

Lagrangian Dynamics and Statistical Geometry in Turbulence

...and Intermittency

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and Y. Li, Z. Xiao,

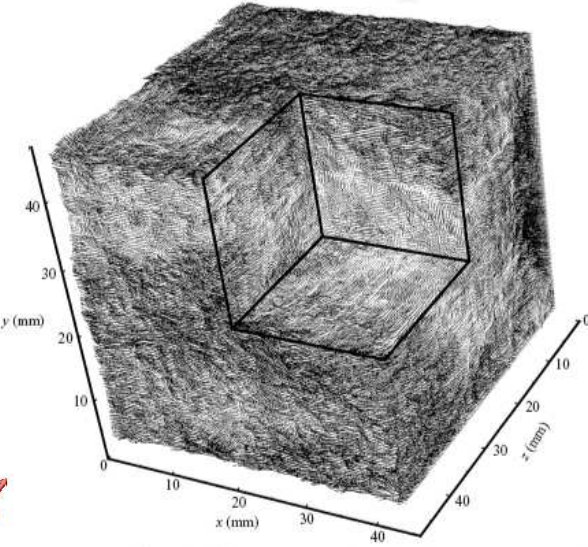
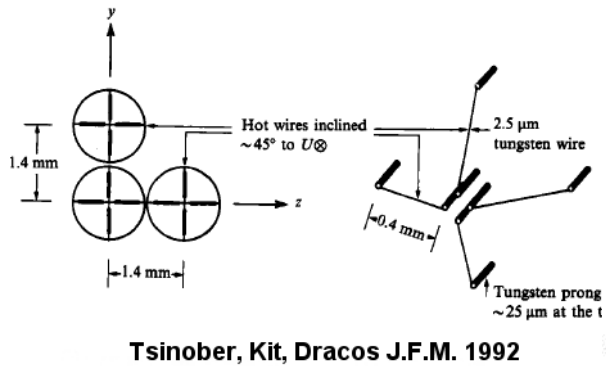
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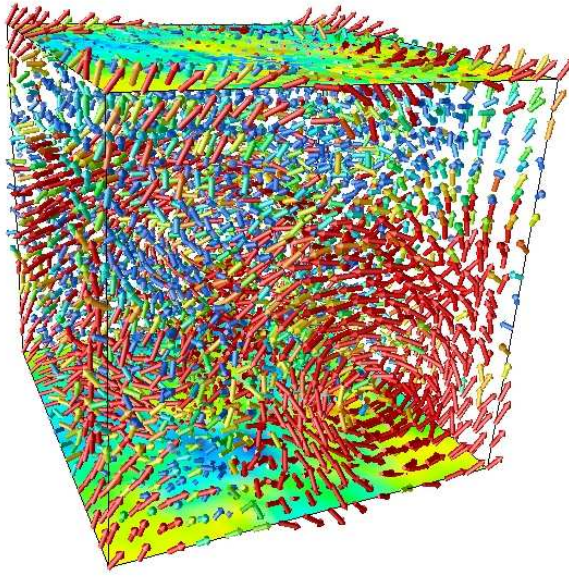
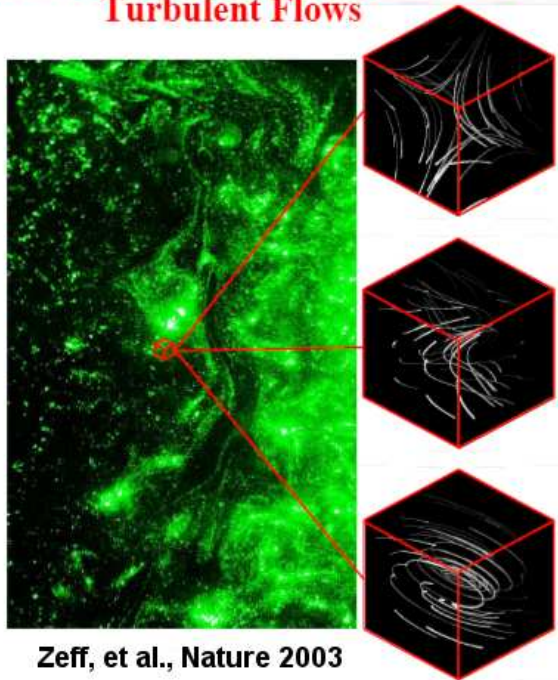
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3D Fluid Turbulence: Full velocity gradients



Intense Rotation and Dissipation in Turbulent Flows



$$A = \begin{pmatrix} \frac{\partial u_x}{\partial x} & \frac{\partial u_x}{\partial y} & \frac{\partial u_x}{\partial z} \\ \frac{\partial u_y}{\partial x} & \frac{\partial u_y}{\partial y} & \frac{\partial u_y}{\partial z} \\ \frac{\partial u_z}{\partial x} & \frac{\partial u_z}{\partial y} & \frac{\partial u_z}{\partial z} \end{pmatrix}$$

3D Nature of Turbulence and Non Locality of Pressure

$$\underbrace{\text{Acceleration}}_{a_i} = \frac{du_i}{dt} = \underbrace{\frac{\partial u_i}{\partial t} + (\mathbf{u} \cdot \nabla) u_i}_{\text{Anti-Correlated}} \stackrel{\text{Require 3D}}{=} \underbrace{-\nabla_i p}_{\text{non local}} + \nu \nabla^2 u_i$$

nonlinear
non local

Incompressibility \leftrightarrow Poisson equation: $\nabla^2 p = -\text{tr}(\mathbf{A}^2)$

Velocity Gradient Tensor: $\mathbf{A} = \begin{pmatrix} \frac{\partial u_x}{\partial x} & \frac{\partial u_x}{\partial y} & \frac{\partial u_x}{\partial z} \\ \frac{\partial u_y}{\partial x} & \frac{\partial u_y}{\partial y} & \frac{\partial u_y}{\partial z} \\ \frac{\partial u_z}{\partial x} & \frac{\partial u_z}{\partial y} & \frac{\partial u_z}{\partial z} \end{pmatrix}$

3D Nature of Turbulence and Non Locality of Pressure

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nonlinear
non local

Incompressibility \leftrightarrow Poisson equation: $\nabla^2 p = -\text{tr}(\mathbf{A}^2)$

$$\rightarrow \text{Pressure Gradient: } \nabla_i p(\mathbf{x}) = \int d\mathbf{y} \underbrace{\mathcal{G}_i(|\mathbf{y} - \mathbf{x}|)}_{\sim \frac{1}{|\mathbf{y} - \mathbf{x}|^2}} \text{tr}(\mathbf{A}^2(\mathbf{y})) \text{ Non Local!!}$$

Green fct.

3D Nature of Turbulence and Non Locality of Pressure

$$\underbrace{\text{Acceleration}}_{a_i} = \frac{du_i}{dt} = \underbrace{\frac{\partial u_i}{\partial t} + (\mathbf{u} \cdot \nabla) u_i}_{\text{Anti-Correlated}} \stackrel{\text{Require 3D}}{=} \underbrace{-\nabla_i p}_{\text{non local}} + \nu \nabla^2 u_i$$

Incompressibility \leftrightarrow Poisson equation: $\nabla^2 p = -\text{tr}(\mathbf{A}^2)$

$$\rightarrow \text{Pressure Hessian: } P_{ij} = \nabla_{ij} p(\mathbf{x}) = \underbrace{-\text{tr}(\mathbf{A}^2(\mathbf{x})) \frac{\delta_{ij}}{3}}_{\text{Local-Isotropic}} + \text{P.V.} \int d\mathbf{y} \underbrace{\mathcal{G}_{ij}(|\mathbf{y} - \mathbf{x}|) \text{tr}(\mathbf{A}^2(\mathbf{y}))}_{\sim \frac{1}{|\mathbf{y} - \mathbf{x}|^3}} \quad \text{Non Local-Anisotropic}$$

See Ohkitani & Kishiba (PoF,95) and Majda, Bertozzi (CUP,01)

