# Two-dimensional turbulences


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Thanks: D. Bernard, G. Boffetta, M. Cencini, G. Falkovich, A. Mazzino, S. Musacchio, M. Vergassola

# Flatland by Edwin A. Abbott (1884)

*Dedication*

To
The Inhabitance of SPACE IN GENERAL
And H.C. IN PARTICULAR
This Work is Dedicated
By a Humble Native of Flatland
In the Hope that
Even as he was Initiated into the Mysteries
Of THREE DIMENSIONS
Having been previously conversant
With ONLY TWO
So the Citizens of that Celestial Region
May aspire yet higher and higher
To the Secrets of FOUR FIVE or EVEN SIX Dimensions
Thereby contributing
To the Enlargment of THE IMAGINATION
And the possible Development
Of that most and excellent Gift of MODESTY
Among the Superior Races
Of SOLID HUMANITY

# Two-dimensional turbulence

Oceans and atmospheres

# 2D, or not 2D ?

## or, does the aspect ratio *alone* ensure the two-dimensional character of the flow ?

• Yes, if $L_z / L_x < \eta$ (viscous scale)

• For $L_z / L_x > \eta$, numerical simulations (Smith & Yakhot PRL ‘94, Musacchio 08) suggest coexistence of 2D and 3D features for $L_z / L_f <\sim 0.5$

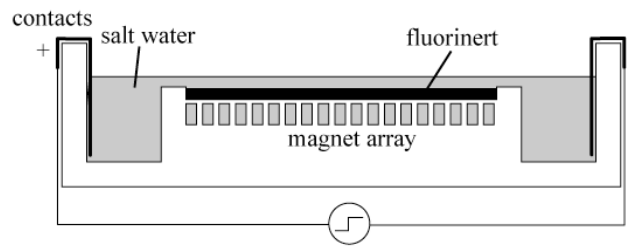
•Analytics and numerics for passive scalar turbulence shows coexistence of different dimensional features for *all* $L_z / L_f$ (AC ‘08)

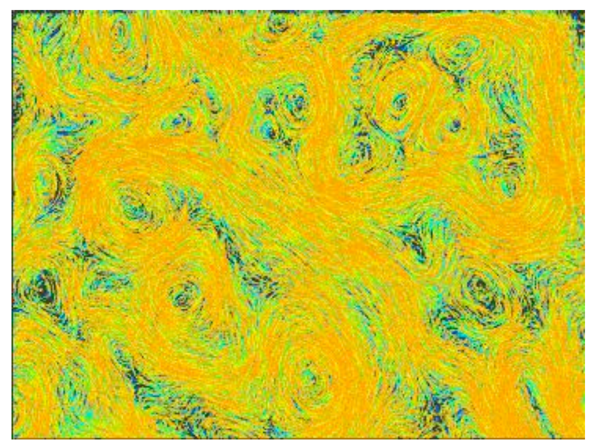
# Two-dimensional turbulence in the lab: electrolyte cell

Shallow layer of electrolyte, about 1 mm deep, possibly stratified, driven by electromagnetic fields.

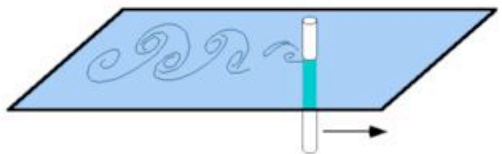
Seeded with small (10 microns) neutrally buoyant particles for visualization

(Sommeria,Tabeling, Ecke)

