla_i \overline{p} + \frac{1}{\mathrm{Re}} \Delta \overline{u}_i + \bar{f}_i - \nabla_j \tau_{ij}$$

Reynolds stress

$$\tau_{ij} = \overline{\overline{u}_i \overline{u}_j} - \overline{u}_i \overline{u}_j + \overline{\overline{u}_i u'_j} + \overline{u'_i \overline{u}_j} + \overline{u'_i u'_j}$$

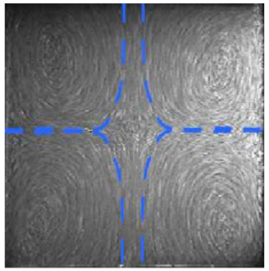
Piloted by large (and fluctuating) scales

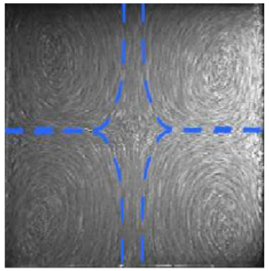
Summary

RANS

LES

kept


(a)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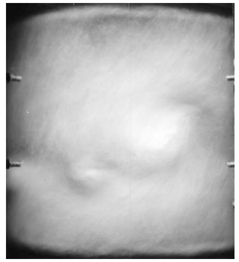
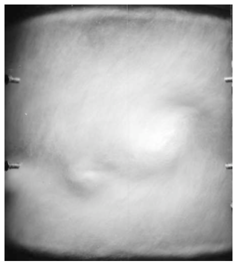
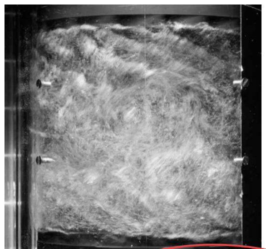


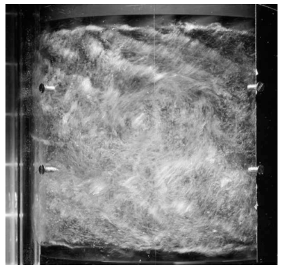
Meca. Stat

SRDT

modeled







delta

Fluctuations

Classical tool: turbulence intensity

$$i = \frac{\overline{\left(u - \bar{u}\right)^2}}{\bar{u}^2}$$

Space independent in isotropic, homogeneous turbulence

From the AMS glossaire

turbulence intensity—The ratio of the root-mean-square of the eddy velocity to the mean wind speed.
In general, it is a quantity that characterizes the intensity of gusts in the airflow.

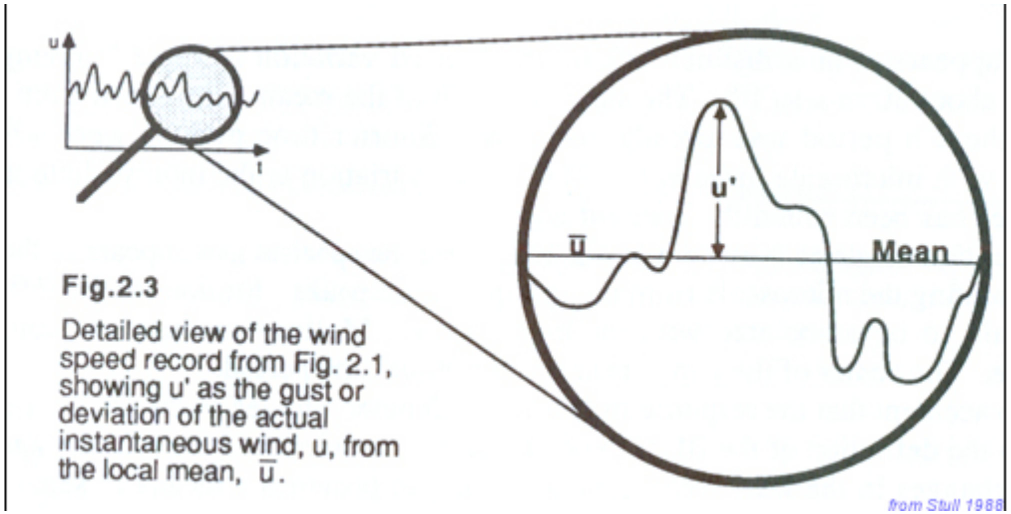
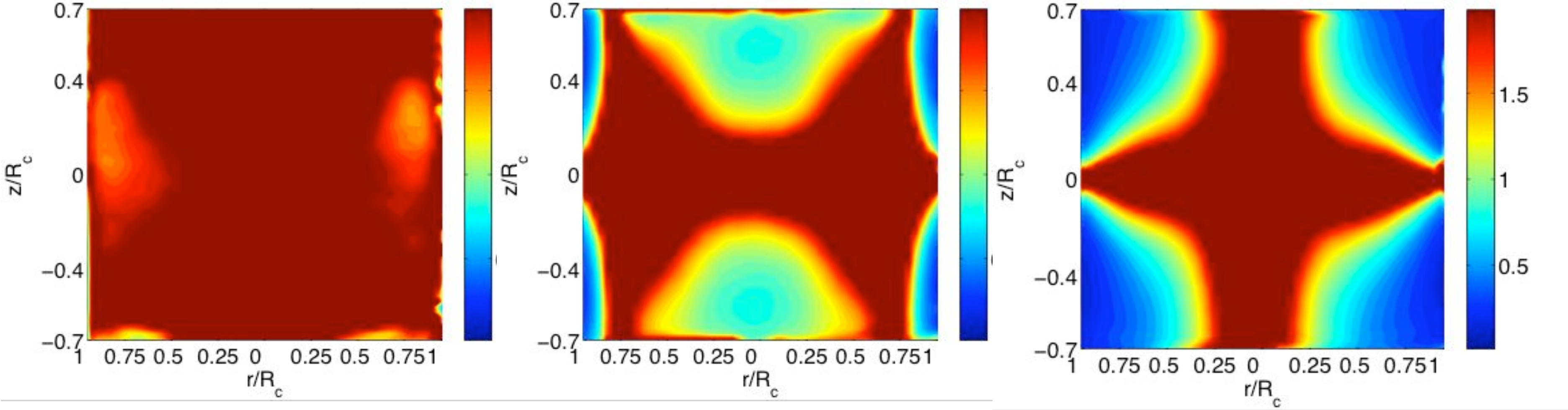


Fig.2.3

Detailed view of the wind speed record from Fig. 2.1, showing u' as the gust or deviation of the actual instantaneous wind, u, from the local mean, ū.