3D Nature of Turbulence and Non Locality of Pressure

$$\underbrace{\text{Acceleration}}_{a_i} = \frac{du_i}{dt} = \underbrace{\frac{\partial u_i}{\partial t} + (\mathbf{u} \cdot \nabla) u_i}_{\text{Anti-Correlated}} = \underbrace{-\nabla_i p}_{\text{non local}} + \nu \nabla^2 u_i$$

Require **3D** nonlinear

$$\begin{array}{c}
 \Downarrow \\
 \frac{\partial}{\partial x_j} \\
 \Downarrow
 \end{array}$$

Time evolution of the velocity gradient tensor $\mathbf{A} = \begin{pmatrix} \frac{\partial u_x}{\partial x} & \frac{\partial u_x}{\partial y} & \frac{\partial u_x}{\partial z} \\ \frac{\partial u_y}{\partial x} & \frac{\partial u_y}{\partial y} & \frac{\partial u_y}{\partial z} \\ \frac{\partial u_z}{\partial x} & \frac{\partial u_z}{\partial y} & \frac{\partial u_z}{\partial z} \end{pmatrix}$

3D Nature of Turbulence and Non Locality of Pressure

$$\underbrace{\text{Acceleration}}_{a_i} = \frac{du_i}{dt} = \frac{\partial u_i}{\partial t} + \underbrace{(\mathbf{u} \cdot \nabla) u_i}_{\text{Anti-Correlated}} = \underbrace{-\nabla_i p}_{\text{non local}} + \nu \nabla^2 u_i$$

Require **3D** nonlinear

$$\begin{array}{c}
 \Downarrow \\
 \frac{\partial}{\partial x_j} \\
 \Downarrow
 \end{array}$$

$$\frac{d}{dt} A_{ij} = \underbrace{-A_{iq} A_{qj}}_{\text{self-stretching term}} - \underbrace{\frac{\partial^2 p}{\partial x_i \partial x_j}}_{\text{Pressure Hessian}} + \underbrace{\nu \frac{\partial^2 A_{ij}}{\partial x_q \partial x_q}}_{\text{Viscous term}}$$

need to be **modeled**

The *Lagrangian* evolution of the *Eulerian* velocity gradient tensor

$$\text{Let } A_{ij} = \frac{\partial u_i}{\partial x_j} \text{ and } \frac{d}{dt} \equiv \frac{\partial}{\partial t} + u_q \frac{\partial}{\partial x_q}$$

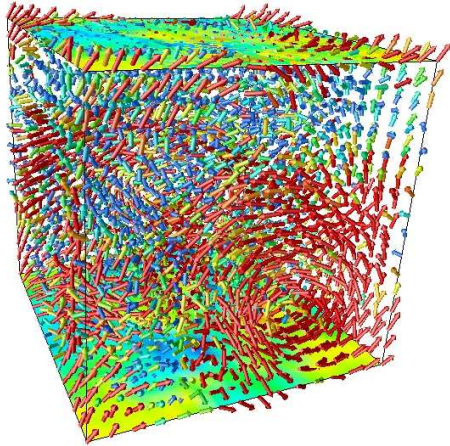
$$A = \begin{pmatrix} \text{Long}_{11} & \text{Trans}_{12} & \text{Trans}_{13} \\ \text{Trans}_{21} & \text{Long}_{22} & \text{Trans}_{23} \\ \text{Trans}_{31} & \text{Trans}_{32} & \text{Long}_{33} \end{pmatrix}$$

Then, along a **fluid trajectory** (Léorat 75, Vieillefosse 82):

$$\nabla(\text{Navier-Stokes}) \Rightarrow \frac{d}{dt} A_{ij} = \underbrace{-A_{iq} A_{qj}}_{\text{self-stretching term}} - \underbrace{\frac{\partial^2 p}{\partial x_i \partial x_j}}_{\text{Pressure Hessian}} + \underbrace{\nu \frac{\partial^2 A_{ij}}{\partial x_q \partial x_q}}_{\text{Viscous term}}$$

need to be **modeled**

Tracking Velocity Gradients along *Lagrangian* trajectories



DNS

- Yeung & Pope (89).
- Girimaji & Pope, J.F.M. (90).
- Pope & Chen , Phys. Fluids (90).

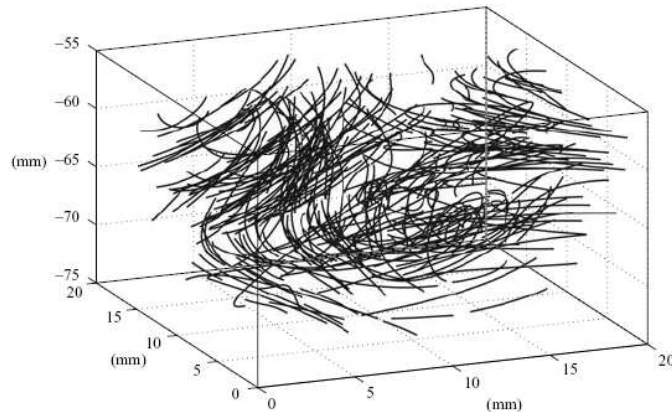


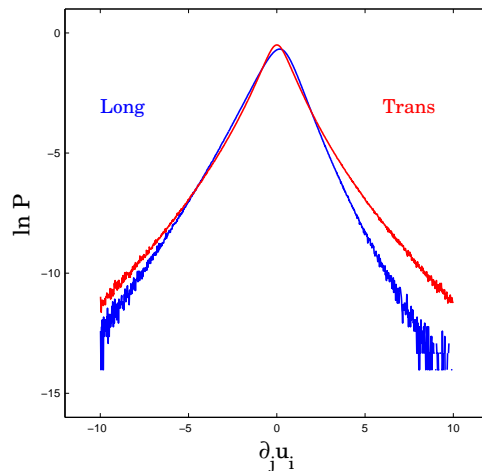
FIGURE 2. Selected particle trajectories as obtained from 3D-PTV.

Experimental

- Zeff *et al.*, Nature (2003).
- Lüthi, Tsinober & Kinzelbach, J.F.M. (2005).

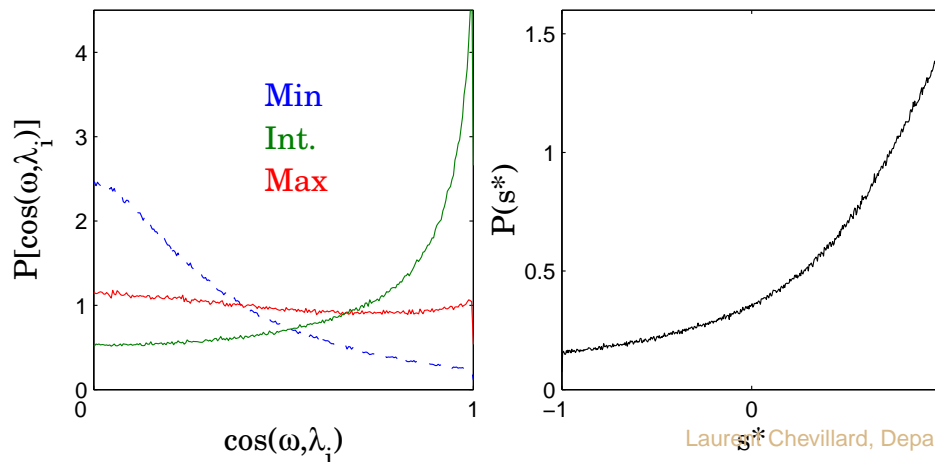
Statistical Intermittency and Geometry in turbulence