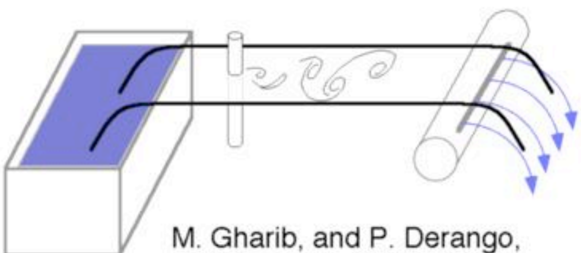


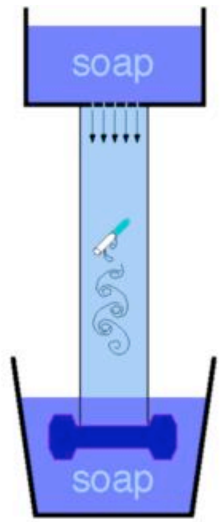
# Two-dimensional turbulence in the lab: soap films



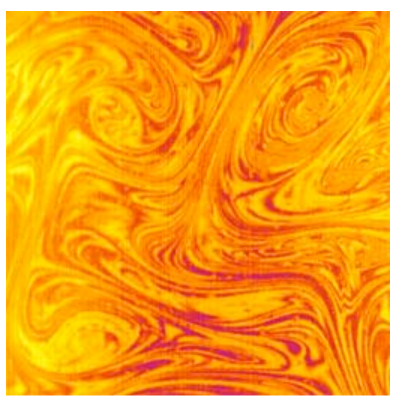
Y. Couder, J. M. Chomaz, and M. Rabaud, Physica **D37**, 384 (1989).



M. Gharib, and P. Derango, Physica **D37**, 406 (1989).

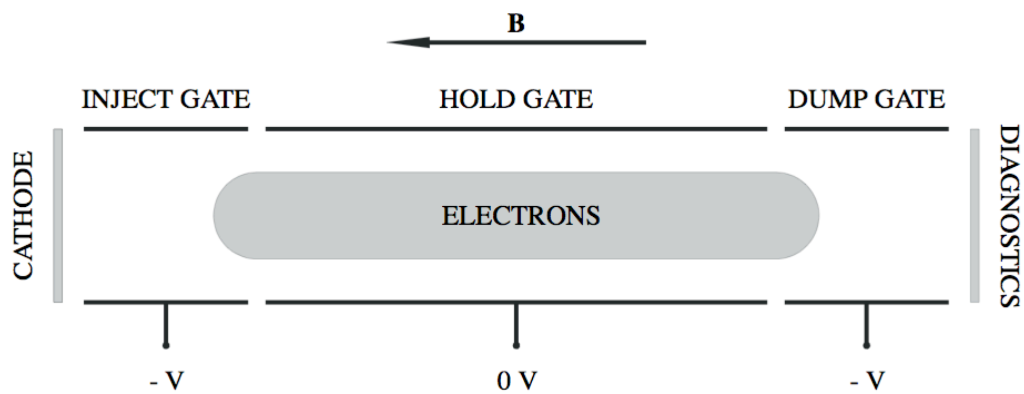


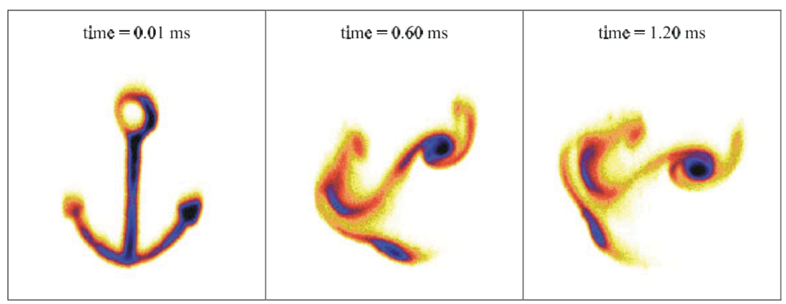

Kellay, Wu, and Goldburg, PRL **74**, 3875 (1995).



