

Multifractals. Eulerian and Lagrangian Statistics

Vienna 2008

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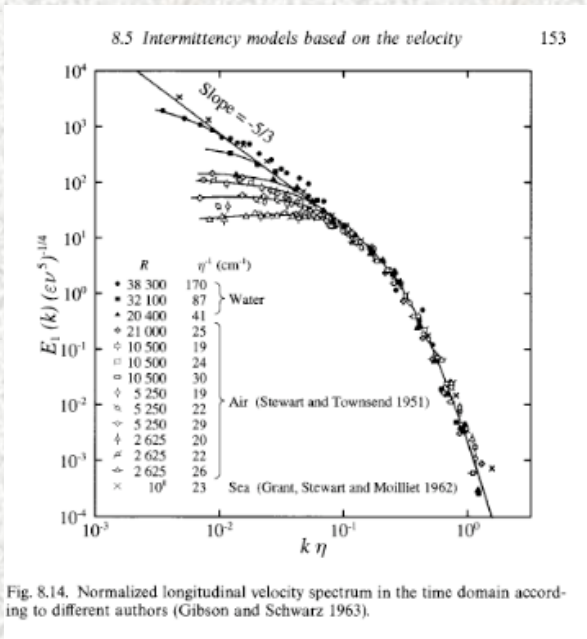


ICTR
Collaboration



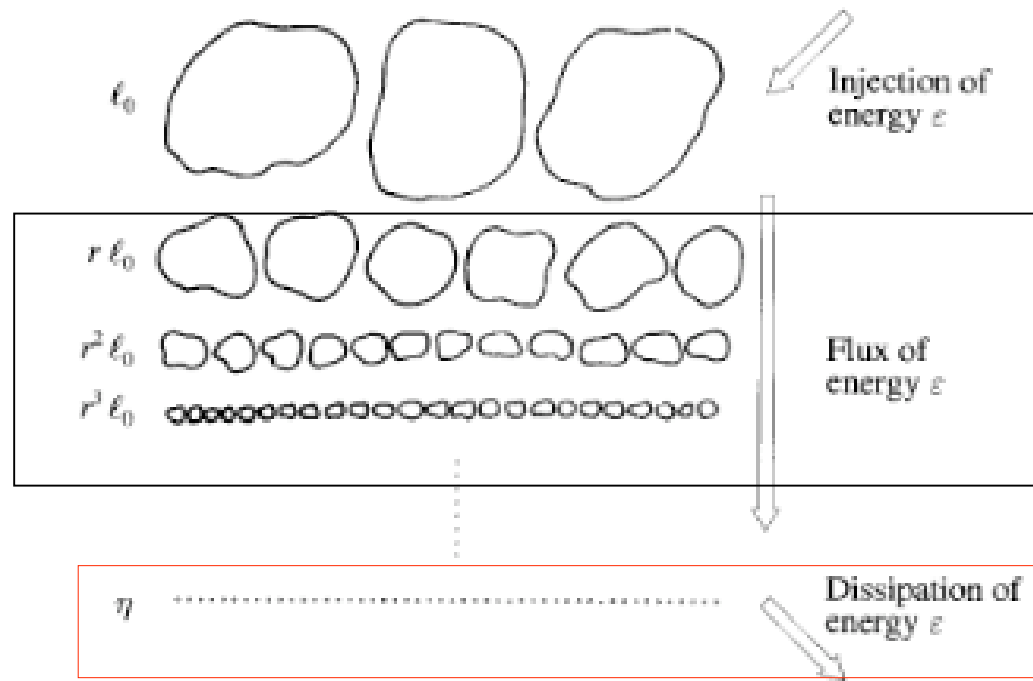
Eulerian Turbulence

$$\partial_t \mathbf{v} + \mathbf{v} \cdot \nabla \mathbf{v} = -\nabla p + \nu \Delta \mathbf{v} + \mathbf{f}$$



Energy flux

104 Phenomenology of turbulence in the sense of Kolmogorov 1941

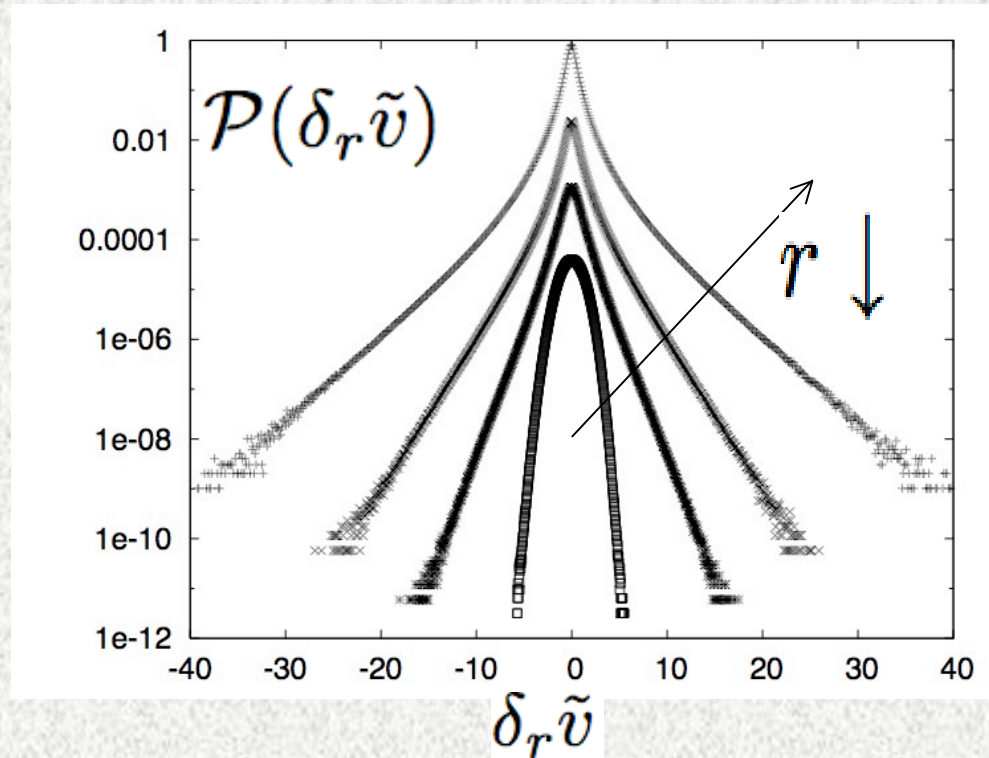


$\eta \ll r \ll L_0$
Inertial range

$$\eta = (\nu^3 / \epsilon)^{1/4}$$

$$\tau_\eta = (\nu / \epsilon)^{1/2}$$

INTERMITTENCY



[DNS $Re_\lambda = 600$
2048³ Benzi et al 2007,
PRL, submitted]

$$\delta_r \tilde{v} = \frac{\delta_r v}{\langle (\delta_r v)^2 \rangle^{1/2}}$$

$$F_4(r) = \frac{\langle (\delta_r v)^4 \rangle}{\langle (\delta_r v)^2 \rangle^2}$$

$$F_4(r) \neq const.$$

The “Standard Model”

$$S_p(r) = \langle [v(\mathbf{x} + \mathbf{r}) - v(\mathbf{x})]^p \rangle \quad \eta \ll r \ll L_0$$

$$\delta_r v \sim v_0 \left(\frac{r}{L_0} \right)^{h(x)} \quad \mathcal{P}_h(r) \sim \left(\frac{r}{L_0} \right)^{3-D(h)}$$

$$S_p(r) = \langle (\delta_r v)^p \rangle \sim \langle v_0^p \rangle \int_I dh \left(\frac{r}{L_0} \right)^{hp+3-D(h)}$$

$$S_p(r) \sim \left(\frac{r}{L_0} \right)^{\zeta_p}$$

Parisi-Frisch 1983

Benzi, Paladin, Parisi & Vulpiani 1984

$$\zeta_p = \inf_h (hp + 3 - D(h))$$

$$F_4(r) \sim r^{\zeta(4) - 2\zeta(2)}$$

$D(h)$

CONNECTION CUMULANTS -- STRUCTURE FUNCTIONS

$$S_p(r) = \langle (\delta_r v)^p \rangle$$

$$\kappa_n(r) = \langle (\log |\delta_r v|)^n \rangle$$

$$S_p(r) = \exp \sum C_n(r) \frac{p^n}{n!}$$

$$\zeta(p)$$

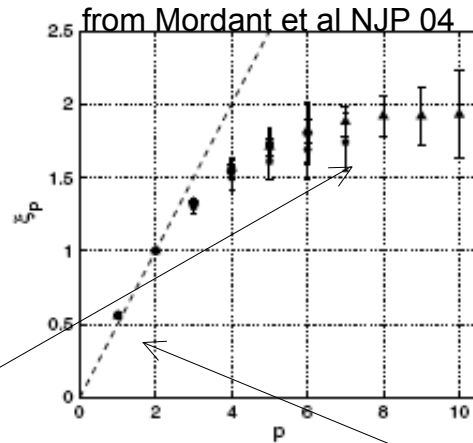


Figure 25. Structure function relative exponents ξ_p : (.....) experiment; (\blacktriangle) DNS $R_\lambda = 75$; (\ast) DNS $R_\lambda = 140$. The dashed line is a K41 prediction for Lagrangian exponents.

$$C_1(r) = \kappa_1(r)$$

$$C_2(r) = \kappa_2(r) - (\kappa_1(r))^2$$

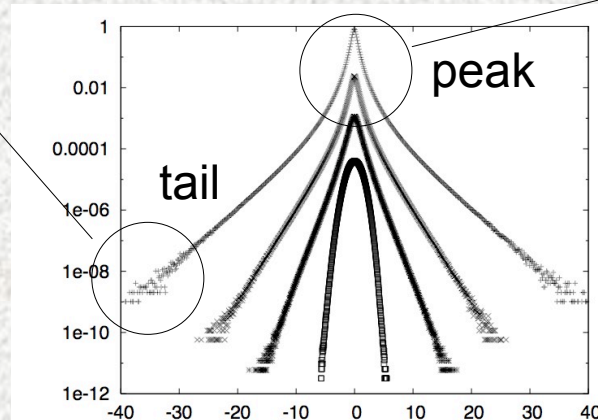
$$C_n(r) = \kappa_n(r) + f(\kappa_{n-1} \dots)$$