DNS Results $\mathcal{R}_\lambda = 150$



Intermittency

- Non-Gaussianity
- Skewness
- Anomalous scaling with Reynolds number

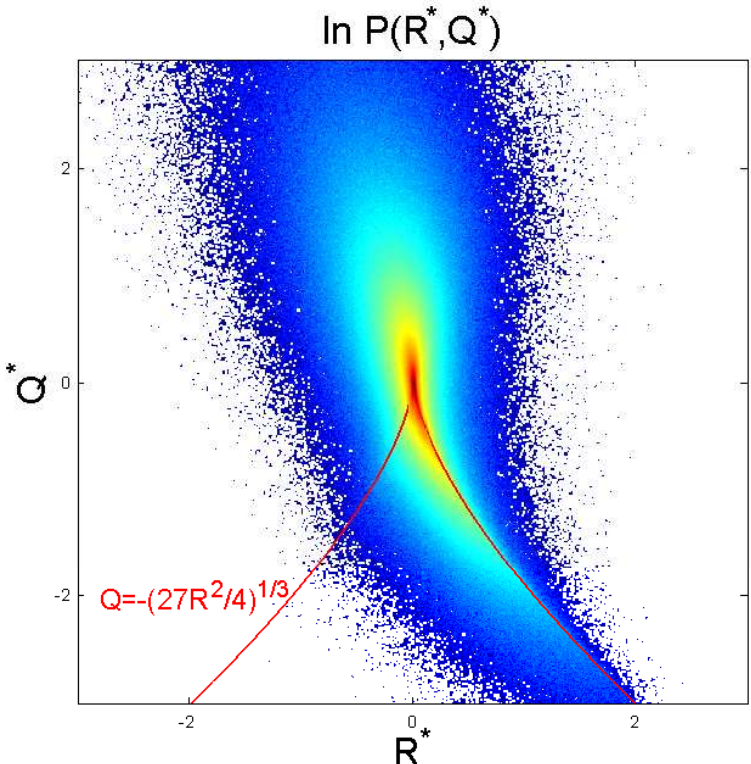
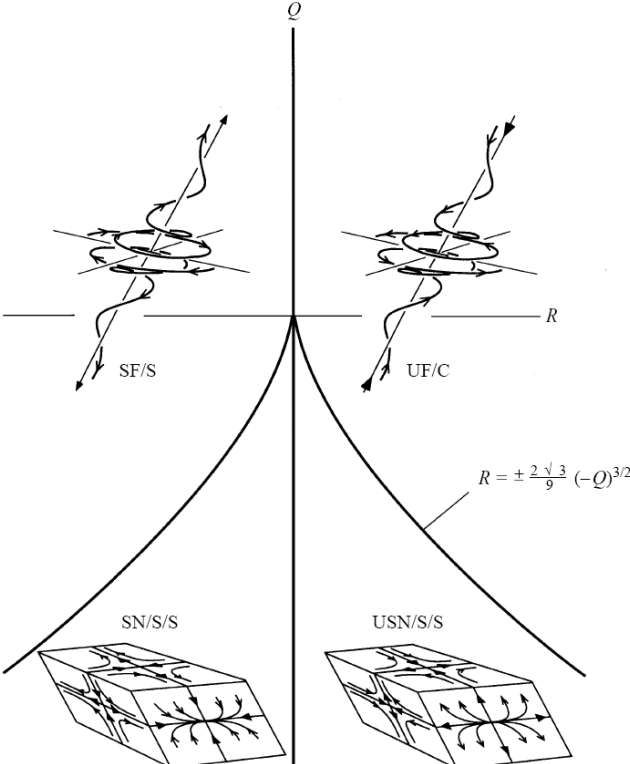


Geometry

- Preferential alignment of vorticity
- Preferential axisymmetric expansion

The RQ plane - Local Topology

See Chong, Perry and Cantwell (90) and Cantwell (93)



$\lambda_i = f(R, Q) \in \mathbb{C}$: Eigenvalues of **A**

- Second invariant: $Q = -\frac{1}{2} \text{Tr}(\mathbf{A}^2) = \frac{1}{4} \underbrace{|\omega|^2}_{\text{Enstrophy}} - \frac{1}{2} \underbrace{\text{Tr}(\mathbf{S}^2)}_{\text{Dissipation}}$
- Third invariant: $R = -\frac{1}{3} \text{Tr}(\mathbf{A}^3) = -\frac{1}{4} \underbrace{\omega_i S_{ij} \omega_j}_{\text{Enstrophy Production}} - \frac{1}{3} \underbrace{\text{Tr}(\mathbf{S}^3)}_{\text{Strain Skewness}}$

Review of various models

$$\frac{d}{dt} A_{ij} = -A_{iq} A_{qj} - \frac{\partial^2 p}{\partial x_i \partial x_j} + \nu \frac{\partial^2 A_{ij}}{\partial x_q \partial x_q}$$

- Restricted Euler Dynamics (Léorat 75-Vieillefosse 84-Cantwell 92)

$$\frac{\partial^2 p}{\partial x_i \partial x_j} = -\frac{\delta_{ij}}{3} \text{Tr}(\mathbf{A}^2) \text{ and } \nu = 0 \rightarrow \text{Finite time singularity}$$

- Lognormality of Pseudo-dissipation $\varphi = \text{Tr}(\mathbf{A}\mathbf{A}^T)$ (Girimaji-Pope 90)
→ Strong *a-priori* assumption

- Linear damping term (Martin *et al.* 98)

$$\frac{\partial^2 p}{\partial x_i \partial x_j} = -\frac{\delta_{ij}}{3} \text{Tr}(\mathbf{A}^2) \text{ and } \nu \frac{\partial^2 \mathbf{A}}{\partial x_q \partial x_q} = -\frac{1}{\tau} \mathbf{A} \rightarrow \text{Finite time singularity}$$

- Delta-vee system (Yi-Meneveau (05))
Projection on Longitudinal $\delta_\ell u$ and Transverse $\delta_\ell v$ increments

Using the material **Deformation (Cauchy-Green Tensor \mathbf{C})**

- Tetrad's model (Chertkov-Pumir-Shraiman 99)

$$\frac{\partial^2 p}{\partial x_i \partial x_j} = -\frac{\text{Tr}(\mathbf{A}^2)}{\text{Tr}(\mathbf{C}^{-1})} C_{ij}^{-1} \text{ and } \nu = 0 \rightarrow \text{Non stationary}$$

- Differential damping term (Jeong-Girimaji 03)

$$\frac{\partial^2 p}{\partial x_i \partial x_j} = -\frac{\delta_{ij}}{3} \text{Tr}(\mathbf{A}^2) \text{ and } \nu \frac{\partial^2 \mathbf{A}}{\partial x_q \partial x_q} = -\frac{\text{Tr}(\mathbf{C}^{-1})}{3\tau} \mathbf{A}$$

→ **Non stationary**

Review of the Restricted Euler approximation

$$\frac{d}{dt} A_{ij} = -A_{iq} A_{qj} - \frac{\partial^2 p}{\partial x_i \partial x_j} + \nu \frac{\partial^2 A_{ij}}{\partial x_q \partial x_q}$$

- Restricted Euler Dynamics (Léorat 75-Vieillefosse 84-Cantwell 92)

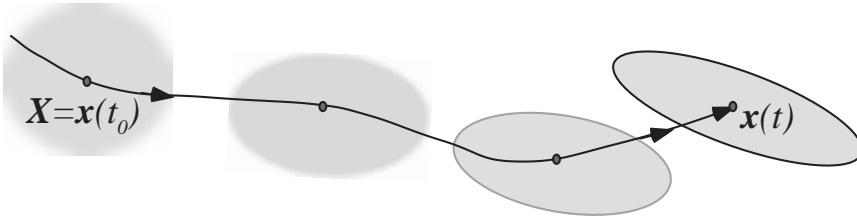
$$\frac{\partial^2 p}{\partial x_i \partial x_j} = -\frac{\delta_{ij}}{3} \text{Tr}(\mathbf{A}^2) \text{ and } \nu = 0$$

$$\frac{d}{dt} \mathbf{A} = - \left(\mathbf{A}^2 - \frac{\delta_{ij}}{3} \text{Tr}(\mathbf{A}^2) \right)$$

→ Non stationary

- Finite time singularity (t^*) of \mathbf{A} for any initial condition
- BUT, at $t \lesssim t^*$
 - Second eigenvalue λ of \mathbf{S} is positive
→ Preferential axisymmetric expansion
 - Vorticity gets aligned with the associated eigenvector u_λ