from Stull 1988

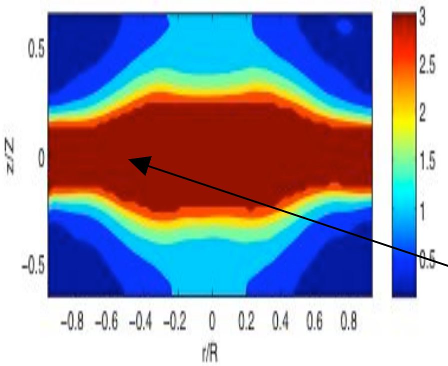
Fluctuations in von Karman

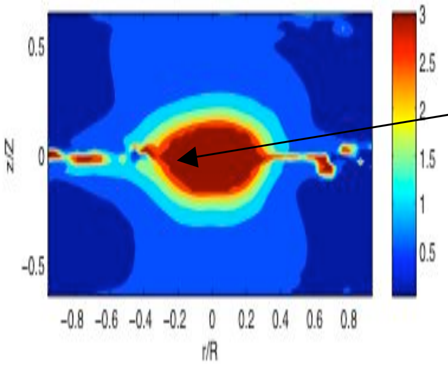


u'_r u'_θ u'_z

Inhomogeneous, anisotropic fluctuations

Turbulence intensity in von Karman





Very large Intensity!

Need for a new global quantity in real flow
(neither homogeneous, nor isotropic)

A new global measure of fluctuations

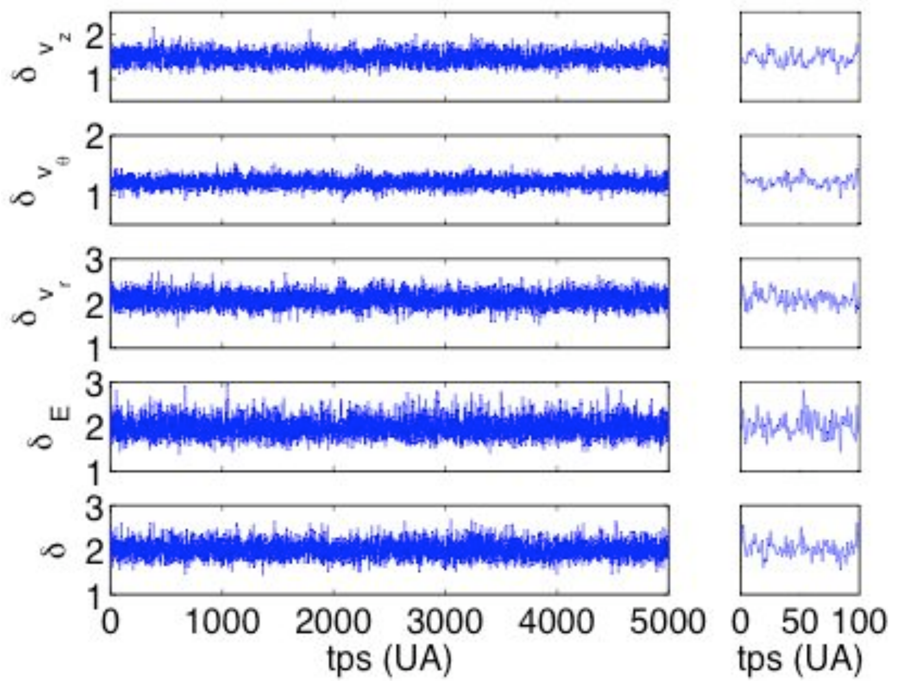
$$\delta(t) = \frac{\left\langle u^2 \right\rangle}{\left\langle \overline{u}^2 \right\rangle}$$

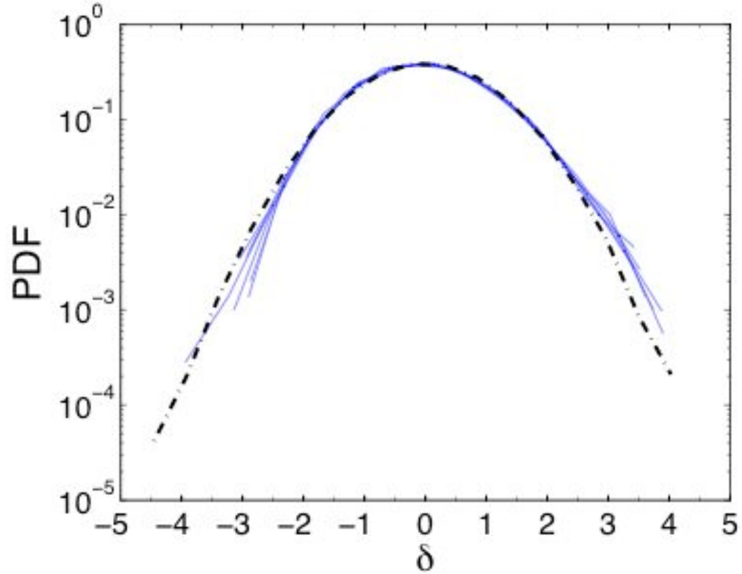
average →

$$\delta = \frac{\overline{\left\langle u^2 \right\rangle}}{\left\langle \overline{u}^2 \right\rangle}$$