[Delour Muzy Arneodo
EPJB 2001]

$$C_n(r) \sim c_n \log(r)$$

$$S_p(r) \sim \left(\frac{r}{L_0} \right)^{\zeta_p}$$

$$c_n = \frac{d^n}{dp^n} \zeta(p) \Big|_{p=0}$$



Velocity field statistics in homogeneous steady turbulence obtained using a high-resolution direct numerical simulation

Toshiyuki Gotoh^{a)}

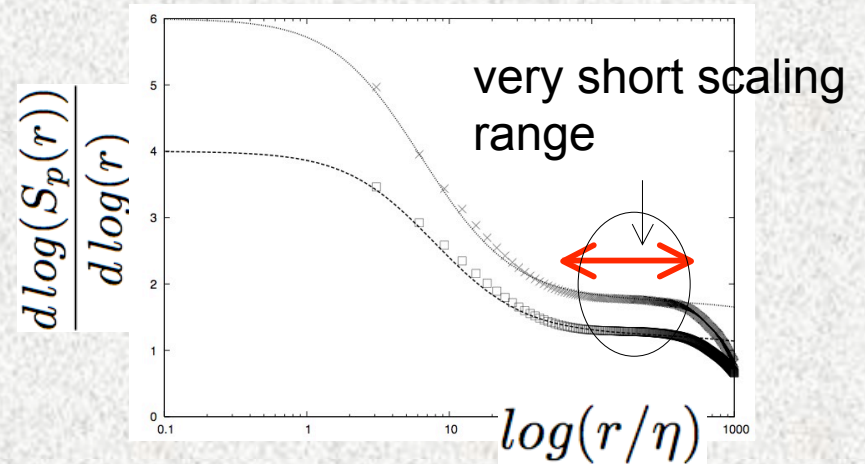
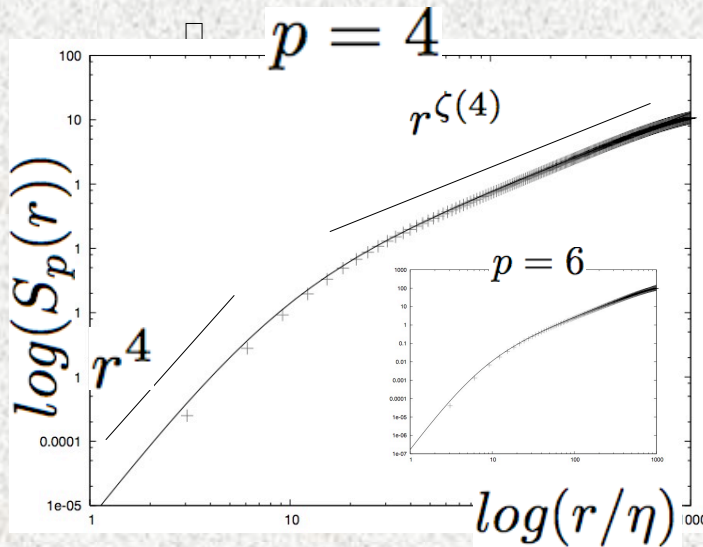
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$$S_p(r) \sim r^{\zeta(p)}$$

$$\zeta_p = \inf_h (hp + 3 - D(h))$$

$$\frac{d \log(S_p(r))}{d \log(r)} \sim \zeta(p)$$

REMOVING FOCUS ON PURE POWER LAW:

TYPICALLY NEVER OBSERVED IN DNS OR CONTROLLED LABORATORY EXPERIMENTS (MODERATE REYNOLDS NUMBERS)

AT HIGH REYNOLDS NUMBERS (ABL, SOLAR WIND ETC..) CONTAMINATION FROM ANISOTROPIES OR/AND NON-HOMOGENEITIES (DIFFICULT TO CONTROL)

IN PRESENCE OF FINITE INERTIAL RANGE EXTENSION:
WHAT TO CONTROL? HOW TO TEST QUANTITATIVELY
INFLUENCE/IMPORTANCE OF VISCOUS AND INTEGRAL
SCALES?

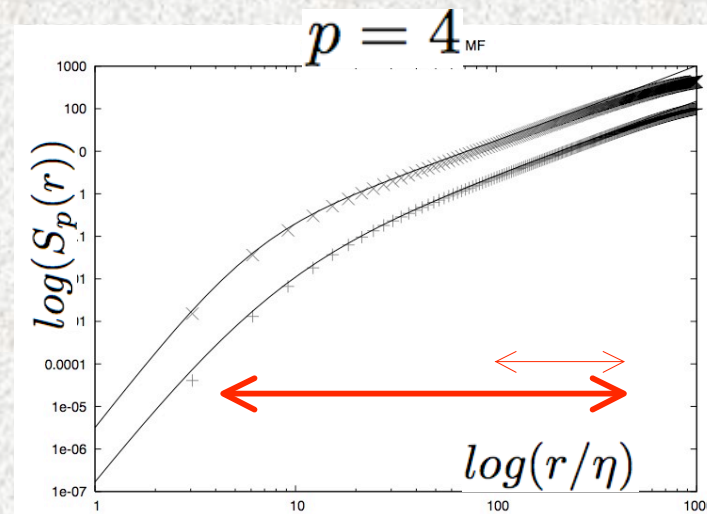
$$\eta \ll r \ll L_0$$

$$\eta \leq r \ll L_0$$

HOW TO CHECK $D(h)$ QUANTITATIVELY CONSIDERING THE NATURAL LIMITATIONS IN THE INERTIAL RANGE EXTENSIONS?

LOOK FOR THE EFFECTS OF VISCOUS SCALES. THE SO-CALLED: INTERMEDIATE DISSIPATIVE RANGE

AND TRY TO TEST MULTIFRACTAL PREDICTION ALSO ON THIS EXTENDED RANGE OF SCALES



Degrees of freedom of turbulence

Giovanni Paladin* and Angelo Vulpiani*

The dissipation scale η can be now determined by imposing that the Reynolds number related to an eddy of length scale l is of order 1,

$$Re(\eta) = \eta \delta_\eta v / \nu \sim \mathcal{O}(1) \tag{5}$$

This is equivalent to the requirement that the dissipative (linear) term of the Navier Stokes equations is able to compete with the nonlinear transfer term.

Inserting Eq. (3) in Eq. (5) we obtain

$$\eta(x) \sim Re^{-1/(1+h(x))} \tag{6}$$

$$Re(r) = \frac{r \delta_r v}{\nu}$$

$$\delta_r v \sim v_0 \left(\frac{r}{L_0}\right)^{h(x)}$$

Multifractal scaling of velocity derivatives in turbulence

Mark Nelkin*

Multifractality in the Statistics of the Velocity Gradients in Turbulence

R. Benzi and L. Biferale

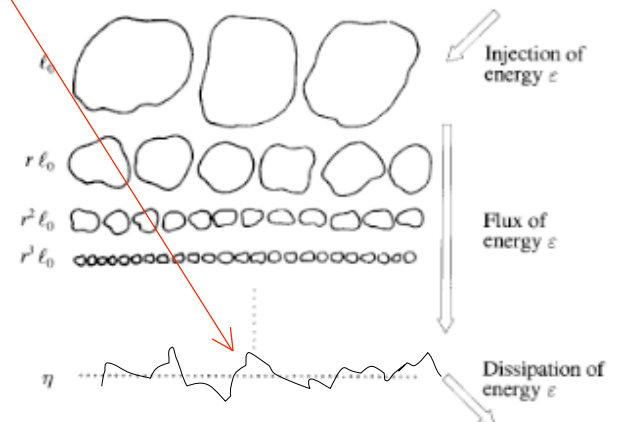
Dipartimento di Fisica, Università "Tor Vergata," via E. Carnevale, I-00173 Roma, Italy

G. Paladin and A. Vulpiani

Dipartimento di Fisica, Università dell'Aquila, I-67010 Coppito, L'Aquila, Italy

M. Vergassola

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A Prediction of the Multifractal Model: the Intermediate Dissipation Range.

U. FRISCH(*) and M. VERGASSOLA(*)(**)

$$S_2(l) = \langle v_l^2 \rangle \sim \int_{\eta(h) < l} d\mu(h) l^{2h+3-D(h)}.$$

The prediction of universal scaling of multifractal type for the energy spectrum can be tested experimentally.