- Fast flowing (up to 10 m/s) soap film, few cm wide, 30 microns thick, driven by combs or rods.
- Visualization by optical intereference or particle seeding
- (Couder, Goldburg, Rutgers, Rivera, Kellay, Ecke)

# Two-dimensional turbulence in the lab: Malmberg-Penning traps

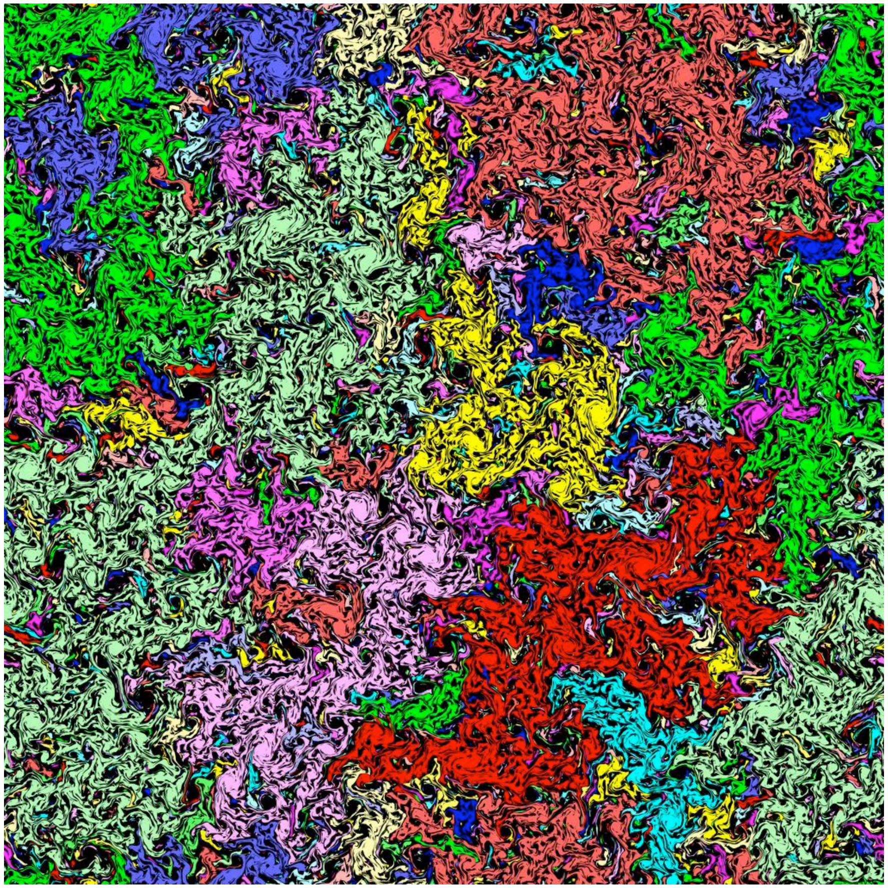



Electrons confined by electromagnetic fields
Decaying turbulence; observation is destructive (!)

# Two-dimensional turbulence *in silico*

- Numerical integration of 2D fluid equations on lattices (up to $16384^2$ points, Bernard *et al* Nature Physics '06, Boffetta JFM '07)