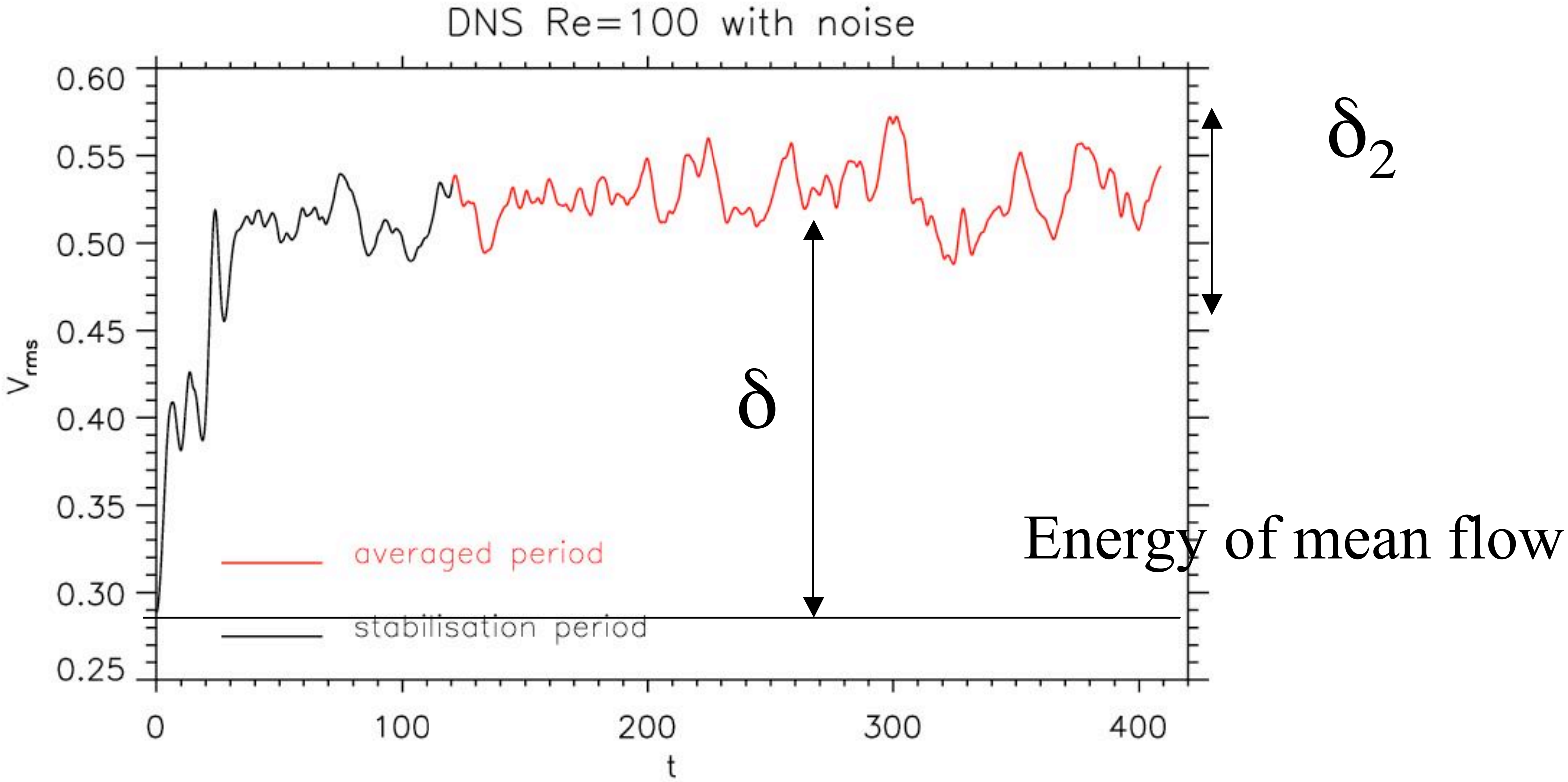
rms →

$$\delta_2 = \frac{\sqrt{\overline{\left\langle u^2 \right\rangle^2} - \overline{\left\langle u^2 \right\rangle}^2}}{\left\langle \overline{u}^2 \right\rangle}$$

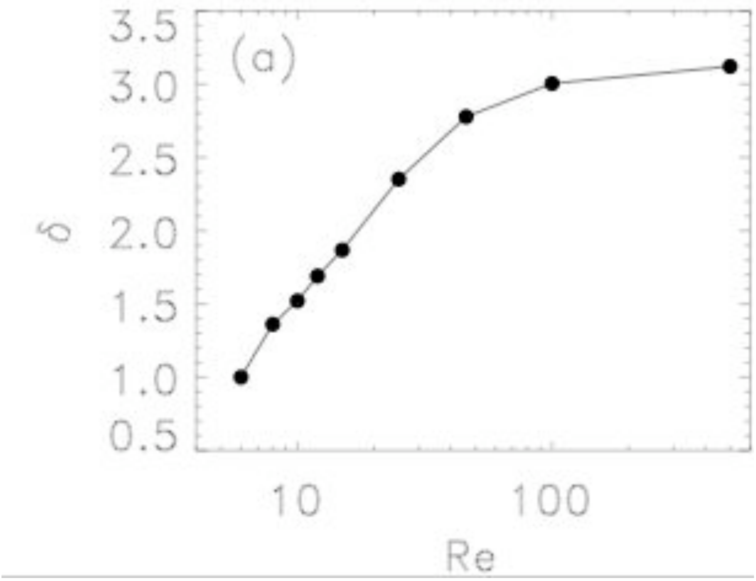


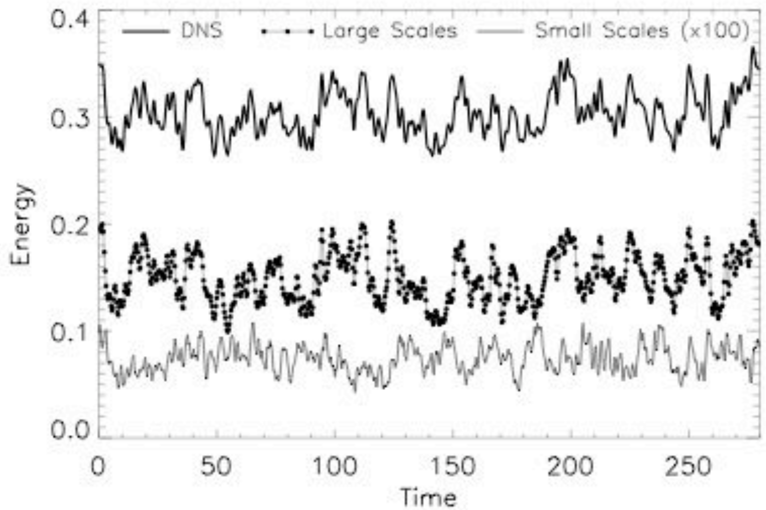


Graphical view on δ and δ_2



Example: TG flow





Large scale: 60% of the instantaneous kinetic energy
Small scales: 0.3 % of the instantaneous kinetic energy