Recently, Gagne and Castaing [16] have analysed a wide sample of turbulence data with Reynolds numbers ranging from (roughly) 10^5 to 10^7 . Remarkable agreement with scaling of multifractal type is obtained over the entire range of available scales. Traditional turbulence measurements involve probes with a length of one millimeter or more which can resolve only the beginning of the intermediate dissipation range. Multifractality predicts that there is considerable life in turbulence well below one-millimeter scales. Smaller probes or high-resolution nonintrusive techniques⁽⁵⁾ should be developed to meet the challenge.

$$\frac{\delta_\eta v \eta}{\nu} \sim \mathcal{O}(1)$$

$$\eta(h) \sim Re^{-\frac{1}{1+h}}$$

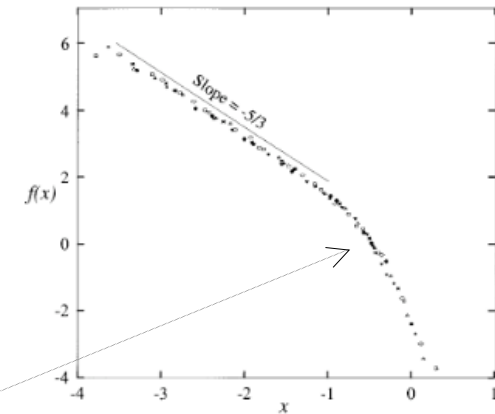
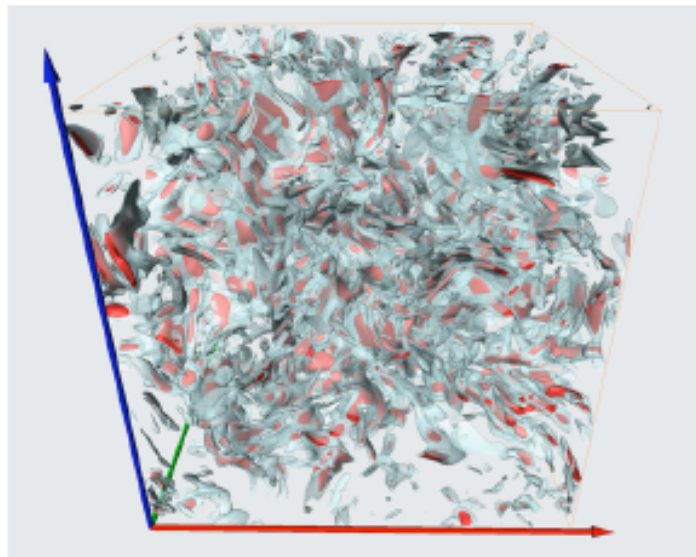
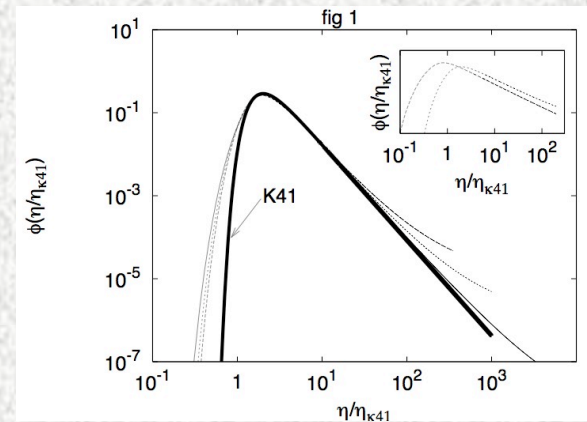


Fig. 8.15. Data in the time domain from nine different turbulent flows with Re 's ranging from 130 to 13000, plotted in log-log coordinates. The wavenumber (horizontal) and the energy spectrum (vertical) have been divided by $\ln(R_i/R_*)$ with $R_* = 75$ and the resulting curves have been shifted to give the best possible superposition (Gagne and Castaing 1991).

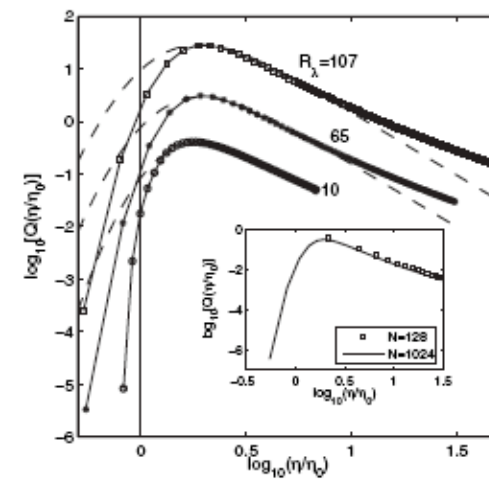
Sub-Kolmogorov-scale fluctuations in fluid turbulence



$$\frac{\delta_\eta v_\eta}{\nu} \sim \mathcal{O}(1)$$



MF prediction
[Biferale PoF
submitted]



Transition between viscous and inertial-range scaling of turbulence structure functions

Charles Meneveau

[10] G. Stolovitzky, K. R. Sreenivasan, and A. Juneja, Phys. Rev. E **48**, R3217 (1993).

[11] L. Sirovich, V. Yakhot, and L. Smith, Phys. Rev. Lett. **72**, 344 (1994).

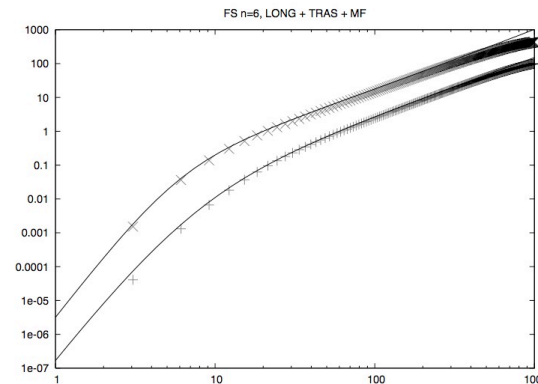
[12] D. Lohse, Phys. Rev. Lett. **73**, 3223 (1994).

BATCHELOR + MF PARAMETRISATION

$$\delta_r v \sim v_0 \frac{r}{\left[\left(\frac{\eta(h)}{L_0} \right)^\alpha + \left(\frac{r}{L_0} \right)^\alpha \right]^{1-h/\alpha}}$$

$$\left\{ \begin{array}{l} \delta_r v \sim r^h \quad \eta \ll r \ll L_0 \\ \delta_r v \sim \frac{\delta_\eta v}{\eta} r \quad r \ll \eta \\ \eta(h) \sim Re^{-\frac{1}{1+h}} \end{array} \right.$$

α Free parameter



See also Chevillard et al Physica D 2006

GOING LAGRANGIAN.....

WHY?

Frisch & Vergassola EPL 1991

The fact that exponents h significantly smaller than $1/3$ have viscous cut-offs much smaller than the Kolmogorov dissipation scale, can be a severe constraint on experimental techniques where multifractality is measured from the statistics of the local dissipation. Such measurements can be spurious for those exponents which have cut-offs less than the probe size [12].

Chevillard et al Physica D 2006

$\langle(\partial_x u)^3\rangle$. Experimentally speaking, measuring gradients is still controversial mainly because hot wire probe sizes are in general of the order of the Kolmogorov scale [28,29,30,31]. We hope that further experimental studies will

- [28] C. W. Van Atta and R. Antonia, Reynolds number dependence of skewness and flatness factors of turbulent velocity derivatives, *Phys. Fluids* **23**, 252 (1980).
- [29] P. Tabeling, G. Zocchi, F. Belin, J. Maurer and H. Willaime, Probability density functions, skewness, and flatness in large Reynolds number turbulence, *Phys. Rev. E* **53**, 1613 (1996).
- [30] K. R. Sreenivasan and R. A. Antonia, The phenomenology of small-scale turbulence, *Annu. Rev. Fluid Mech.* **29**, 435 (1997).
- [31] H.S. Kang, S. Chester and C. Meneveau, Decaying turbulence in an active-grid-generated flow and comparisons with large-eddy simulation, *J. Fluid Mech.* **480**, 129 (2003).

Lagrangian turbulence?

Is the multifractal formalism able to describe also the phenomenology of Lagrangian turbulence ?

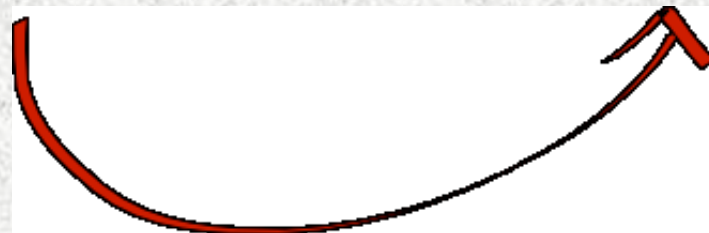
“....Unfortunately, there are no significant lagrangian measurements of velocity, acceleration, etc., to test the multifractal predictions. ...”

M.S. Borgas, “The Multifractal Lagrangian Nature of Turbulence”, Phyl. Trans: Phys. Sciences and Eng. Vol. 342 (1993) 379.

Recently things are changing !

Eulerian MF

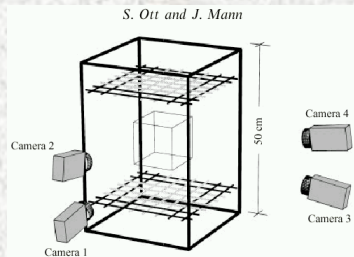
Lagrangian MF



With some surprise...