Cauchy-Green Tensor: Tracking the volume deformation



Deformation gradient: $D_{ij}(t) = \frac{\partial x_i}{\partial X_j}(t)$

$$\text{Dyn.: } \frac{d}{dt} \mathbf{D} = \mathbf{A} \mathbf{D}$$

$$\Leftrightarrow \mathbf{D}(t) = \underbrace{\prod_{t_0}^t e^{\mathbf{A}(\xi) d\xi}}_{\text{Time ordered exponential}}$$

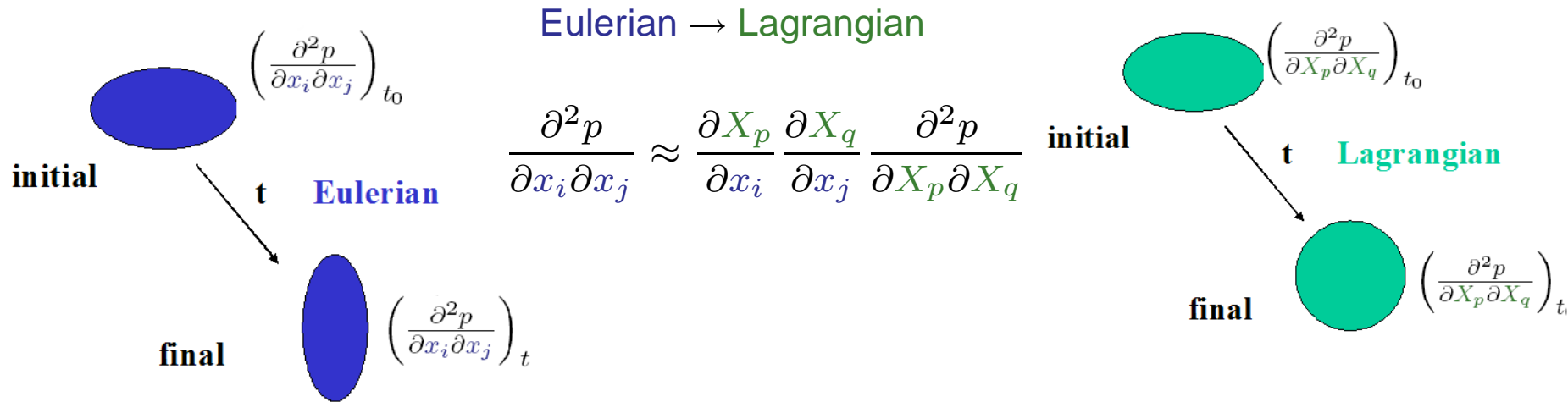
Time ordered exponential

Cauchy-Green Tensor: $\mathbf{C} = \mathbf{D} \mathbf{D}^T$ or $C_{ij}^{-1} = \frac{\partial X_p}{\partial x_i} \frac{\partial X_p}{\partial x_j}$

Non Stationary

- Monin-Yaglom 75
- Girimaji-Pope 90: DNS $\mathcal{R}_\lambda = 90$
- Lüthi, Tsinober, Kinzelbach 05: Exp. $\mathcal{R}_\lambda = 50$

Re-Interpretation of the Chertkov et al. **Tetrad** Model



Isotropic

hyp.:
$$\frac{\partial^2 p}{\partial X_p \partial X_q} = \frac{\delta_{pq}}{3} \frac{\partial^2 p}{\partial X_m \partial X_m} \stackrel{\text{Poisson equation}}{=} -\delta_{pq} \frac{\text{Tr}(\mathbf{A}^2)}{\text{Tr}(\mathbf{C}^{-1})}$$

Chertkov, Pumir and Shraiman (99)

$$\frac{\partial^2 p}{\partial x_i \partial x_j} = -\frac{\text{Tr}(\mathbf{A}^2)}{\text{Tr}(\mathbf{C}^{-1})} C_{ij}^{-1}$$

Non Stationary

The Jeong *et al.* *Lagrangian Linear Diffusion Model*

Eulerian → Lagrangian

$$\frac{\partial^2 \mathbf{A}}{\partial x_m \partial x_m} = \frac{\partial X_p}{\partial x_m} \frac{\partial X_q}{\partial x_m} \frac{\partial^2 \mathbf{A}}{\partial X_p \partial X_q}$$

hyp.: **Linear damping** in the Lagrangian frame $\nu \frac{\partial^2 \mathbf{A}}{\partial X_p \partial X_q} = -\frac{\delta_{pq}}{3} \frac{\mathbf{A}}{\Theta}$

$$\nu \frac{\partial^2 \mathbf{A}}{\partial x_m \partial x_m} = -\frac{\text{Tr}(\mathbf{C}^{-1})}{3\Theta} \mathbf{A} \rightarrow \Theta ??$$

Jeong and Girimaji (03) **Non Stationary**

Chevillard-Meneveau (06) → Self-consistent time-scale estimation:

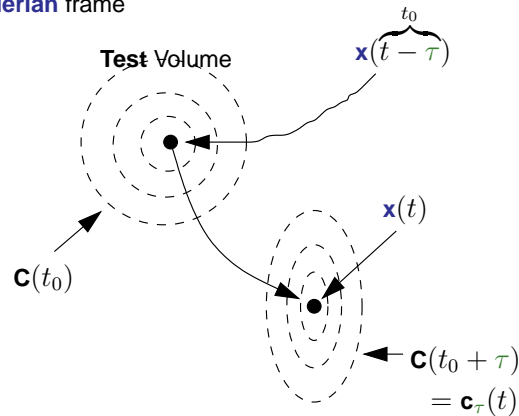
$$\frac{1}{\Theta} \sim \nu \frac{\delta^2}{\delta X^2} \sim \frac{\nu}{(\text{distance traveled during } \tau_K)^2} \sim \underbrace{\frac{\nu}{\lambda^2}}_{\text{Taylor}} \sim \frac{1}{T} \text{ Integral time scale}^{-1}$$

Stationary Cauchy-Green Tensor

$$\mathbf{D}(t) = \frac{\partial \mathbf{x}}{\partial \mathbf{X}}(t) = \prod_{t_0}^t e^{\mathbf{A}(\xi)d\xi} = \underbrace{\mathbf{d}_\tau(t)}_{\text{present}} \overbrace{\mathbf{D}(t-\tau)}^{\text{old history}}$$

Over a **dissipative** time scale $\tau = \begin{cases} \tau_K & \text{Kolmogorov} \\ 1/\sqrt{\text{Tr}(2\mathbf{S}^2)} & \text{Local} \end{cases}$

Eulerian frame



$$\text{with } \mathbf{d}_\tau(t) = \prod_{t-\tau}^t e^{\mathbf{A}(\xi)d\xi} \approx e^{\int_{t-\tau}^t \mathbf{A}(\xi)d\xi} \approx e^{\tau \mathbf{A}(t)}$$

Recent deformation

Let \mathbf{c}_τ the **stationary** "Cauchy-Green" Tensor

$$\mathbf{c}_\tau = \mathbf{d}_\tau \mathbf{d}_\tau^T$$

See Chevillard-Meneveau 06

A *Stochastic* model for the *velocity gradient tensor*

Chevillard-Meneveau, Physical Review Letters 97, 174501 (2006)