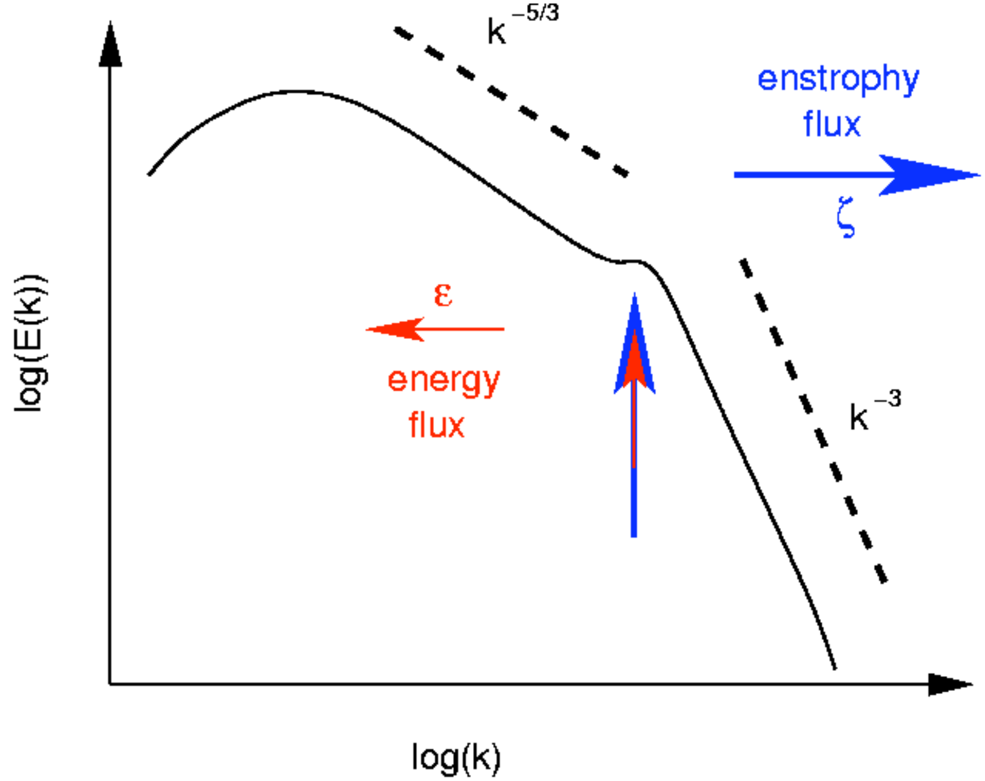


# 2D Navier-Stokes turbulence

$$\partial_t \omega + v \nabla \omega = \nu \nabla^2 \omega + f$$

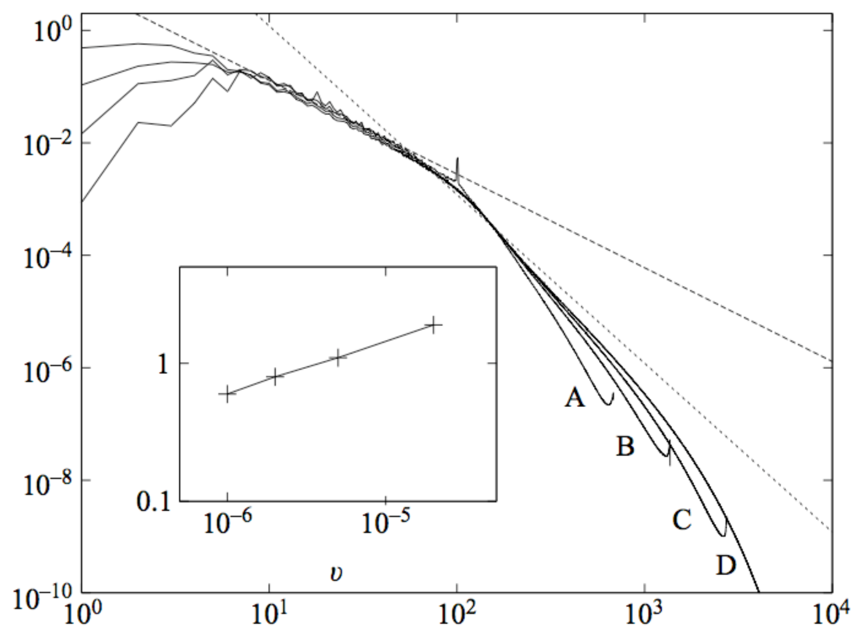
$$v = z \times \nabla \psi \qquad \omega = - \nabla^2 \psi$$

- Double cascade (Kraichnan - Batchelor - Leith ~ 67-71)



# Was Kraichnan right ?

- Claims against BKL scenario
- Exact bounds (Foias, Doering, Tran, Dritschel) for *finite* systems
- Finite size effects or deviations ?
- Present numerical and experimental data support Kraichnan's theory




Boffetta JFM '07

# 2D Navier-Stokes turbulence

Perspectives:

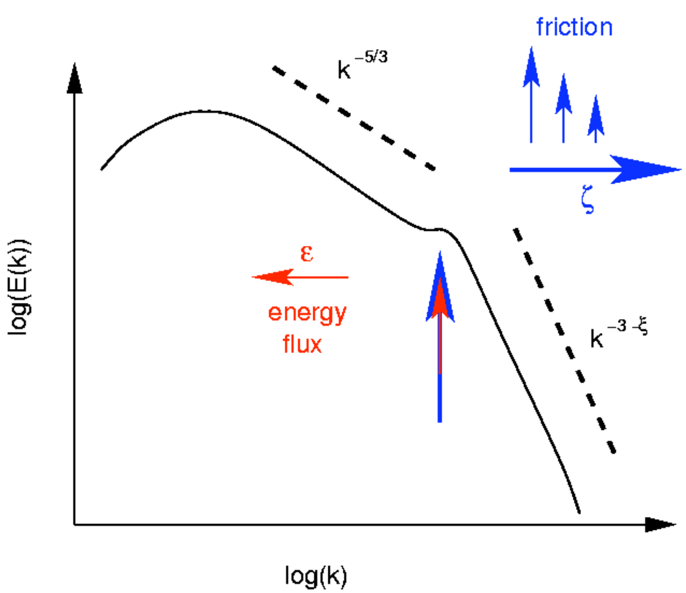
- Conformal invariance
- Relation to non-equlibrium stat mech
- Point vortices

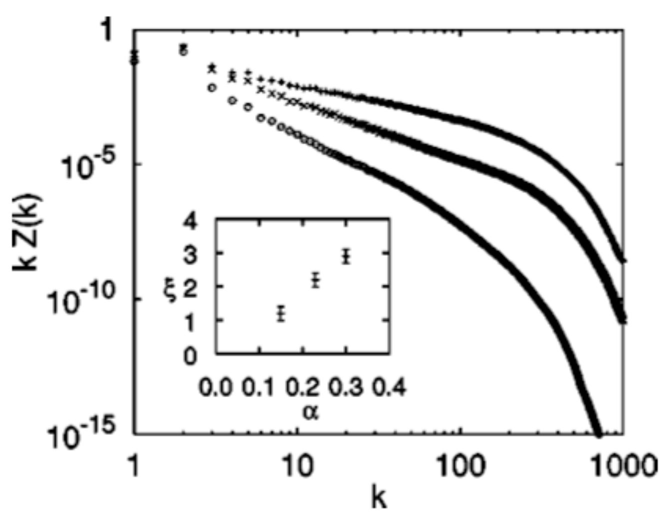
# Ekman-Navier-Stokes turbulence

$$\partial_t \omega + v \nabla \omega = \nu \nabla^2 \omega + f - \alpha\omega$$

$$v = z \times \nabla \psi \qquad \omega = -\nabla^2 \psi$$

- Friction reduces enstrophy transfer (Lilly '71, Ott '00, Chertkov '99 for passive scalar, AC et al '02)