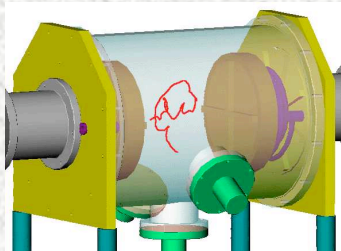
Experiments

Experimental Lagrangian measurements are intrinsically difficult: one has to follow (many) Lagrangian trajectories for long time at high Reynolds (i.e. high sampling frequency)

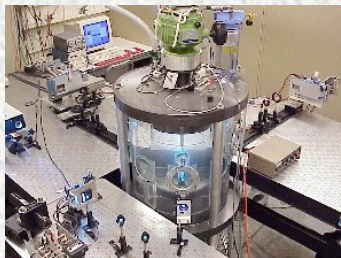


**Ott and Mann
experiment at Risø**
conventional 3D PTV -
 $Re_\lambda = 100-300$

Luthi, Tsinober et al
3D PTV and 3D scanning PTV for
velocity gradients



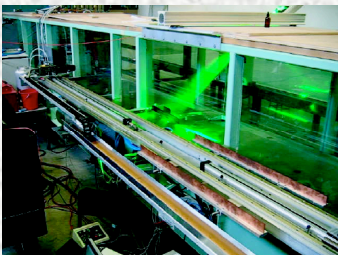
Pinton et al ENSL
Acoustic/Laser
Doppler tracking -
 $Re_\lambda \sim 800$ (single
particle tracking)



**Bodenschatz et al at
Cornell-MPI**
silicon strip detectors
(now also CCD) $Re_\lambda \approx 1000-$
1500

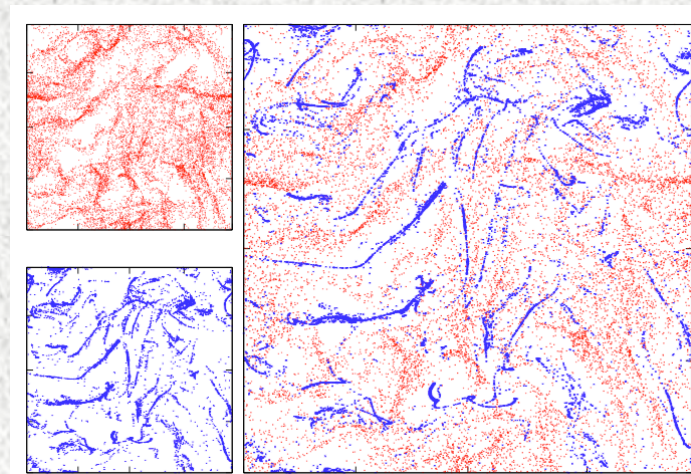
non intrusive tracking down to

$$\tau \sim \tau_\eta$$

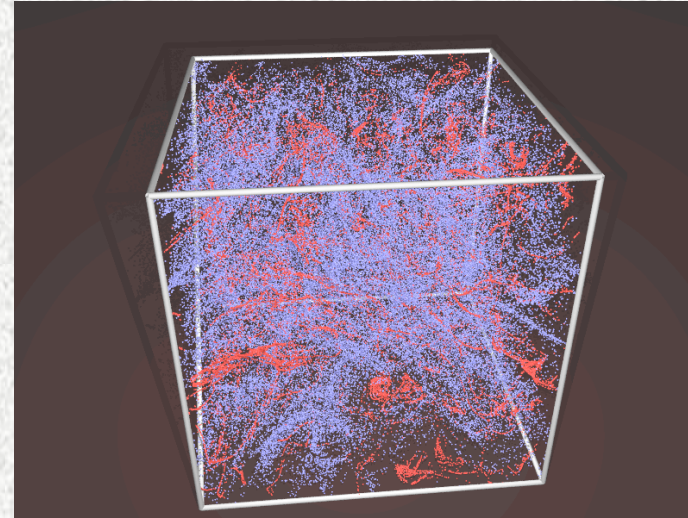


**Warhaft et al
experiment at
Cornell**
Fast moving camera
 $Re_\lambda \approx 300$

DNS



+s and -s



- low to moderate Reynolds numbers, Re
- computationally expensive (Cpu time $\propto Re_\lambda^6$)
- memory demanding (ram $\propto Re_\lambda^{9/2}$)

- + high time resolution and long tracking
- + large Lagrangian statistics
- + multiparticle tracking
- + simultaneous Eulerian-Lagrangian statistics

$$\tau \ll \tau_\eta$$

N	Re_λ	η	L	T_L	τ_η	T	δx	N_p
512	183	0.01	3.14	2.1	0.048	5	0.012	$0.96 \cdot 10^6$
1024	284	0.005	3.14	1.8	0.033	4.4	0.006	$1.92 \cdot 10^6$
2048	400	0.0025	3.14	1.8	0.02	5.9	0.003	$2 \cdot 10^9$

Pseudo spectral code - dealiased 2/3 rule - normal viscosity -

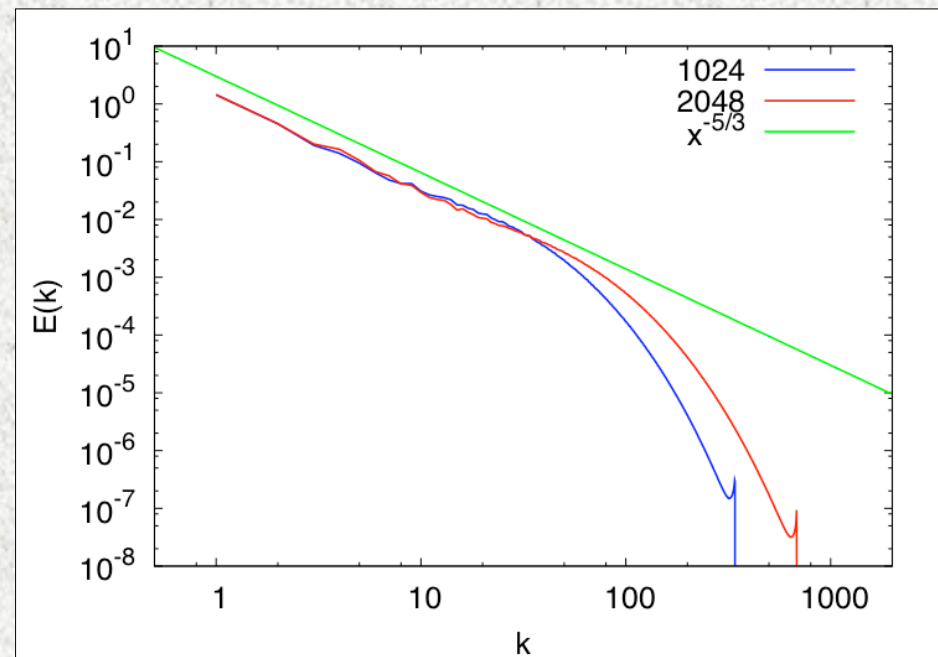
2 millions of passive tracers- code fully parallelized with

MPI+FFTW - Platform IBM SP4 (sust. Performance

150Mflops/proc) - 50000 cpu hours -

duration of the run: 40 days

Energy spectrum



PHYSICS OF FLUIDS 17, 021701 (2005)
Particle trapping in three-dimensional fully developed turbulence
 PHYSICS OF FLUIDS 17, 115101 (2005)
 Lagrangian statistics of particle pairs in homogeneous isotropic turbulence
 PHYSICS OF FLUIDS 17, 111701 (2005)
 Multiparticle dispersion in fully developed turbulence
 PHYSICS OF FLUIDS 18, 091702 (2006)
 Lyapunov exponents of heavy particles in turbulence

Lagrangian velocity statistics

$$S_p(r) = \langle (u(x+r) - u(x))^p \rangle \sim r^{\zeta_E(p)}$$

$$S_p(\tau) = \langle (v(t+\tau) - v(t))^p \rangle \sim \tau^{\zeta_L(p)}$$

$$\tau_\eta \ll \tau \ll T_L$$

Does it exist and how to estimate $\zeta_L(p)$?

In Eulerian turbulence we have $\zeta_E(p) = \inf_h (hp + 3 - D(h))$

Let's try to make a **predictions**

$$\delta_{\tau} v \sim \delta_r u$$

We assume that r and τ are linked by the typical eddy turn over time at the given spatial scale

$$\tau_r \sim r / \delta_r u$$

Bridge between Eulerian and Lagrangian description:

$$\tau \sim \frac{L_0^h}{v_0} r^{1-h}$$

[Borgas (1993); Boffetta et al (2002)]

$$S_p(r) = \langle (\delta_r v)^p \rangle \sim \langle v_0^p \rangle \int_I dh \left(\frac{r}{L_0} \right)^{hp+3-D(h)}$$

EULERIAN

Multifractal prediction for the Lagrangian structure functions

$$S_p(\tau) \sim \langle v_0^p \rangle \int_{h \in I} dh \left(\frac{\tau}{T_L} \right)^{\frac{hp+3-D(h)}{1-h}}$$

where

$$\zeta_L(p) = \inf_h \left(\frac{hp+3-D(h)}{1-h} \right)$$

Same $D(h)$ of
the Eulerian field !!

WARNING: NO EXACT RESULTS SUPPORTING THE
EXISTENCE OF SCALING LAWS IN LAGRANGIAN
FRAMEWORK

BATCHELOR-MENEVEAU -> LAGRANGIAN

[CHEVILLARD ET AL PRL 2003]

$$\delta_\tau v = v_0 \frac{\tau/T_L}{\left[\left(\frac{\tau}{T_L}\right)^\beta + \left(\frac{\tau_\eta}{T_L}\right)^\beta \right]^{\frac{1-2h}{\beta(1-h)}}$$

$$\delta_\tau v \sim \left(\frac{\tau}{T_L}\right)^{\frac{h}{1-h}}$$

$$\tau \gg \tau_\eta$$

$$\delta_\tau v \sim \tau \frac{\delta_{\tau_\eta} v}{\tau_\eta} \sim a\tau$$

$$\tau \ll \tau_\eta$$

β free parameter

$$\mathcal{P}_h(\tau, \tau_\eta) \sim \left[\left(\frac{\tau_\eta}{T_L}\right)^\beta + \left(\frac{\tau}{T_L}\right)^\beta \right]^{\frac{3-D(h)}{\beta(1-h)}}$$