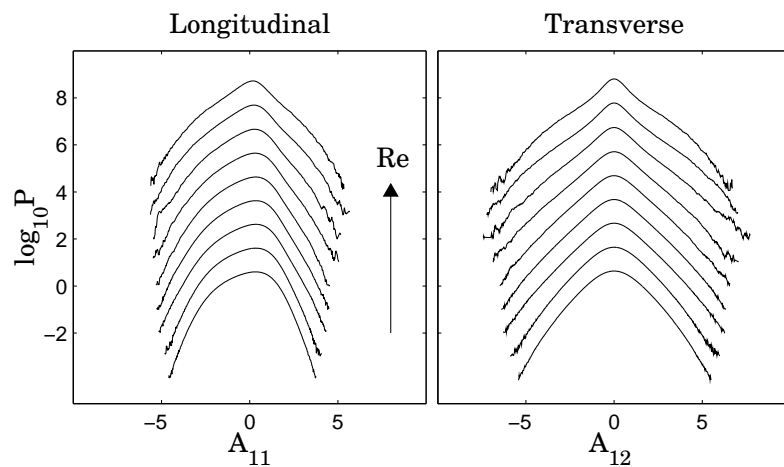
$$d\mathbf{A} = \left(\underbrace{-\mathbf{A}^2}_{\text{Self-stretching}} + \underbrace{\frac{\mathbf{c}_\tau^{-1}}{\text{Tr}(\mathbf{c}_\tau^{-1})} \text{Tr}(\mathbf{A}^2)}_{\text{Pressure Hessian}} - \underbrace{\frac{\text{Tr}(\mathbf{c}_\tau^{-1})}{3T} \mathbf{A}}_{\text{Viscous}} \right) dt + \underbrace{d\mathbf{W}}_{\text{Forcing}}$$

- Simplest **white-in-time Gaussian forcing**
→ Tracefree-Isotropic-Homogeneous-Unit variance
- **Explicit** Reynolds number \mathcal{R}_e dependence when $\tau = \tau_K$
- **Fluctuating** (Local) dissipative time scale $\tau = \Gamma(\mathcal{R}_e) / \sqrt{\text{Tr}(2\mathbf{S}^2)}$

$$\mathcal{R}_e \text{ effects} \rightarrow \begin{cases} \text{Isotropization of Pressure Hessian} \\ \text{Weakening Viscous term} \end{cases}$$

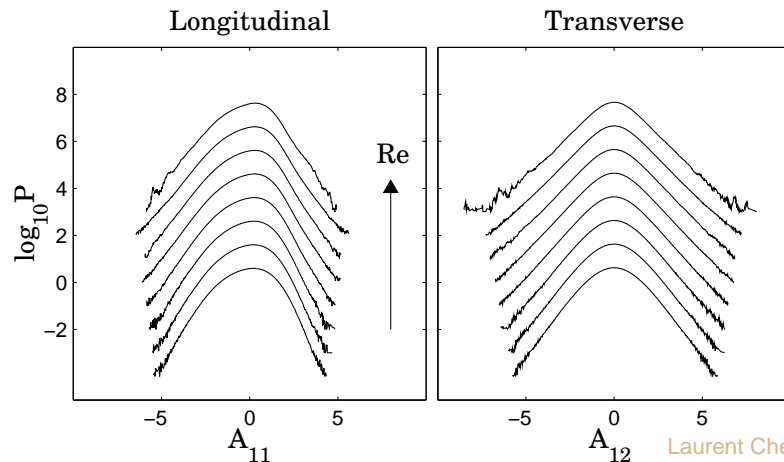
Prediction of *Intermittency* (I): Deformation of PDFs

Chevillard & Meneveau, C.R. Mécanique 335, 187 (2007).



$$\tau = \tau_K$$

- **Continuous** deformation \leftrightarrow **Intermittency**
- At high $\mathcal{R}_e \rightarrow$ Not realistic

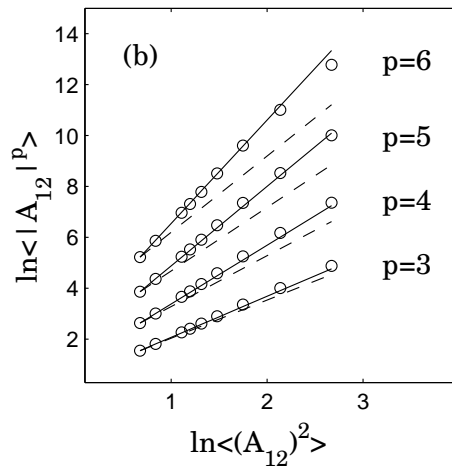
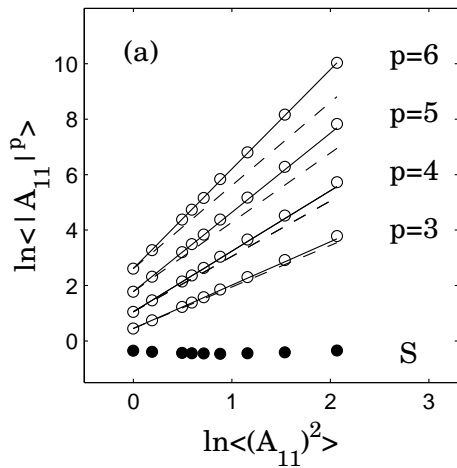


$$\tau = \Gamma(\mathcal{R}_e) / \sqrt{\text{Tr}(2\mathbf{S}^2)}$$

- **Continuous** deformation \leftrightarrow **Intermittency**
- Very realistic but $\Gamma > \Gamma_c$ for regularization

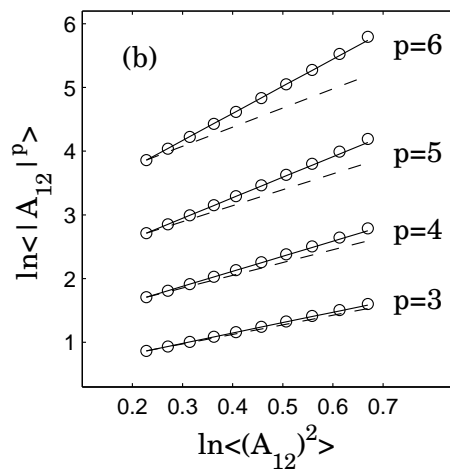
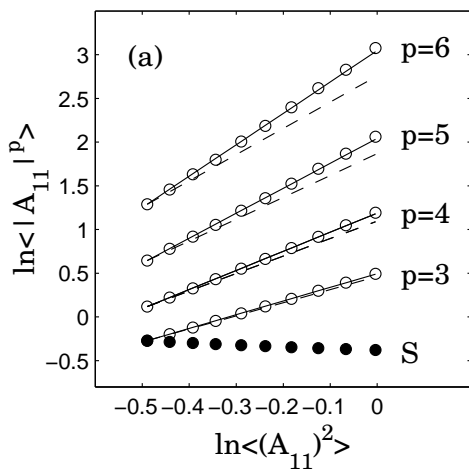
Prediction of *Intermittency* (II): Relative scalings

- - (K41 Monofractal) and — Nelkin's **Multifractal** predictions (Lognormal $\rightarrow c_2$)



$$\tau = \tau_K$$

- c_2 Long $\approx 0.025 \rightarrow \mu \approx 0.22$
- c_2 Trans ≈ 0.040
- Skewness $\approx -0.35 \rightarrow -0.5$



$$\tau = \Gamma(\mathcal{R}_e) / \sqrt{\text{Tr}(2\mathbf{S}^2)}$$

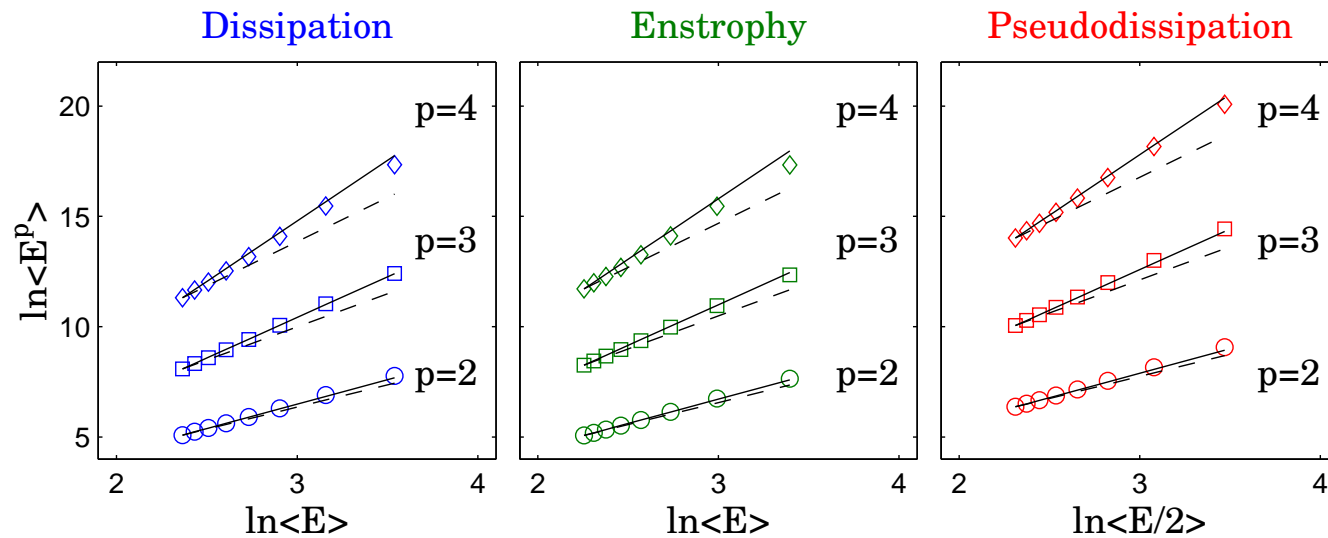
- c_2 Long ≈ 0.025
- c_2 Trans ≈ 0.045
- Skewness $\approx -0.35 \rightarrow -0.5$