Transport: vector

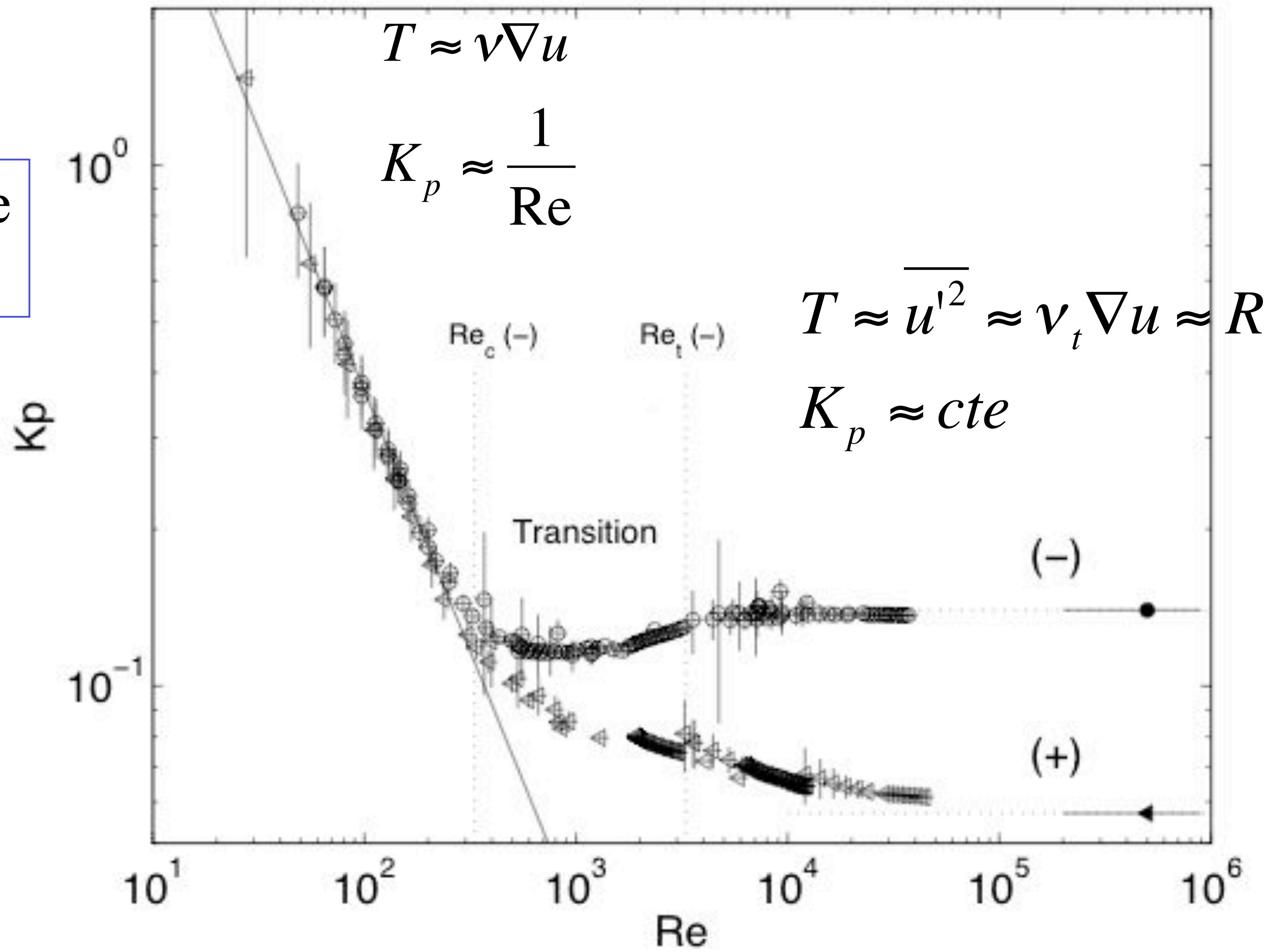
Torque
In VK

$$T \approx \nu \nabla u$$

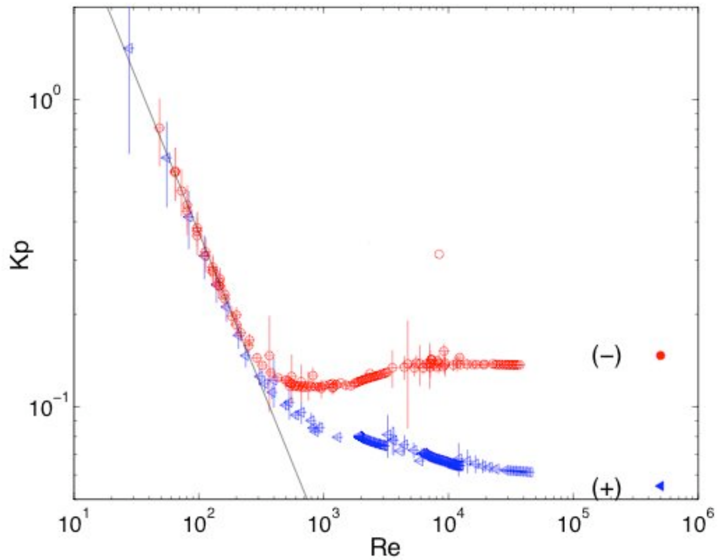
$$K_p \approx \frac{1}{\mathrm{Re}}$$

$$T \approx \overline{u'^2} \approx \nu_t \nabla u \approx R^2 F \nabla u$$

$$K_p \approx cte$$

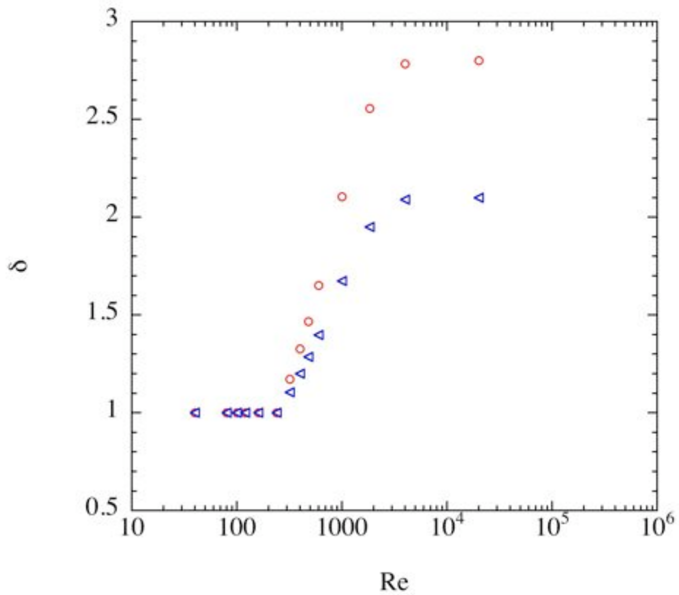


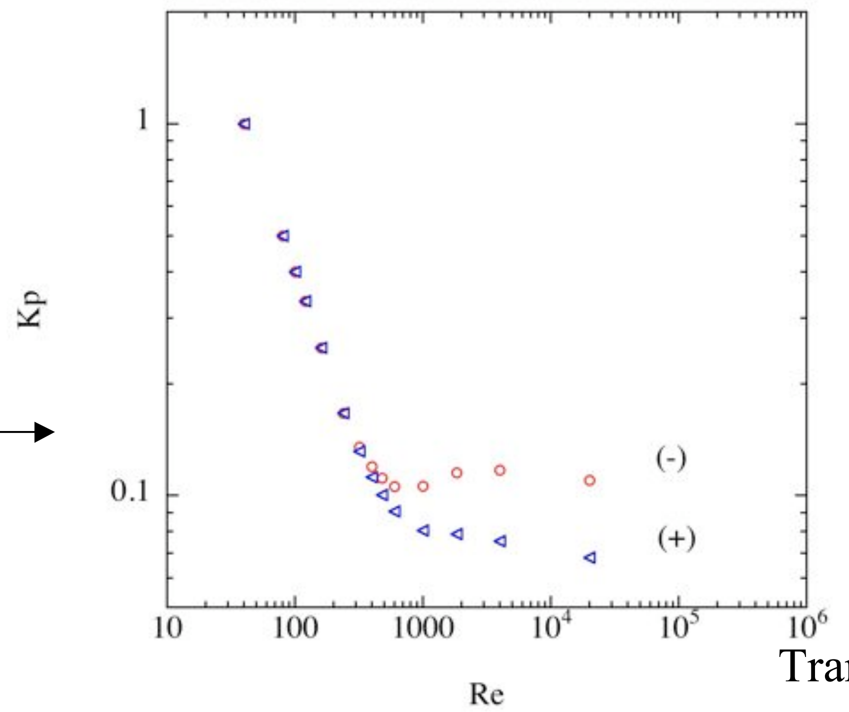