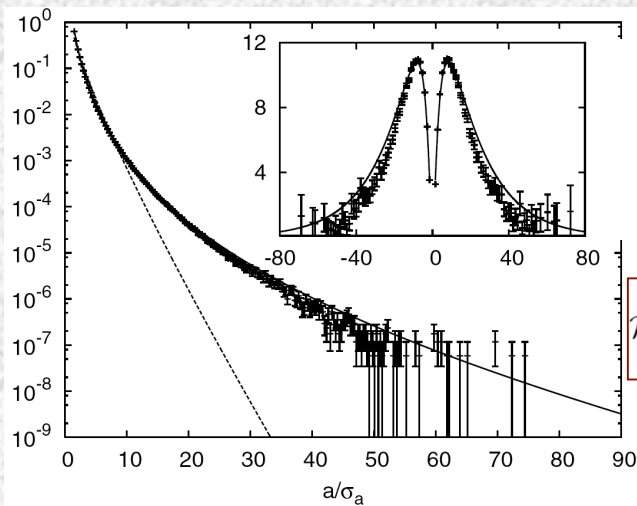
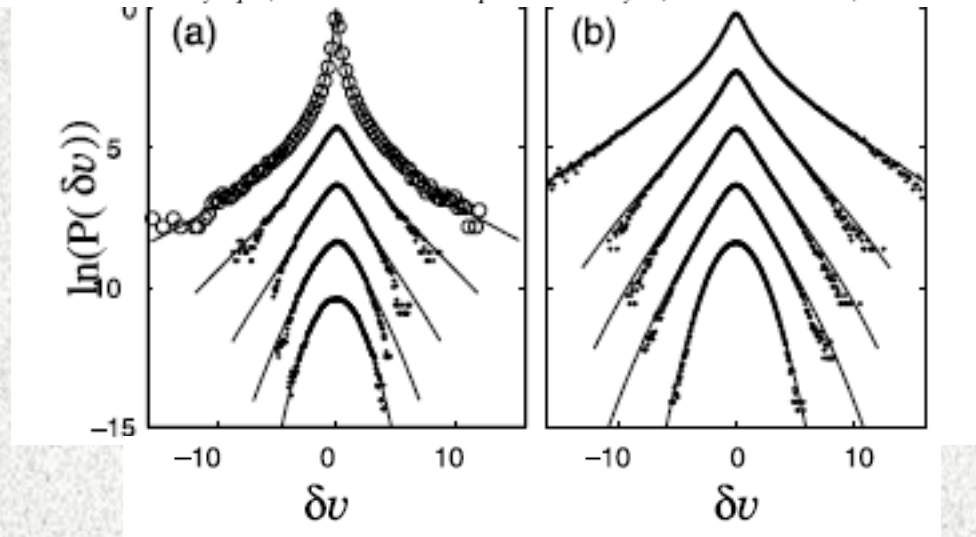
Start from Eulerian $D(h)$

but: dissipative time fluctuates (as the dissipative scale)

$$\tau_\eta(h) \sim Re_\lambda^{\frac{2(h-1)}{1+h}}$$

Lagrangian Velocity Statistics in Turbulent Flows: Effects of Dissipation

L. Chevillard, S.G. Roux, E. Levêque, N. Mordant,* J.-F. Pinton, and A. Arneodo
Laboratoire de Physique, École Normale Supérieure de Lyon, 46 allée d'Italie, F-69007 Lyon, France



Multifractal Statistics of Lagrangian Velocity and Acceleration in Turbulence

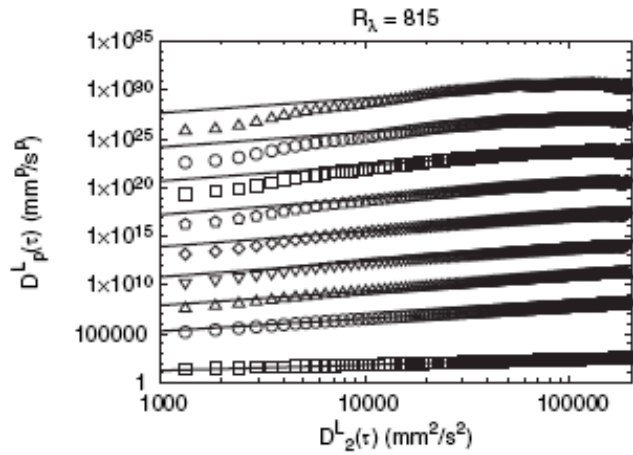
L. Biferale,¹ G. Boffetta,^{2,6} A. Celani,³ B.J. Devenish,^{1,7} A. Lanotte,⁴ and F. Toschi⁵

$$P(a) \sim \int_{h \in I} dh a^{\frac{h-5+D(h)}{3}} \nu^{\frac{7-2h-2D(h)}{3}} L_0^{D(h)+h-3} \sigma_v^{-1} \times \exp\left(-\frac{a^{\frac{2(1+h)}{3}} \nu^{\frac{2(1-2h)}{3}} L_0^{2h}}{2\sigma_v^2}\right)$$

$$\delta_\tau v \sim \tau \frac{\delta_{\tau\eta} v}{\tau_\eta} \sim a\tau$$

High Order Lagrangian Velocity Statistics in Turbulence

Haitao Xu,^{1,2} Mickaël Bourgoïn,³ Nicholas T. Ouellette,¹ and Eberhard Bodenschatz^{1,2,*}



GLOBAL FIT? -> POWER LAW?

$$S_p(\tau) \sim \langle v_0^p \rangle \int_{h \in I} dh \left(\frac{\tau}{T_L} \right)^{\frac{hp+3-D(h)}{1-h}}$$

$$S_p(\tau) \sim \tau^{\zeta_L(p)}$$

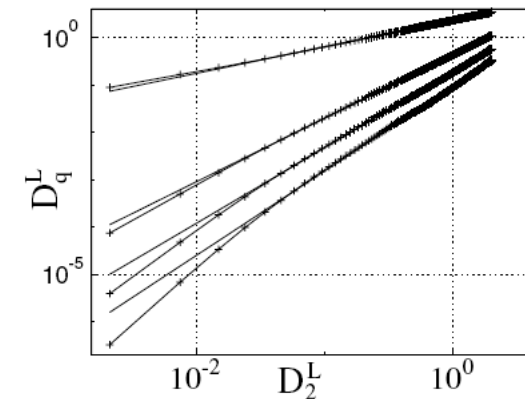


FIG. 5. ESS plots of the structure function variation (in double log coordinates). The solid curves are best linear fits with slopes equal to $\xi_q^L = 0.56 \pm 0.01, 1.34 \pm 0.02, 1.56 \pm 0.06,$ and 1.8 ± 0.2 for $p = 1, 3, 4,$ and 5 from top to bottom. Coordinates in arbitrary units.

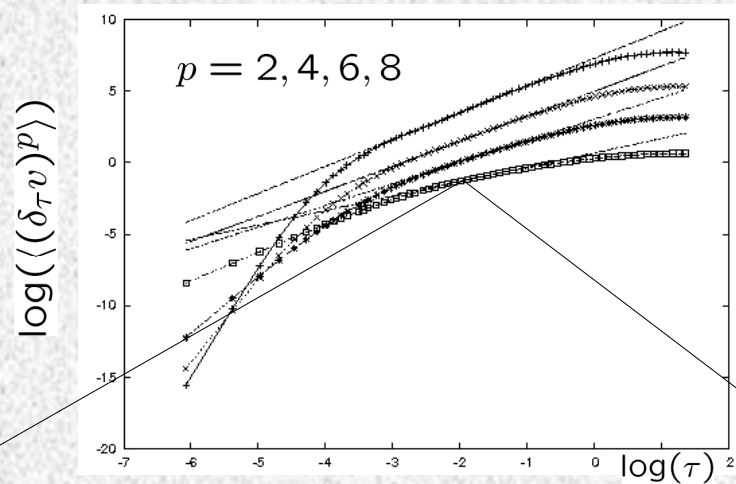
Measurement of Lagrangian Velocity in Fully Developed Turbulence

N. Mordant,¹ P. Metz,¹ O. Michel,² and J.-F. Pinton¹

¹CNRS & Laboratoire de Physique, École Normale Supérieure, 46 allée d'Italie, F-69007 Lyon, France

²Laboratoire d'Astrophysique, Université de Nice, Parc Valrose, F-06108, Nice, France

(Received 26 March 2001; published 2 November 2001)

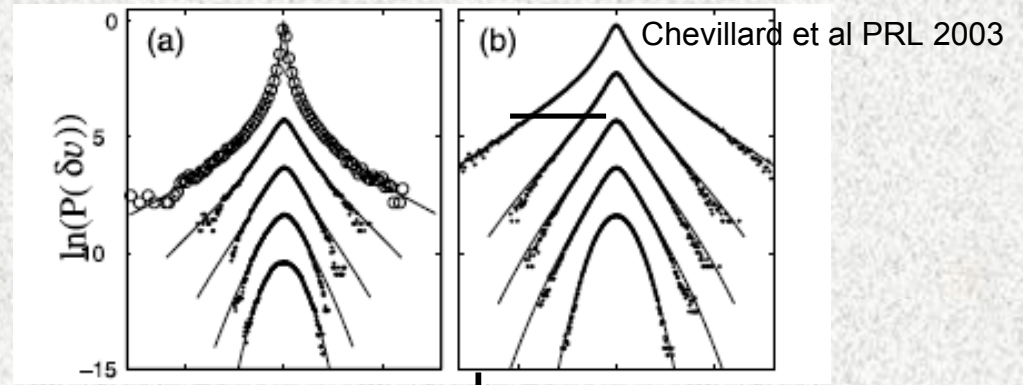
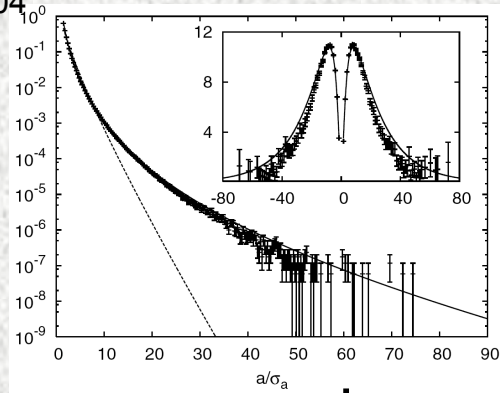


$$\zeta_p(\tau) = \frac{d \log (S_p(\tau))}{d \log (\tau)}$$

The local exponents $\zeta_p(\tau)$ act as **magnifying glass**, probing locally the value of the scaling exponents.