\Rightarrow **Robustness (Universality)**

Prediction of *Intermittency* (III): Relative scalings of *Measures*

- - (K41) and — Nelkin-Meneveau's **Multifractal** predictions (Lognormal $\rightarrow \mu$)

$$\mathbf{A} = \underbrace{\mathbf{S}}_{\text{Deformation}} + \underbrace{\mathbf{\Omega}}_{\text{Rotation}} \left\{ \begin{array}{l} \text{Dissipation } \epsilon = \text{Tr}(\mathbf{S}^2) \rightarrow \mu^\epsilon \\ \text{Enstrophy } \zeta = -\text{Tr}(\mathbf{\Omega}^2) \rightarrow \mu^\zeta \\ \text{Pseudodissipation } \varphi = \text{Tr}(\mathbf{A}\mathbf{A}^T) \rightarrow \mu^\varphi \end{array} \right.$$



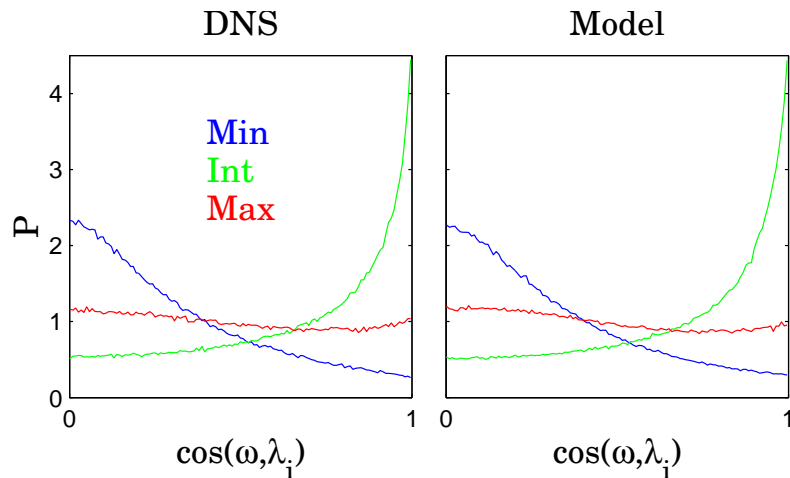
Same intermittency

Conclusions: $\mu^\epsilon = \mu^\zeta = \mu^\varphi = \mu \approx 0.25 \approx \underbrace{c_2^{\text{Long}}}_{\times 9}$

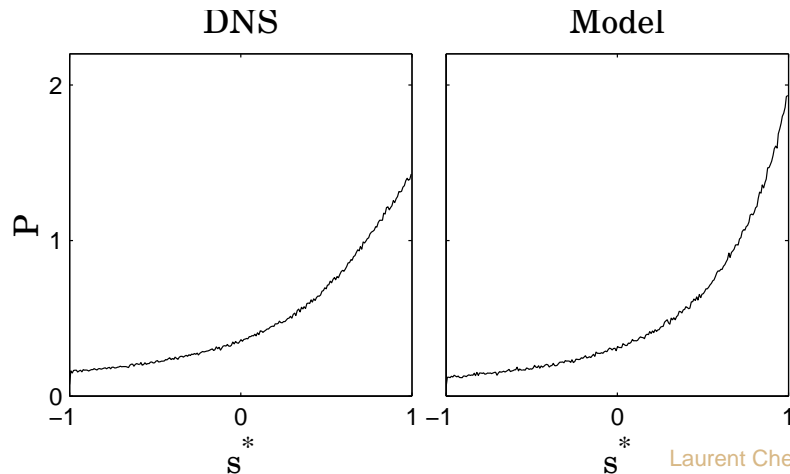
Refined Similarity Hypothesis

DNS comparisons (I)

- **DNS** 256^3 : $\mathcal{R}_\lambda = 150$
- **Model**: $\tau_K/T = 0.1$ (Consistent with Yeung *et al.* JoT 06)



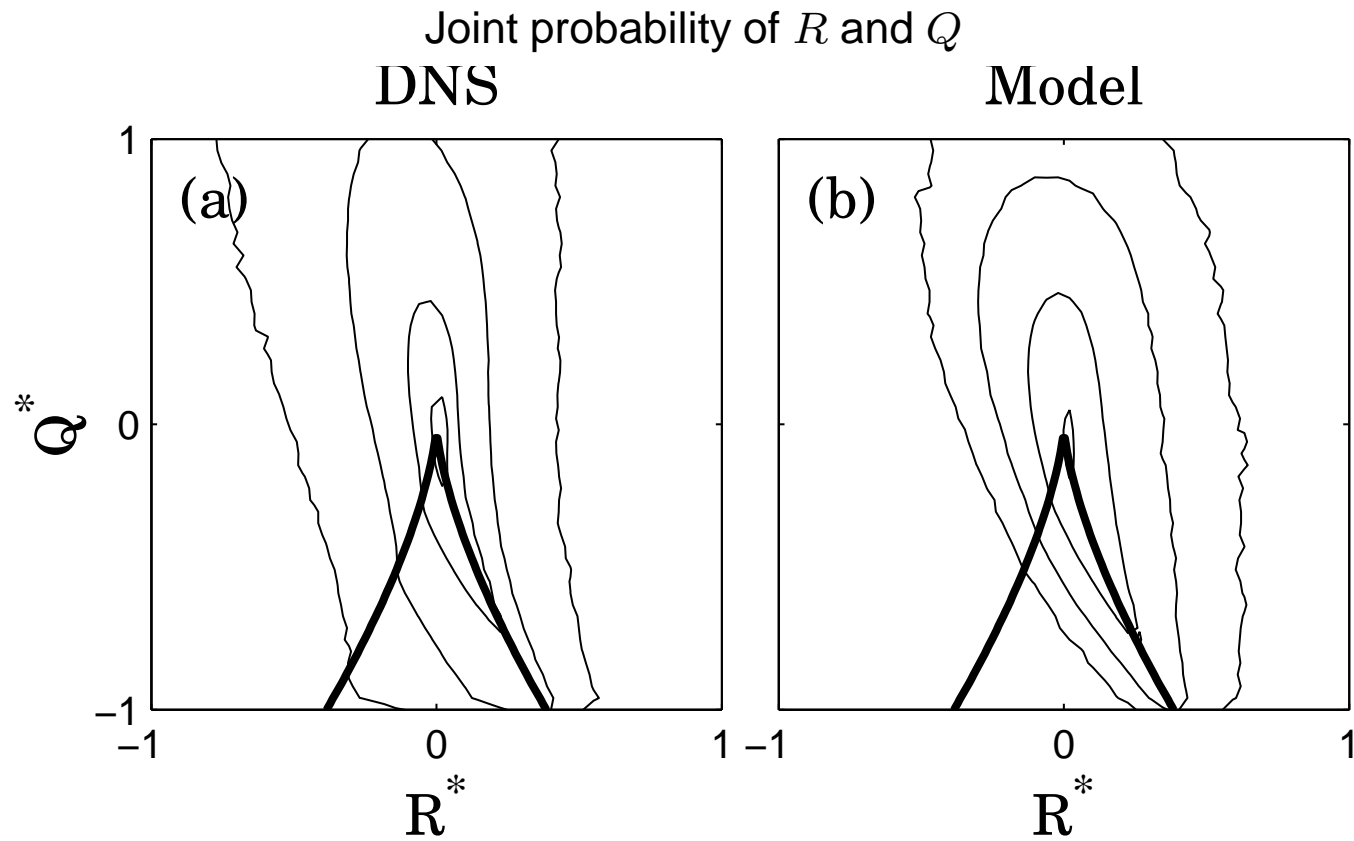
Alignment of vorticity with eigenvectors of strain **S**
 → Preferential **alignment**