Transport and Fluctuations



$$T \approx \nu \nabla u + \overline{u'^2}$$

$$K_p \approx \frac{a}{\mathrm{Re}} + b(\delta - 1)$$





Transport

RANS: what we learned

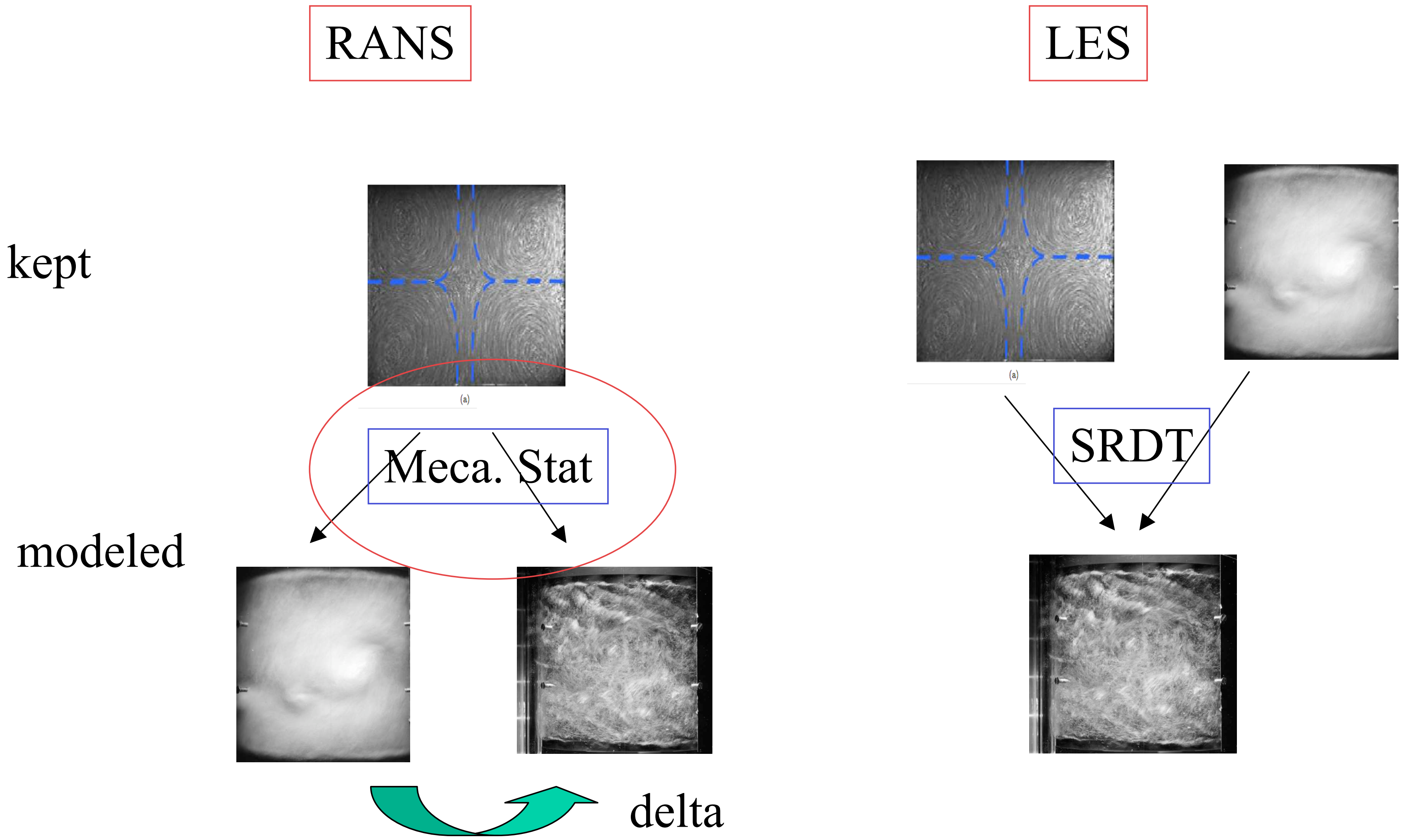
$$\partial_t \overline{u}_i + \overline{u}_j \nabla_j \overline{u}_i = -\nabla_i \overline{p} + \frac{1}{\mathrm{Re}} \Delta \overline{u}_i + \bar{f}_i - \nabla_j \tau_{ij}$$