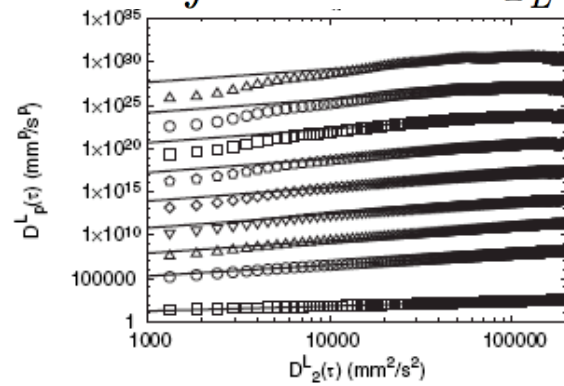
-) Power law scaling -> plateaux for **local scaling exponents**
-) Comparing results from different components: estimate of anisotropy

Biferale et al prl 2004



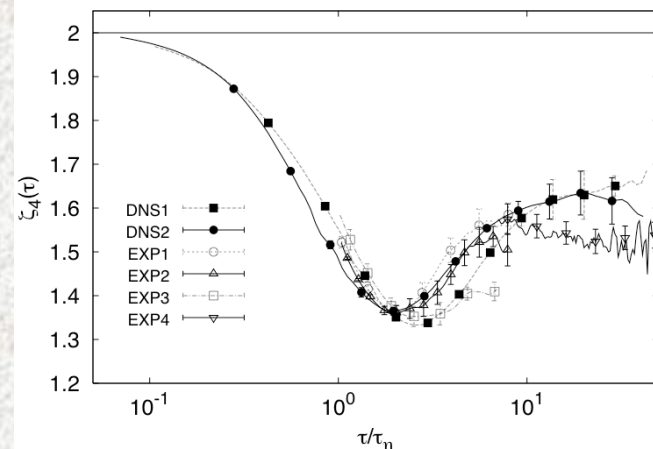
$$\langle (\delta_\tau v)^p \rangle \sim \int dh (\delta_\tau v)^p \mathcal{P}_h\left(\frac{\tau}{T_L}, \frac{\tau_\eta}{T_L}\right)$$

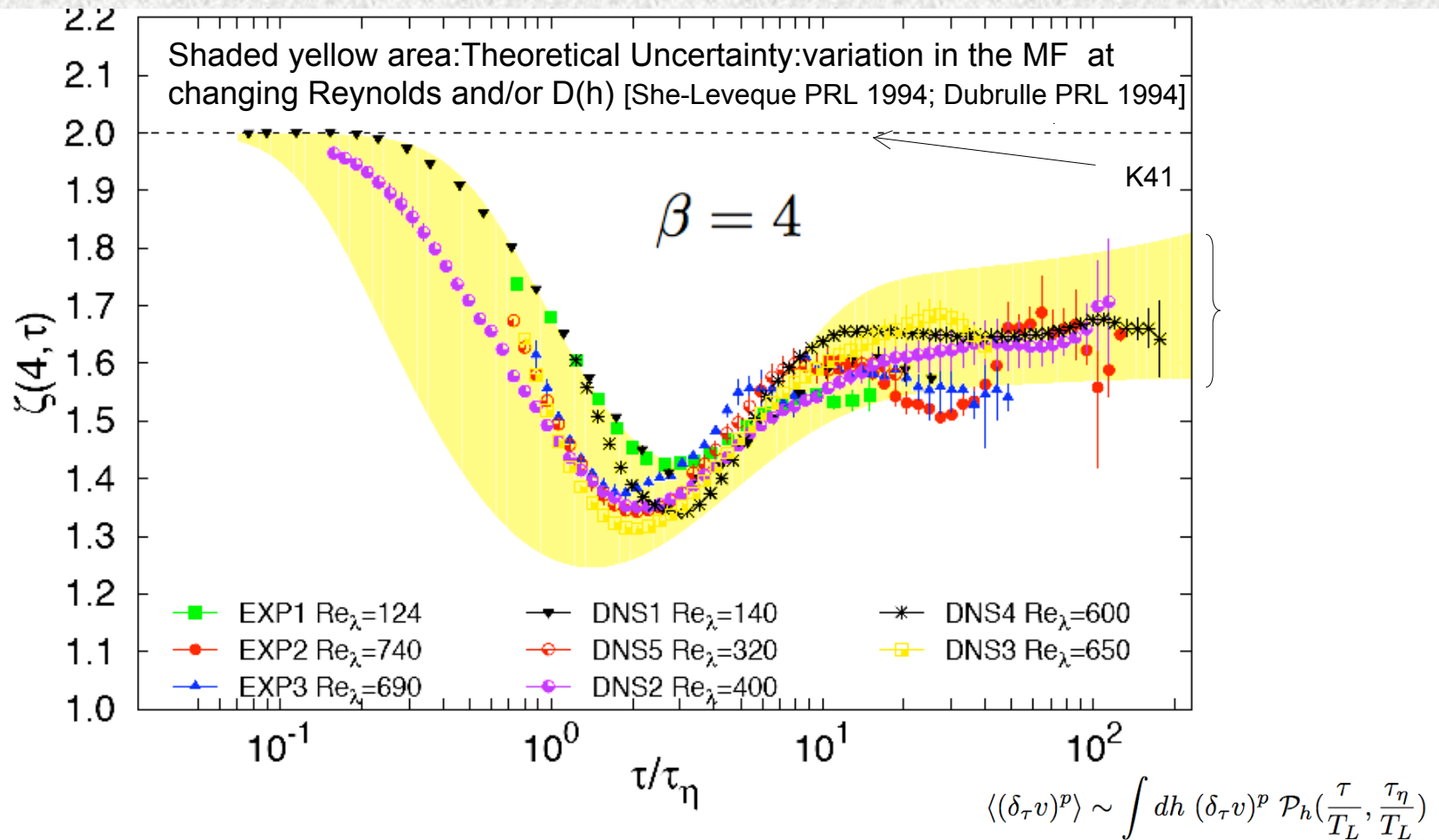
Xu et al PRL 2006



$$\zeta_p(\tau) = \frac{d \log (S_p(\tau))}{d \log (S_2(\tau))}$$

Biferale, Bodenschatz, Cencini et al, PoF 2007 submitted





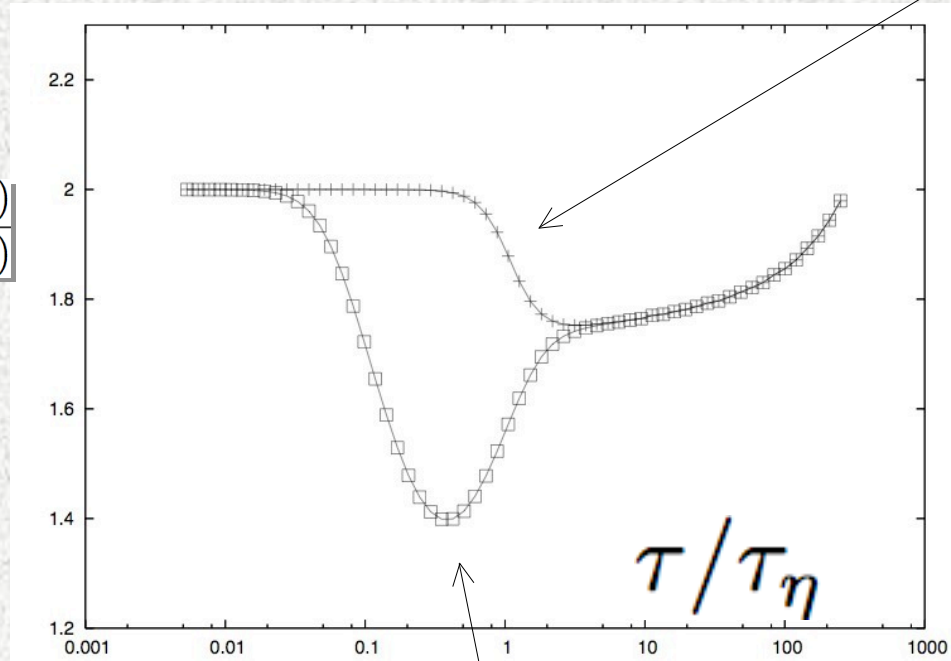
International Collaboration for Turbulence Research, A. Arneodo,¹ J. Berg,² R. Benzi,³ L. Biferale,³ E. Bodenschatz,⁴ A. Busse,⁵ E. Calzavarini,⁶ B. Castaing,¹ M. Cencini,⁷ L. Chevillard,¹ R. Fisher,⁸ R. Grauer,⁹ H. Homann,⁹ D. Lamb,⁸ A.S. Lanotte,¹⁰ E. Leveque,¹ B. Lüthi,¹¹ J. Mann,² N. Mordant,¹² W.-C. Müller,⁵ S. Ott,² N. Ouellette,¹³ J.-F. Pinton,¹ S.B. Pope,¹⁴ S.G. Roux,¹ F. Toschi,^{15, 16} H. Xu,⁴ and P.K. Yeung¹⁷

WE LEARN ABOUT:
 (i) INTERMITTENCY; (ii) UNIVERSALITY; (iii) ANISOTROPY

MultiFractal WITHOUT DISSIPATIVE FLUCTUATING

$\tau_\eta(h)$

$$\zeta_p(\tau) = \frac{d \log (S_p(\tau))}{d \log (S_2(\tau))}$$



τ / τ_η

MultiFractal WITH DISSIPATIVE FLUCTUATING

$\tau_\eta(h)$

WHAT HAPPENS AROUND DISSIPATIVE TIME?