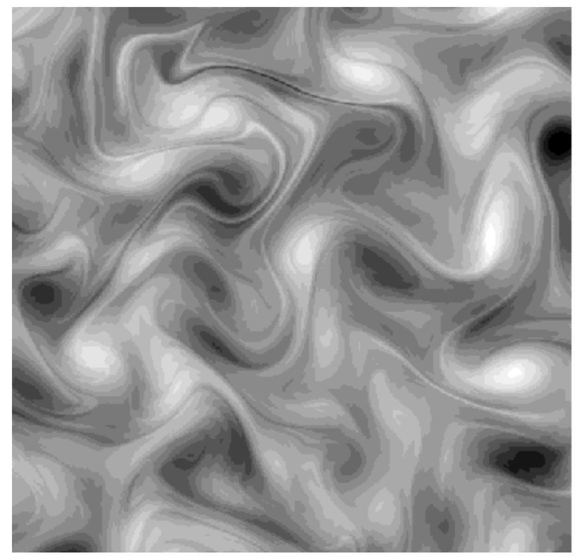


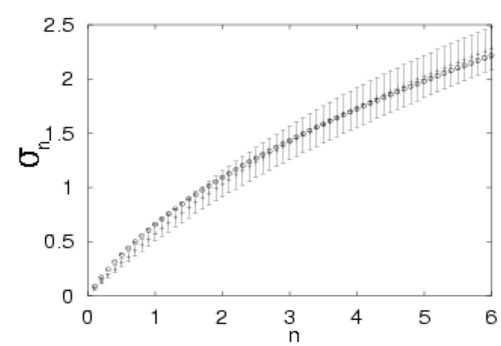



# Ekman-Navier-Stokes turbulence

$$\partial_t \omega + v \nabla \omega = \nu \nabla^2 \omega + f - \alpha\omega$$

- Competition between friction and stretching
- Lyapunov exponents
- Intermittency of vorticity
- Nonuniversal behavior
- Limit of strong friction analyticallly solvable ?

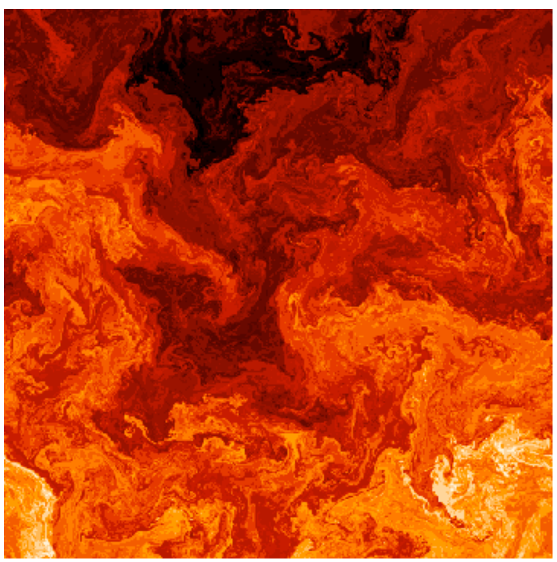




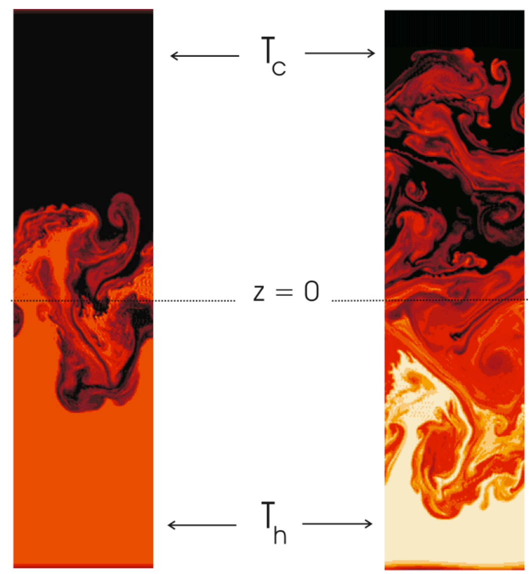
# 2D Boussinesq turbulence

$$\partial_t v + v\,\nabla\, v = -\,\nabla p + \nu \nabla^2 v - \alpha v \; - \beta\, g\, T$$
$$\partial_t T + v\,\nabla T = \kappa \nabla^2 T + f$$

Gradient-driven (f=-v.g)



Rayleigh-Taylor (f=0)