Reynolds stress

$$\tau_{ij} = +\overline{u'_i u'_j}$$

Piloted by LS fluctuations

Summary



Statistical mechanics of axisymmetric truncated Euler

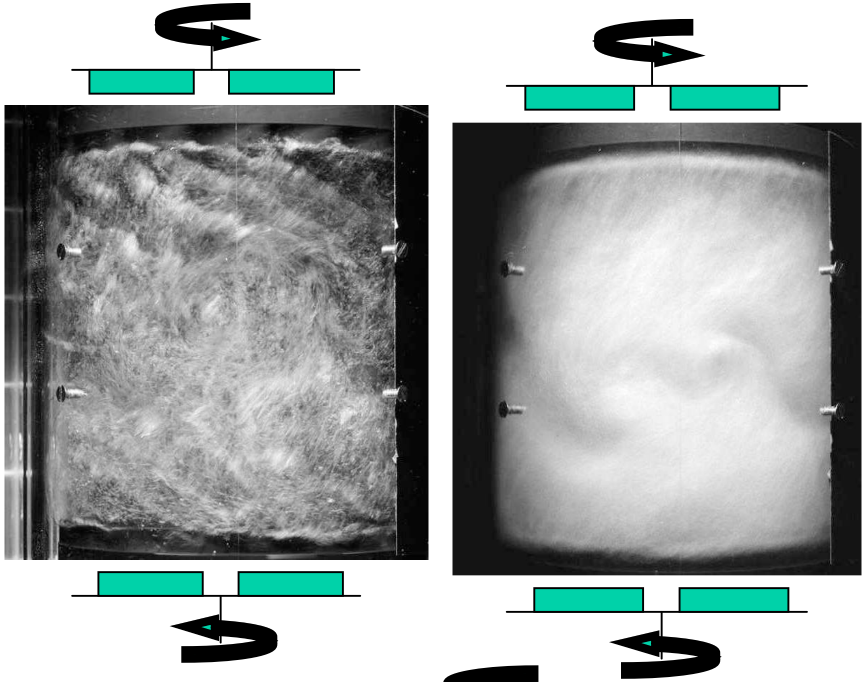
Basic equations

$$\partial_t \sigma + \{\psi, \sigma\} = \nu \Delta \sigma$$

$$\partial_t \xi + \{\psi, \xi\} = \partial_z \left(\frac{\sigma}{r^4} \right) + \nu \Delta \xi$$

$$\sigma = r u_\theta \qquad \xi = \omega_\theta / r$$

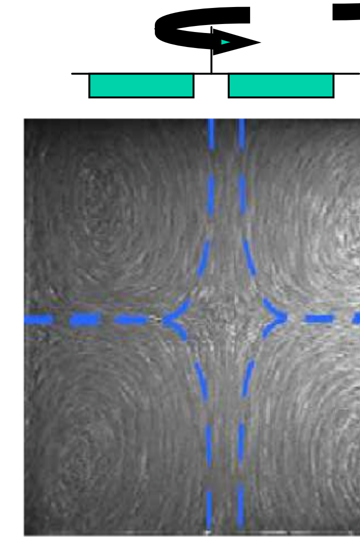
$$\vec{u}_P = -\overrightarrow{e_z} \times \vec{\nabla} \psi \qquad \omega = -\Delta \psi$$



Conservation laws

$$E = \frac{1}{2} \int u^2 = \frac{1}{2} \int \xi \psi \, dx + \frac{1}{2} \int \frac{\sigma^2}{r^2} dx$$

$$H = \int u \omega = \int \sigma \; \xi \, dx$$



(a)

Statistical description

Probability distribution to observe
σ and ξ at point r

$$\rho(r,\sigma,\xi)$$

Entropie $S = -\int \rho(r,\sigma,\xi)\ln\rho(r,\sigma,\xi)\,d^2r\,d\sigma\,d\xi$

Coarse grained A. M. $\overline{\sigma} = \int \sigma \int \rho(r,\sigma,\xi)\,d^2r\,d\sigma\,d\xi$

Coarse grained vort $\overline{\xi} = \int \xi \int \rho(r,\sigma,\xi)\,d^2r\,d\sigma\,d\xi$

Maximisation of S under conservation constraints

Gibbs distribution

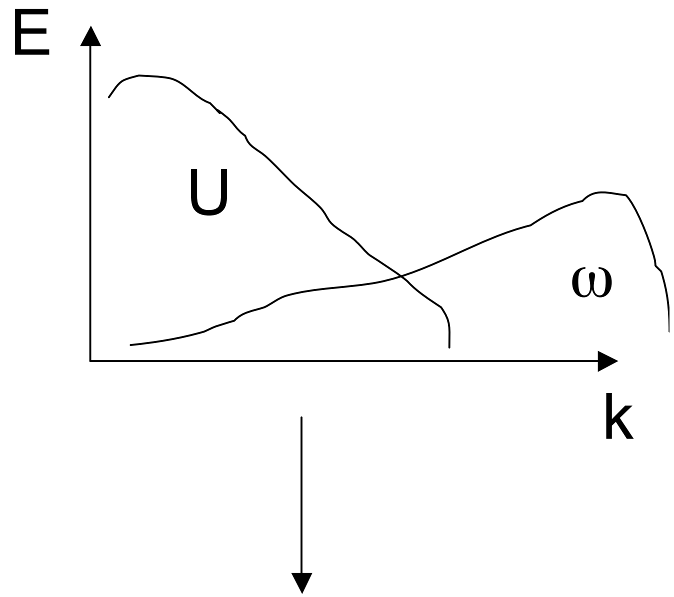
$$\rho(\sigma) = \frac{e^{-\beta(\xi\psi+\sigma^2/r^4)-\mu\sigma\xi}}{Z}$$

Mean and fluctuations

1st moment: Equilibrium state

$$\beta_\sigma \bar{\psi} + \mu_\sigma \bar{\sigma} = 0$$

$$\beta_\xi \frac{\bar{\sigma}}{r^2} + \mu_\xi \bar{\xi} = 0$$



2nd moment: fluctuations

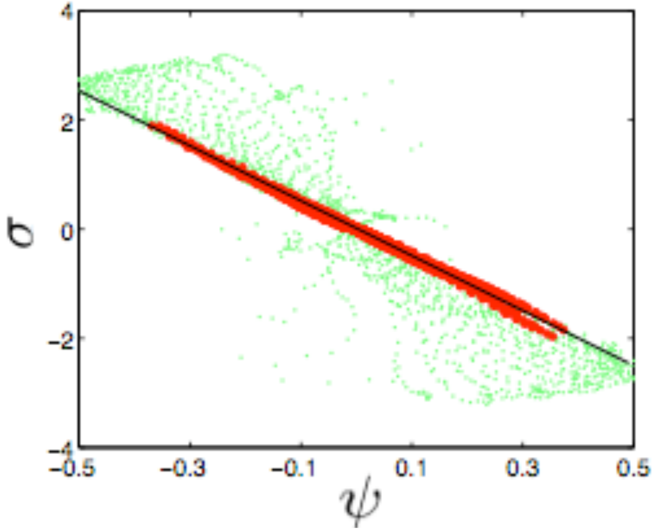
$$\overline{\sigma^2} - \bar{\sigma}^2 = -\frac{1}{\mu_\sigma} \frac{\delta \bar{\sigma}}{\delta \bar{\xi}} = \frac{1}{\beta_\sigma} r^2$$