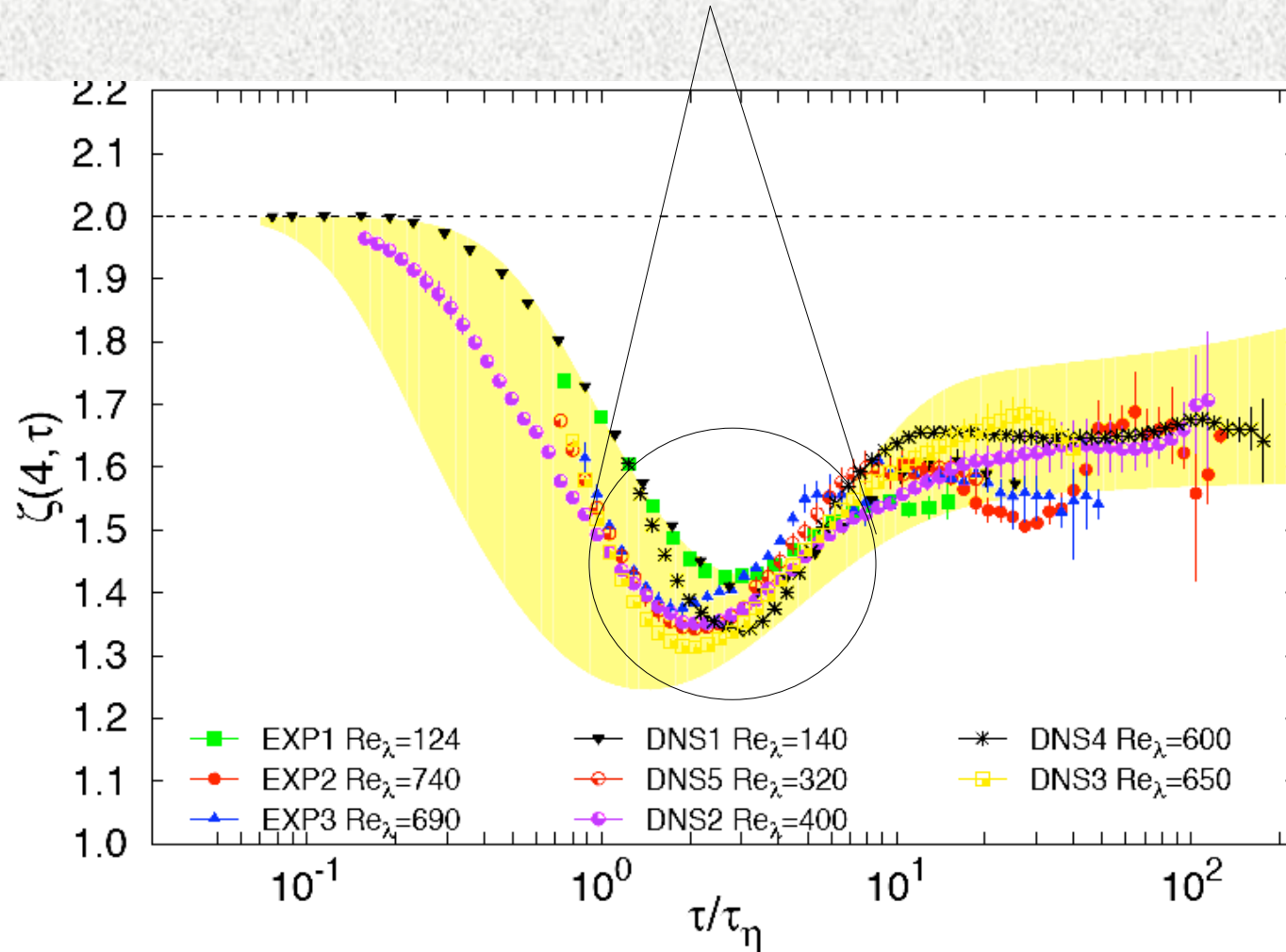
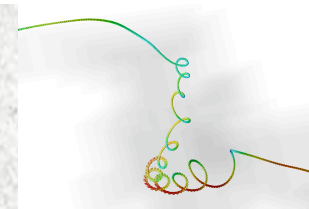
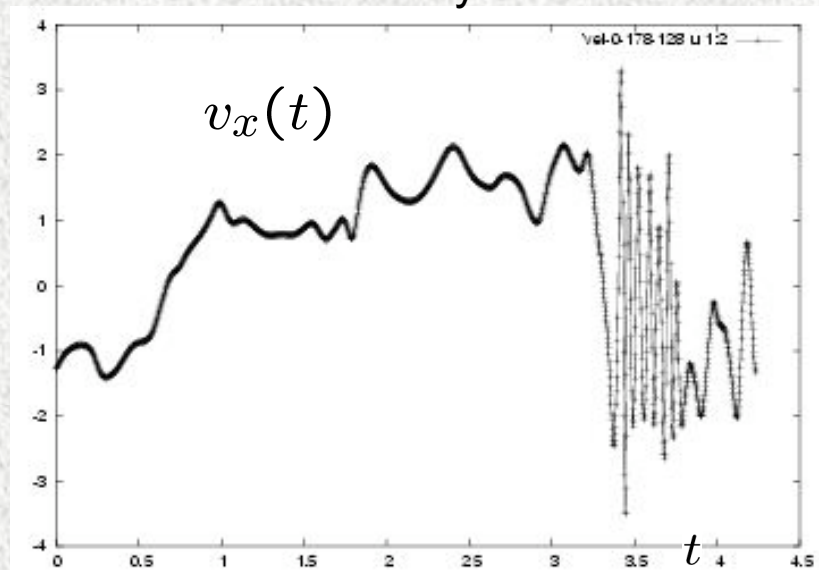


FIG. 1: Log-Lin plot of the local exponent for the fourth moment, $\zeta(4, \tau)$, averaged over the three velocity components, as a function of the normalised time lag τ/τ_η . Data sets come from three experiments (EXP) (see table 1) and five direct numerical simulations (DNS) (see table 2). Error bars are estimated out of the spread between the three components, but for EXP1 and EXP3 where only two components have been considered because of large systematic anisotropic effects in the third one. Each data set is plotted only in the time range where the known experimental/numerical limitations are certainly not affecting the

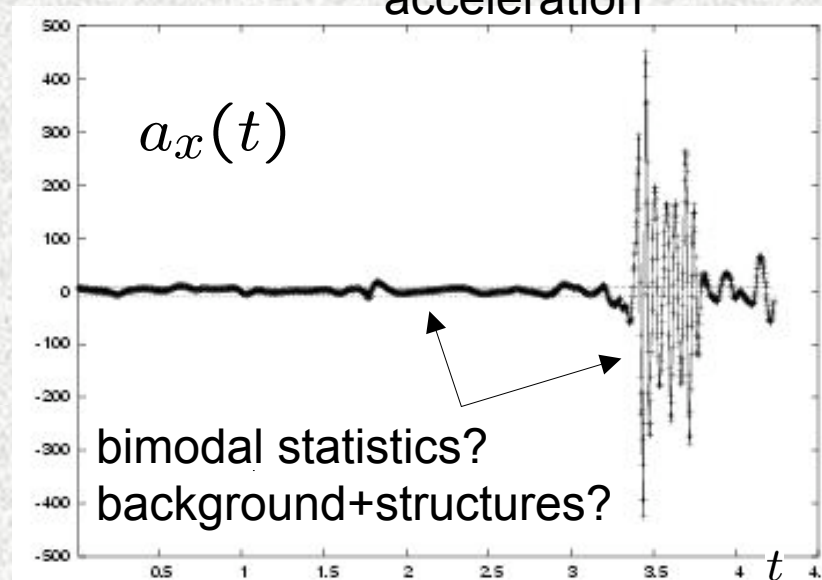
TRAPPING INTO VORTEX FILAMENTS



velocity



acceleration



Particle trapping in three-dimensional fully developed turbulence

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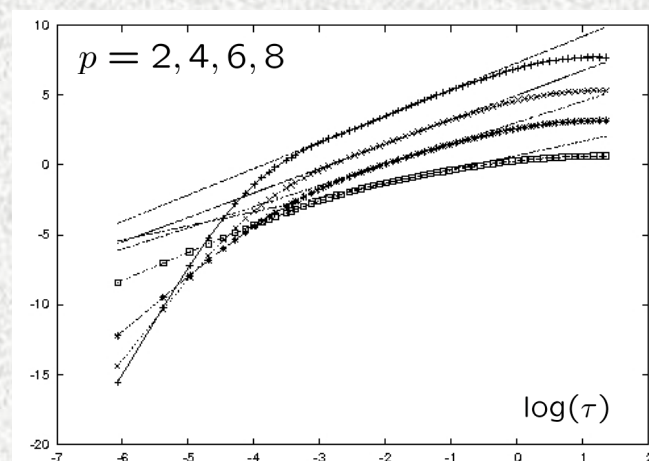
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F. Toschi

Istituto per le Applicazioni del Calcolo, CNR, Viale del Politecnico 137, 00161 Roma, Italy

[see also La Porta et al Nature 2001]]

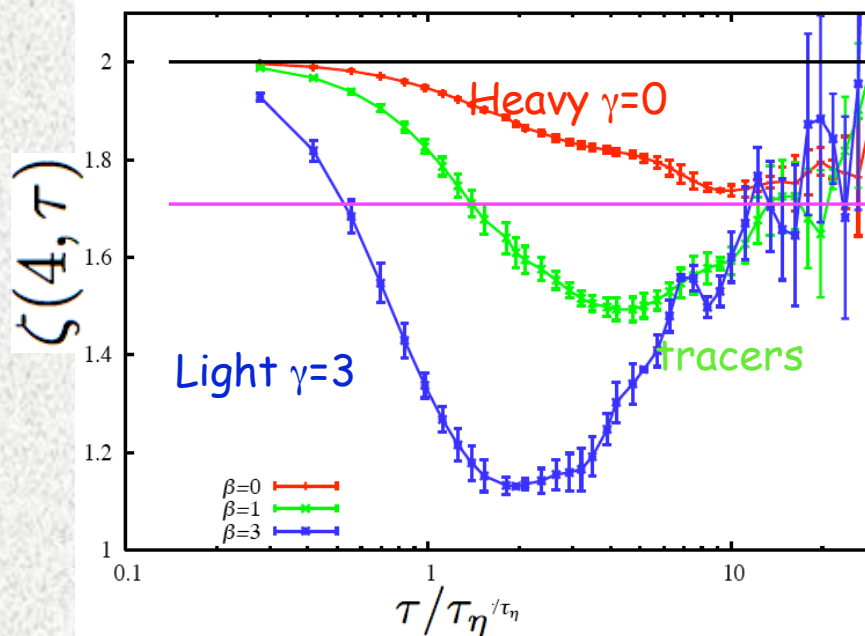


WHERE ARE THESE ANIMALS?