Bolgiano '59 scaling (not observed in 3D !) PRL '02, PRL '06

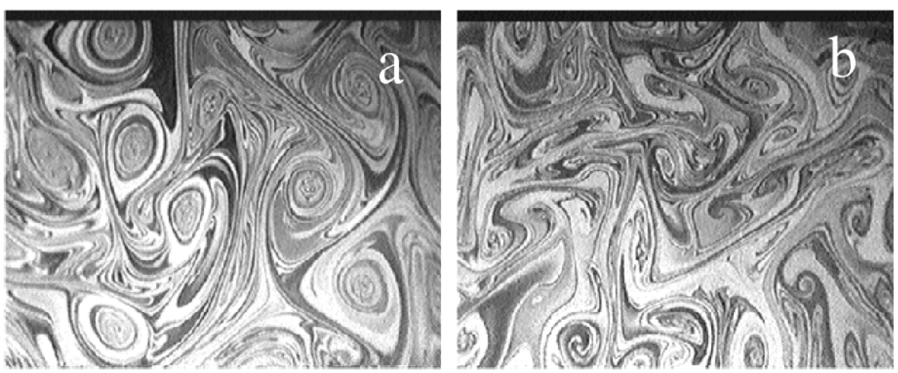
# 2D viscoelastic turbulence

$$\partial_t v + v \nabla v = -\nabla p + \nu \nabla^2 v - \alpha v + f + \eta \nu \nabla \sigma / \tau$$

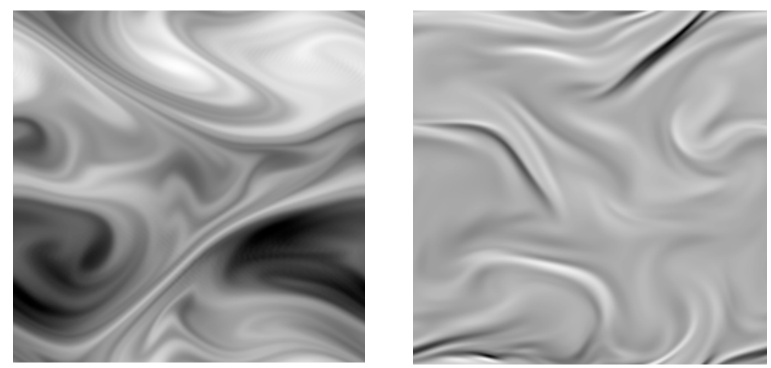
$$\partial_t \sigma + v \nabla \sigma = \sigma^t \nabla v + \nabla v \sigma - 2(\sigma - 1)/\tau$$

$$\sigma = \langle R R \rangle$$

- Polymer solutions (soap film Kellay, Goldburg, em cell Bonn)
- Suppression of large-scale structures