PDF of rate of **Strain** s^* :

$$s^* = \frac{-3\sqrt{6}\alpha\beta\gamma}{(\alpha^2 + \beta^2 + \gamma^2)^{3/2}}$$
 → Preferential **axisymmetric expansion**

DNS comparisons (II)



DNS comparisons (III)

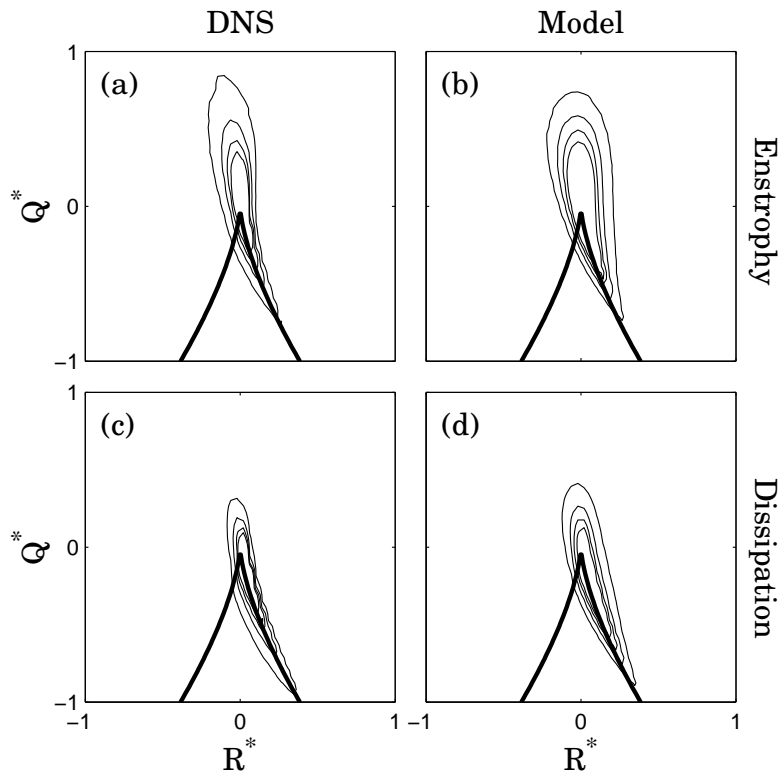
Focussing on Enstrophy-Dissipation dominated regions (see Chertkov *et al.* 99):

$$Q = -\frac{1}{2}\text{Tr}(\mathbf{A}^2) = \frac{1}{4} \overbrace{|\boldsymbol{\omega}|^2}^{\text{Enstrophy}} - \frac{1}{2} \overbrace{\text{Tr}(\mathbf{S}^2)}^{\text{Dissipation}}$$

Conditional average:

$$\langle |\boldsymbol{\omega}|^2 | R, Q \rangle \mathcal{P}(R, Q)$$

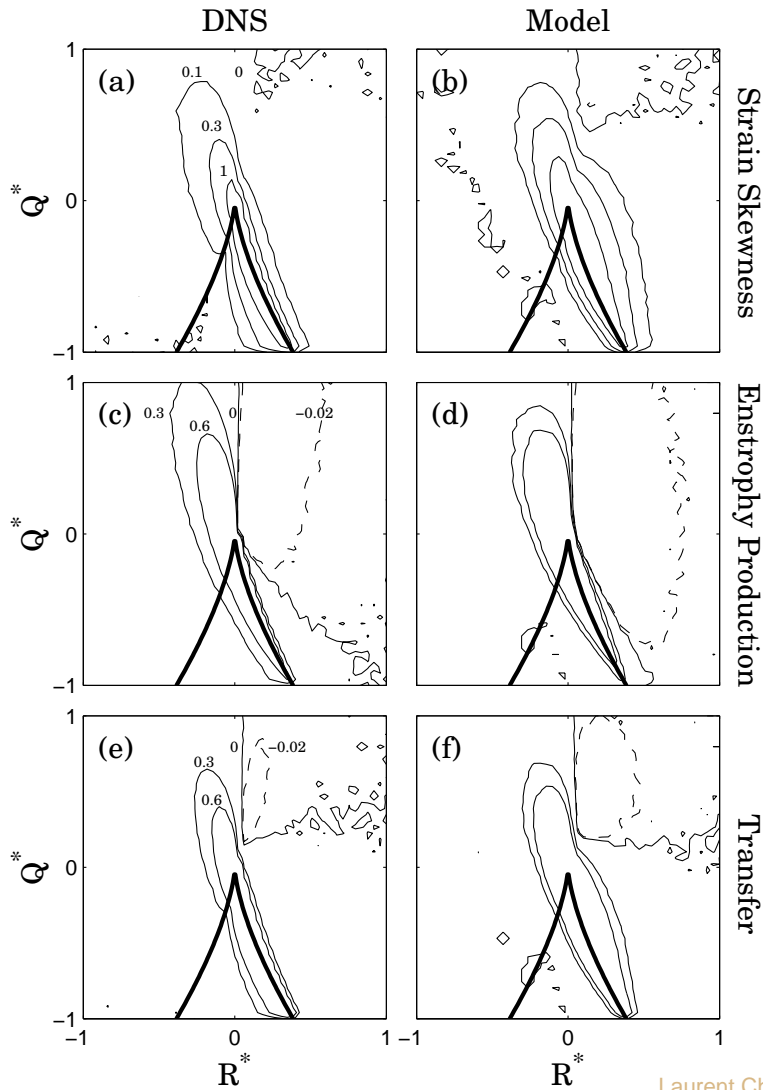
$$\langle \text{Tr}(\mathbf{S}^2) | R, Q \rangle \mathcal{P}(R, Q)$$



DNS comparisons (IV)

$$R = -\frac{1}{3}\text{Tr}(\mathbf{A}^3) = -\frac{1}{4} \underbrace{\omega_i S_{ij} \omega_j}_{\text{Enstrophy Production}} - \frac{1}{3} \underbrace{\text{Tr}(\mathbf{S}^3)}_{\text{Strain Skewness}}$$

Conditional average (Chertkov *et al.* 99):