$$\mathbf{v}_L(t) = \dot{\mathbf{x}}_L(t) = \mathbf{v}_E(\mathbf{x}_L(t), t)$$

$$\frac{d\mathbf{v}(t)}{dt} = \gamma \frac{D\mathbf{u}(\mathbf{x}, t)}{Dt} - \frac{1}{\tau} [\mathbf{v}(t) - \mathbf{u}(\mathbf{x}(t), t)]$$

plot from E. Calzavarini



St=0.6

CAN WE PREDICT IT USING A SUITABLE MF?

$$\delta_\tau v = v_0 \frac{\tau/T_L}{[(\frac{\tau}{T_L})^\beta + (\frac{\tau_\eta}{T_L})^\beta]^{\frac{1-2h}{\beta(1-h)}}}$$

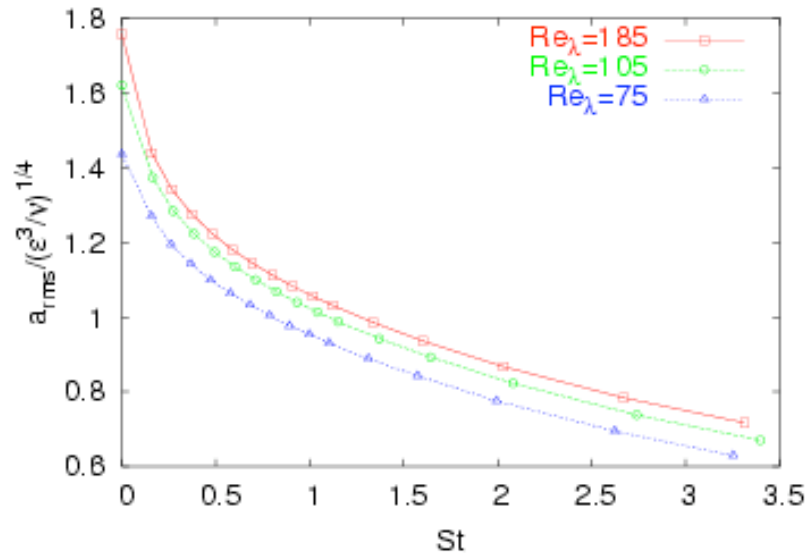
$\beta(St)?$

Not so simple, where is preferential concentration?

Acceleration statistics

Acceleration statistics of heavy particles in turbulence

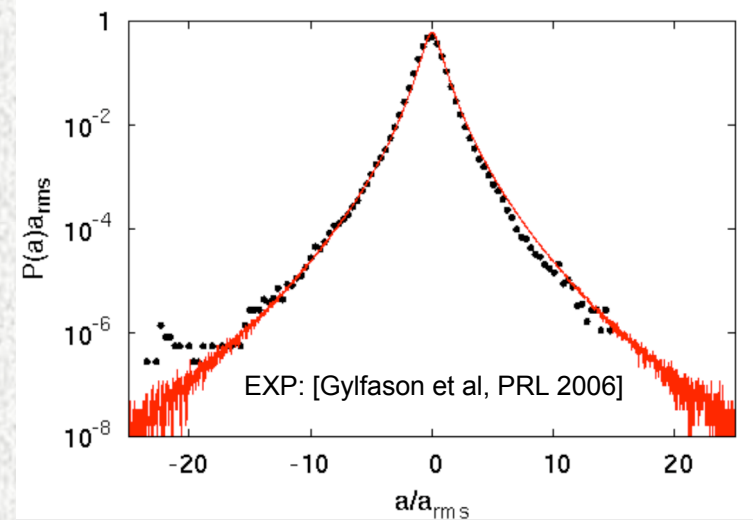
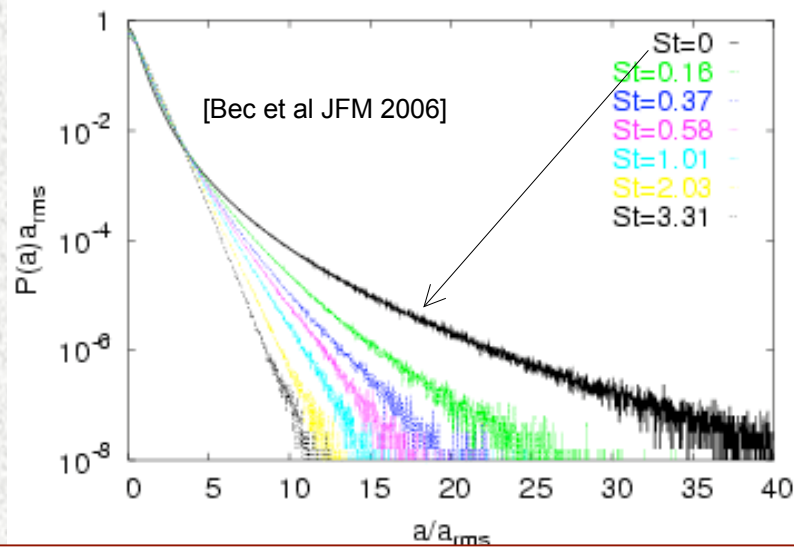
By J. BEC¹, L. BIFERALE², G. BOFFETTA³, A. CELANI⁴,
M. CENCINI⁵, A. LANOTTE⁶, S. MUSACCHIO⁷
AND F. TOSCHI⁸



- ◆ At increasing St : strong depletion of both rms acc. and pdf tails.
- ◆ Residual dependence on Re very similar to that observed for tracers. [Sawford et al (2003); Borgas (1993)]

CAN WE
PREDICT USING
A SUITABLE MF?

$\beta(St)?$



$$p(a) \sim \int_{h \in I} dh a^{\frac{h-5+D(h)}{3}} \nu^{\frac{7-2h-2D(h)}{3}} L_0^{D(h)+h-3} \sigma_v^{-1} \times \exp\left(-\frac{a^{\frac{2(1+h)}{3}} \nu^{\frac{2(1-2h)}{3}} L_0^{2h}}{2\sigma_v^2}\right)$$

OPEN PROBLEMS

HOW TO EXTEND THE MF PREDICTION TO INCLUDE PREFERENTIAL CONCENTRATION.

PROBE EULERIAN \leftrightarrow LAGRANGIAN MF CONNECTION TO HIGHER ORDER STATISTICS. IS LAGRANGIAN REALLY FULLY INCLUDED IN EULERIAN? (AND VICEVERSA)

OBSERVED MISMATCH BETWEEN LONGITUDINAL-TRANSVERSE SCALING IN EULERIAN SF, AND BETWEEN ENSTROPY AND ENERGY DISSIPATION. FINITE REYNOLDS EFFECT?

PROBE REFINED KOLMOGOROV HYPOTHESIS IN LAGRANGIAN DOMAIN \rightarrow IMPORTANT FOR MODELISATION [preliminary results \rightarrow OK]

PROBE MULTISCALE-MULTITIME CORRELATION FUNCTIONS $\langle |\delta_r v(t)|^p |\delta_R v(t + \tau)|^q \rangle$

INCLUDE IN MF DESCRIPTION KNOWN EXACT RESULTS (KARMAN-HOWART EQ)

ANISOTROPIC FLUCTUATIONS: IS THERE A CASCADE, IS THERE INTERMITTENCY? IS IT UNIVERSAL?

BUILD UP SEQUENTIAL MF SURROGATES $v_i(\mathbf{x}(t), t); \partial_j v_i(\mathbf{x}(t), t)$ TO FEED STOCHASTIC MODELS FOR PARTICLE ADVECTION IN TURBULENCE.

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V. S. L'vov and I. Procaccia Phys. Rev. Lett 1996

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Multiscale Model of Gradient Evolution in Turbulent Flows

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A conditionally cubic-Gaussian stochastic

Lagrangian model for acceleration in isotropic turbulence

A.G. LAMORGESE and S. B. POPE and P. K. YEUNG and B. L. SAWFORD J. Fluid Mech. 2007

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G. Stolovitzky and K. R. Sreenivasan Rev. Mod. Phys. 1994

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C. Meneveau Physical Review E 1996

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L. Biferale, G. Boffetta, A. Celani, A. Crisanti and A. Vulpiani
Phys. Rev. E 1998.

***Unified multifractal description of velocity increments statistics
in turbulence: Intermittency and skewness***

L. Chevillard and B. Castaing and E. Leveque and A. Arneodo Physica D2006

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F. Toschi, H. Xu.

and many others:

R. Benzi, E. Calzavarini, L. Chevillard, B. Devenish, R. Fisher, T. Gotoh, L. Kadanoff,
D. Lamb, S. Musacchio, N. Ouellette.

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This is the web site of the free CFD database, kindly hosted by [Cineca](#) supercomputing center (Bologna, Italy).

The administrator of the site, in charge of registering new users on the site is: [Federico Toschi](#). Please refer to him any question or comment regarding this web site.

If you wish to contribute to this DNS database of fluid-dynamics data by sharing your raw data or computer codes you are interested in; please contact the administrator of the site.

Thank you



upcoming events

Computational Physics and New Perspectives in Turbulence
Nagoya - Japan,
2006-09-11

March 2006

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