LS fluctuations

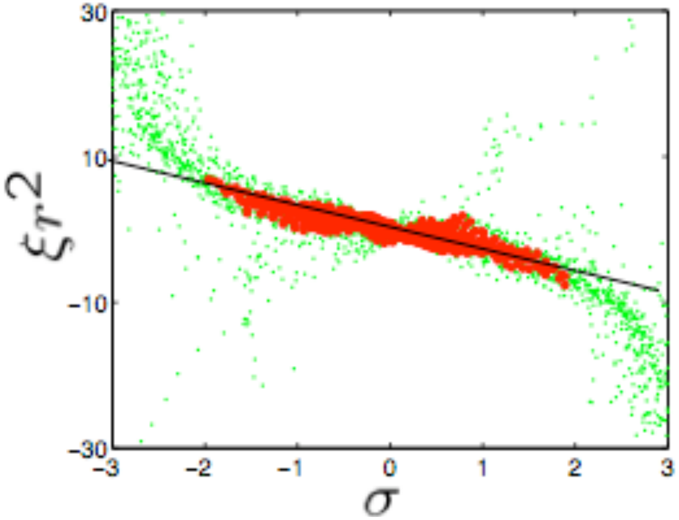
$$\overline{\xi^2} - \bar{\xi}^2 = -\frac{1}{\mu_\xi} \frac{\delta \bar{\xi}}{\delta \bar{\sigma}} = \frac{\beta_\sigma}{\mu_\sigma \mu_\xi} \frac{1}{r^2}$$

SS fluctuations

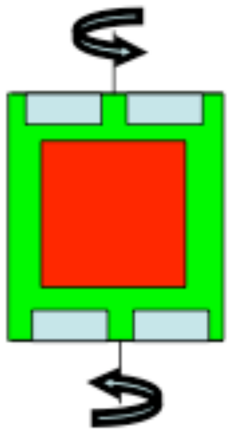
response dissipation



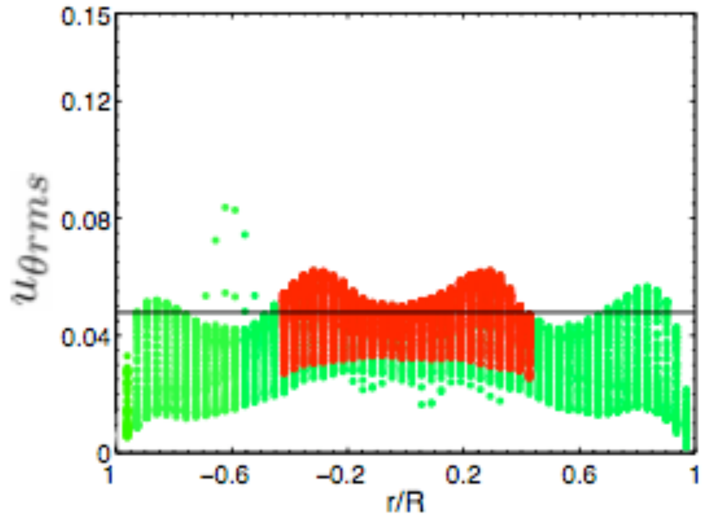
$$\frac{\beta_\sigma}{\mu_\sigma} \approx \frac{\beta_\xi}{\mu_\xi}$$



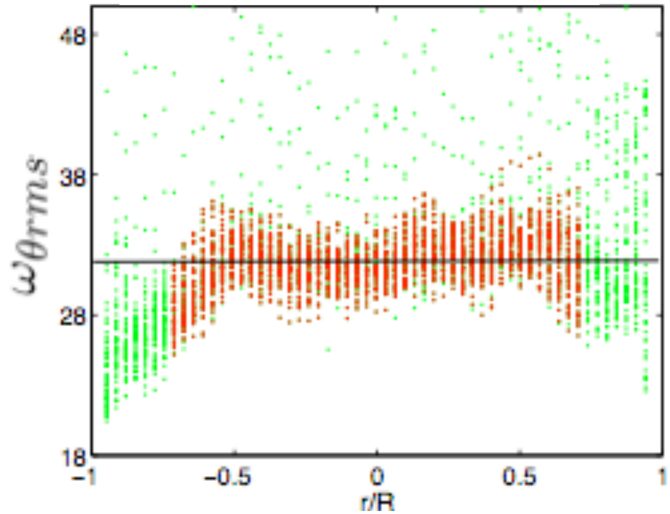
$$\beta_\xi \overline{\psi} + \mu_\xi \overline{\sigma} = 0$$



$$\frac{\beta_\sigma \overline{\sigma}}{r^2} + \mu_\sigma \overline{\xi} = 0$$



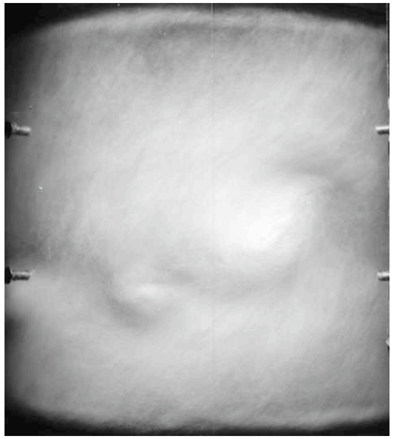
$$\beta_\sigma \neq \beta_\xi$$



$$\overline{{u_\theta}^2} - \overline{u_\theta}^2 = \frac{1}{\beta_\xi}$$

$$\overline{{\omega_\theta}^2} - \overline{\omega_\theta}^2 = \frac{\beta_\sigma \beta_\xi}{\mu_\sigma \mu_\xi} \frac{1}{\beta_\sigma}$$

Cascades lambdastated by mean flow?
(cf Brissaud et al)