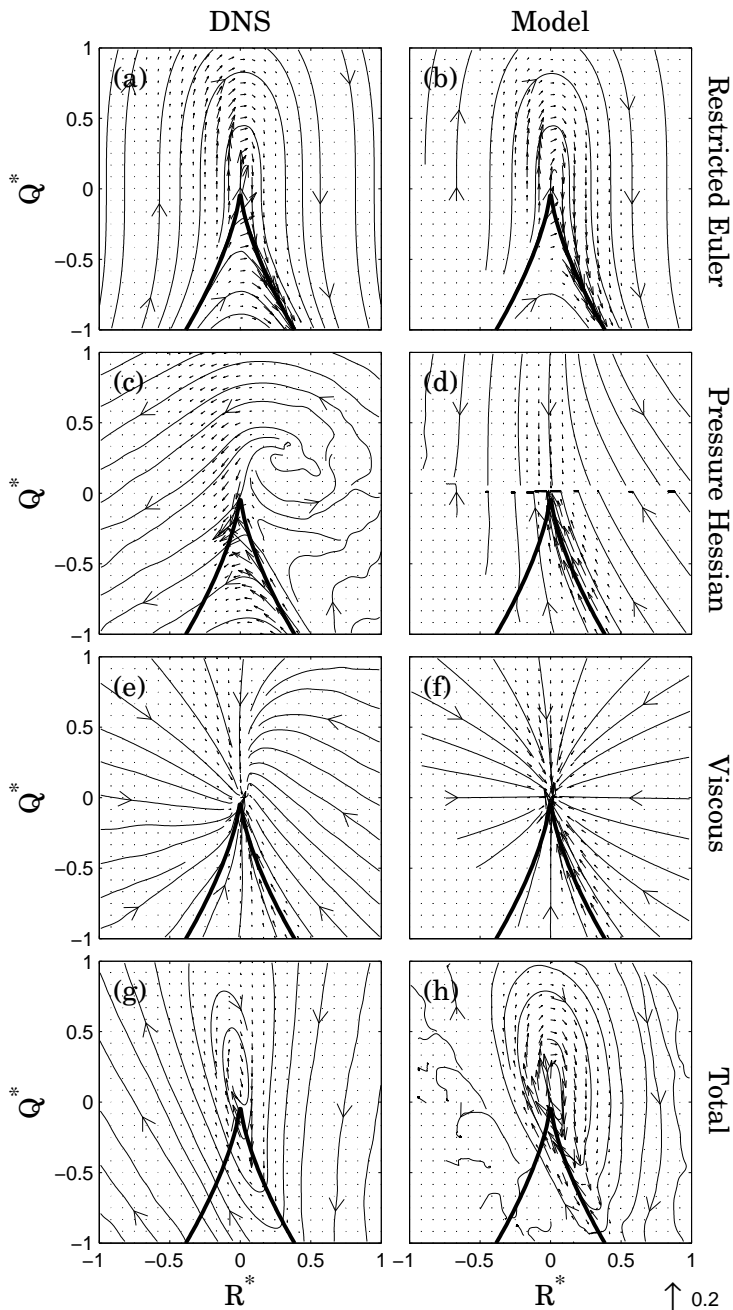


$$\langle -\text{Tr}(\mathbf{S}^3) | R, Q \rangle \mathcal{P}(R, Q)$$

$$\langle \omega_i S_{ij} \omega_j | R, Q \rangle \mathcal{P}(R, Q)$$

$$\langle -\text{Tr}(\mathbf{A}^T \mathbf{A}^2) | R, Q \rangle \mathcal{P}(R, Q)$$

DNS comparisons (V): *Focussing* on Pressure Hessian and Viscous effects



Cond. average (van der Bos *et al.* 02):

$$\left\langle \begin{pmatrix} dR/dt \\ dQ/dt \end{pmatrix}_{\text{RE}} \middle| R, Q \right\rangle \mathcal{P}(R, Q)$$

$$\left\langle \begin{pmatrix} dR/dt \\ dQ/dt \end{pmatrix}_{\text{PH}} \middle| R, Q \right\rangle \mathcal{P}(R, Q)$$

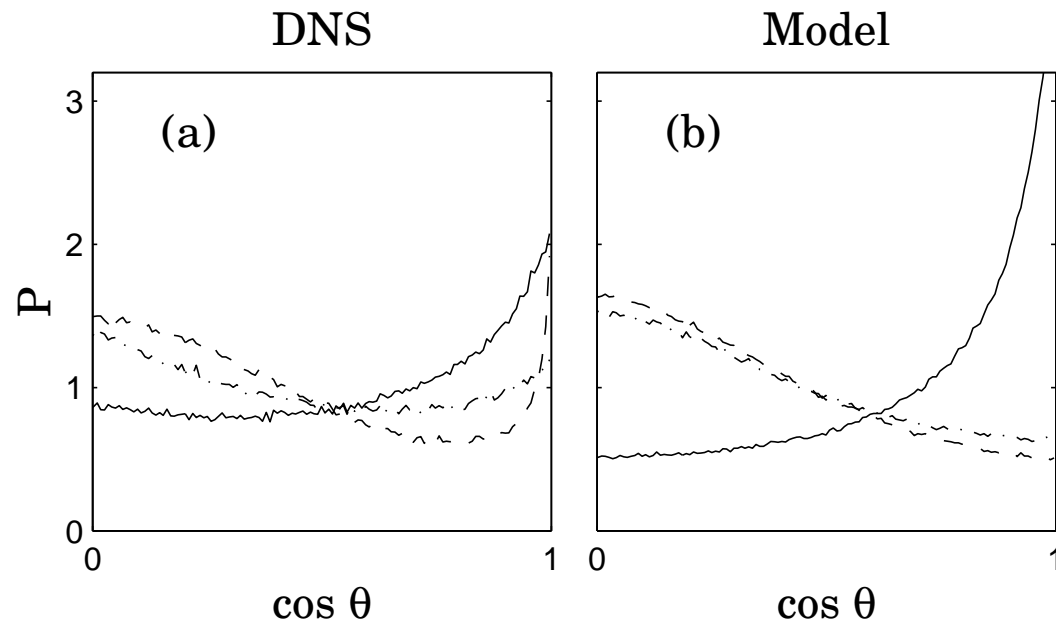
$$\left\langle \begin{pmatrix} dR/dt \\ dQ/dt \end{pmatrix}_{\text{Viscous}} \middle| R, Q \right\rangle \mathcal{P}(R, Q)$$

$$\left\langle \begin{pmatrix} dR/dt \\ dQ/dt \end{pmatrix}_{\text{Total}} \middle| R, Q \right\rangle \mathcal{P}(R, Q)$$

Alignment of Vorticity with Pressure Hessian eigenvectors

Euler equations

$$\overbrace{\text{Ohkitani (93), Gibbon et al. (97, 06, 07)}} \Rightarrow \begin{cases} \frac{d\omega_i}{dt} = S_{ij}\omega_j & \text{Vorticity stretching} \\ \frac{d^2\omega_i}{dt^2} = -P_{ij}\omega_j & \text{Ertel's Theorem (42)} \end{cases}$$

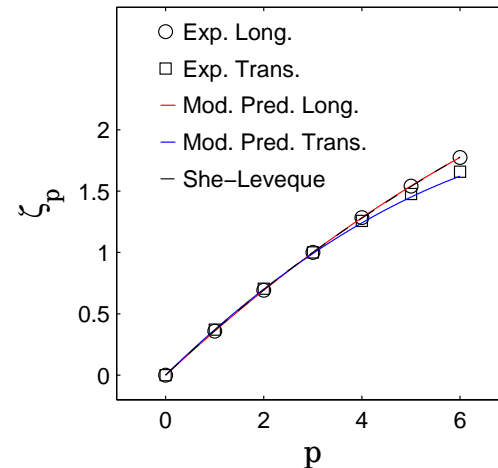


Alignments with "intermediate" eigenvector reproduced
 Alignments with "smallest" eigenvector NOT reproduced

Conclusions

8 independent **ODEs** $d\mathbf{A}/dt = -\mathbf{A}^2 - \mathbf{P} + \nu \Delta \mathbf{A}$

- A **new stationary** stochastic model for \mathbf{A} including closures for
 - Pressure Hessian \mathbf{P}
 - Velocity gradient Laplacian $\nu \Delta \mathbf{A}$
 → Physics of **Recent** deformation
- Well-known properties of turbulence (**vorticity** alignments, RQ-plane, skewness) well reproduced. Discrepancies in **Enstrophy** dominated regions.
- **Prediction** of Intermittency
 - **quantitative** agreement with standard data
 - Transverse more intermittent than Longitudinal
 - Dissipation and Enstrophy scale the same



Perspectives

- Improving **rotation-vorticity stretching** dominated regions
- Reaching **very high** \mathcal{R}_e (see Biferale et al., PRL **98**, 214501 (2007).)
- Modeling **Subgrid**-scale stress tensor (See Chevillard, Li, Eyink, Meneveau)