Soap film experiments

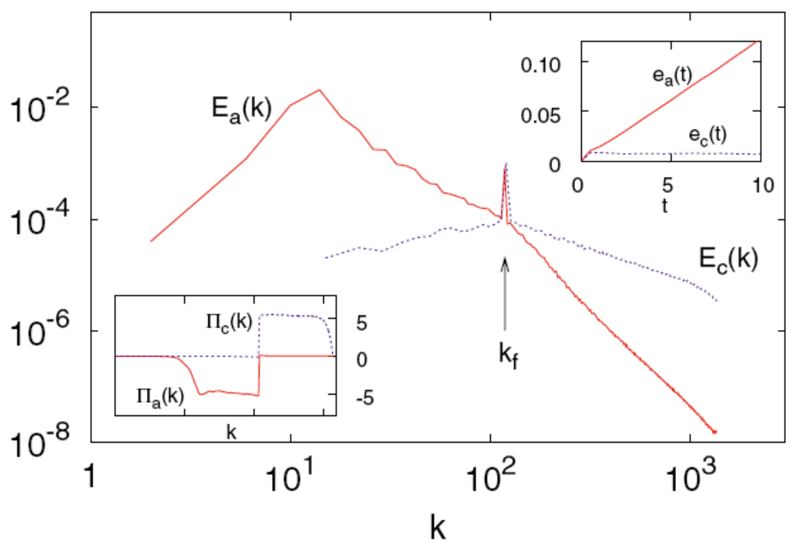


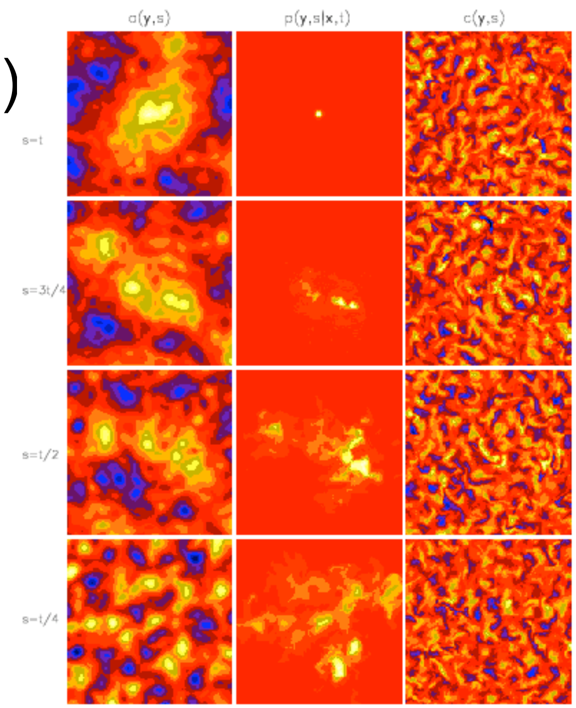
Numerics (Oldroyd B, PRL'03)

# 2D magnetohydrodynamic turbulence

$$\partial_t v + v \nabla v = - \nabla p + \nu \nabla^2 v - \nabla \phi \nabla^2 \phi$$
$$\partial_t \phi + v \nabla \phi = \kappa \nabla^2 \phi + f$$
$$b = z \times \nabla \phi$$

- Double cascade
- Active vs passive scalar (PRL '02)



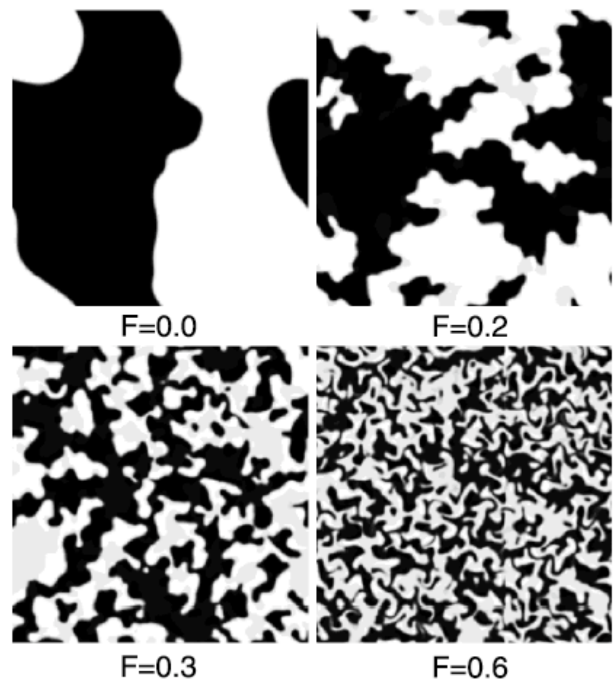
# 2D binary fluid turbulence

$$\partial_t v + v \nabla v = -\nabla p + \nu \nabla^2 v - \nabla \phi \nabla^2 \phi + f$$

$$\partial_t \phi + v \nabla \phi = \lambda \nabla^2 (\delta \mathcal{F} / \delta \phi)$$

$$\mathcal{F} = \int [\tfrac{1}{2}(\nabla \phi)^2 - \tfrac{1}{2}\phi^2 + \tfrac{1}{4}\phi^4]\, dx$$

- Stirring suppresses coarsening (Berti et al, PRL 05)



# Analogies

- MHD, binary fluids, polymer solutions: similar feedback on NS
- Similar effect: energy inverse cascade is hindered
- Kinetic energy transfer to other modes: magnetic field, line tension, polymer elasticity
- Waves: Alfvèn, capillary, elastic
- Task: complete the dictionary ….