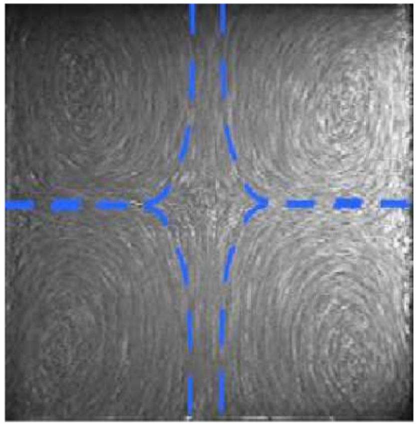


$$u_\theta rms = \frac{1}{\beta_\xi}$$

Cold
Helical

$$\frac{1}{\beta_\xi} << \frac{1}{\beta_\sigma}$$

$$\frac{1}{\mu_\xi} >> \frac{1}{\mu_\sigma}$$

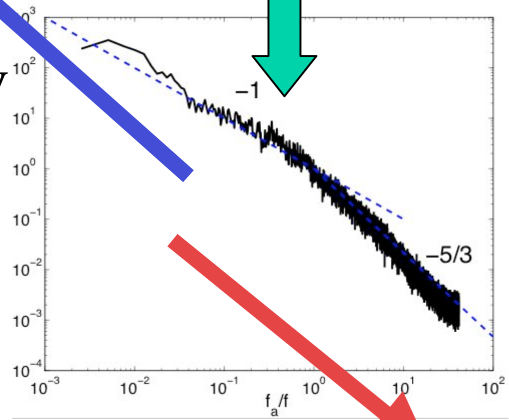


(a)

Conserved
(lambdastat)

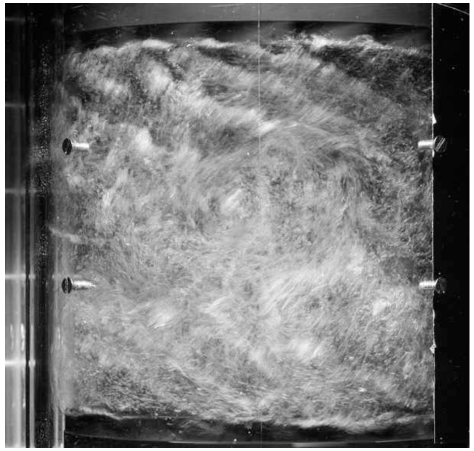
$$\frac{\mu_\sigma}{\beta_\sigma} = \frac{\mu_\xi}{\beta_\xi} = \lambda$$

Energy



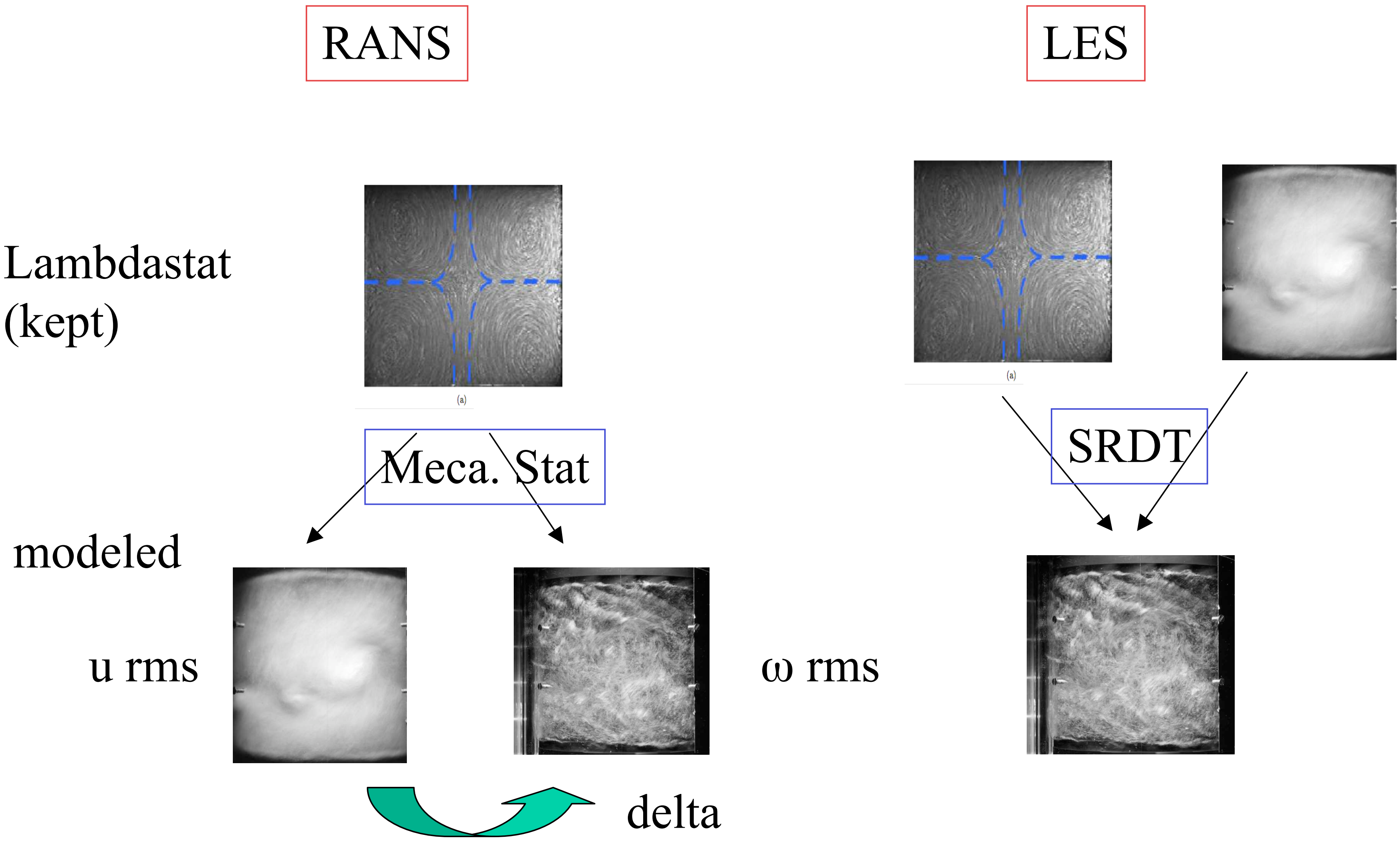
Helicity

Warm
Non Helical



$$\omega_\theta rms = \frac{1}{\beta_\sigma \lambda^2}$$

Summary


RANS
LES
Lambdastat
(kept)
(a)
(a)
Meca. Stat
SRDT
modeled
u rms
ω rms
delta