

# Plasmoid Reconnection

DAMTP 23.05.11

BBag seminar 19.10.10.

Let us start from basics: magnetic fields in astrophysical situations tend to cross (as they are moved around by the fluid) and can then reconnect, releasing magnetic energy into heat or plasma flows:

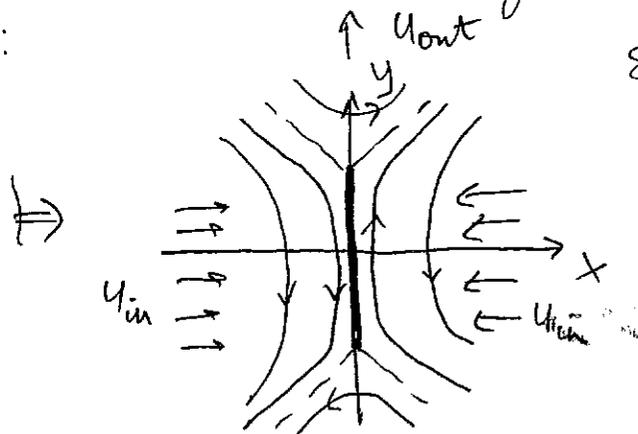
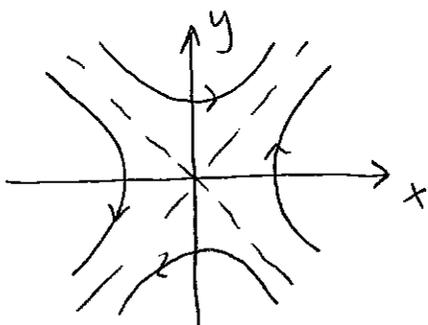


$$\frac{\partial \vec{B}}{\partial t} = \nabla \times (\vec{u} \times \vec{B}) + \eta \nabla^2 \vec{B}$$

Astrophysical examples: SW + magnetosphere      Dungey  
 magnetotail                                      Sweet  
 solar flares

## Fundamental theoretical fact # 1

X points collapse into current sheets (can move within ideal MHD):



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Syrovatskii  
 Chapman & Kendall

i.e. MHD wants to have an equilibrium that has a singular segment - where the ideal assumption breaks down.  $\mathcal{R} \sim L_{CS} \sim \text{size of the system}$ ,

Fundamental theoretical fact #2

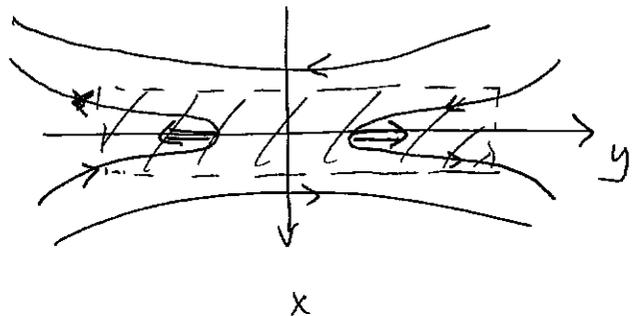
Sweet-Parker theory of resistive reconnection  
(qualitative, but has been shown to hold in resistive MHD)

From induction eqn: 
$$u_{in} B_{in} \sim \eta j \sim \eta \frac{B_{in}}{\delta}$$
 (el. field)      (Ohmic dissipation)      (width of sheet)

So 
$$u_{in} \sim \frac{\eta}{\delta}$$

Mass conservation: 
$$u_{in} L \sim u_{out} \delta$$

Momentum equation: plasma accelerated by magnetic tension - or by pressure



$$\frac{\partial \vec{u}}{\partial t} + \vec{u} \cdot \nabla \vec{u} = -\nabla p + \vec{B} \cdot \nabla \vec{B}$$

$\hat{y}$ : 
$$\frac{u_{in}}{\delta} u_{out} + \frac{u_{out}^2}{L} \sim \frac{B_x}{\delta} B_{in} + \frac{B_{in}^2}{L}$$

So, either way,      solenoidality  $\frac{B_x}{\delta} \sim \frac{B_{in}^2}{L}$

$$u_{out} \sim B_{in} \equiv V_A$$

NB: Alfvénic outflow!

Combine all this:

$$u_{in} \sim \frac{\eta}{\delta} \sim u_{out} \frac{\delta}{L} \sim v_A \frac{\delta}{L} \Rightarrow \boxed{\delta \sim \sqrt{\frac{\eta L}{v_A}} \sim \frac{L}{\sqrt{S}}}$$

$$S = \frac{v_A L}{\eta}$$

↓  
Lundquist #

Therefore  $u_{in} \sim \frac{v_A}{\sqrt{S}}$

or  $\frac{\partial \psi}{\partial t} \sim u_{in} B_{in} \sim \eta E$

or, rec. rate

$$\frac{\partial B_{in}}{\partial t} \sim \frac{u_{in} B_{in}}{L} \sim \left( \frac{1}{\sqrt{S}} \frac{v_A}{L} \right) B_{in}$$

slow time scale.

Solar flare:  $S \sim 3 \cdot 10^{12}$

- $\tau_A \sim 40 \text{ sec}$   $L/v_A$
- $\tau_{sp} \sim 7 \cdot 10^7 \text{ sec}$  (2 yrs)  $(L/v_A) \sqrt{S}$
- $\tau_{res} \sim 10^{14} \text{ sec}$  ( $\sim 10^6$  yrs)  $(L/v_A) S$
- $\tau_{flare} \sim 10^3 - 10^4 \text{ sec}$

So, SP reconnection is too slow!

### Fundamental theoretical fact # 3

Current sheets are unstable for  $S > S_c \sim 10^4$

(plasmoid instability: Loureiro, AAS, Cowley, PR 14, 100703)  
and break super-Alfvénically fast into a huge # of  
plasmoids (secondary islands) - a type of tearing  
instability.

$$\gamma \sim S^{1/4} v_A / L, \quad \# \text{ of plasmoids} \sim S^{3/8}$$

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Numerical simulations have now been through the  $S \sim 10^4$  barrier and everyone is deep plasmoids.

Question: is plasmoïd reconnection fast? YES

There are numerical indications that it ~~is~~ is.

Note:  $\tau_c \sim \tau_A \cdot \sqrt{S_c} \sim 100 \tau_A \sim 4 \cdot 10^3$  sec about right for solar flares.

Question: how do we characterize plasmoïd reconnection quantitatively?

New work: Uzdensky, Loureiro, AAS 45Xiv:1008.330.

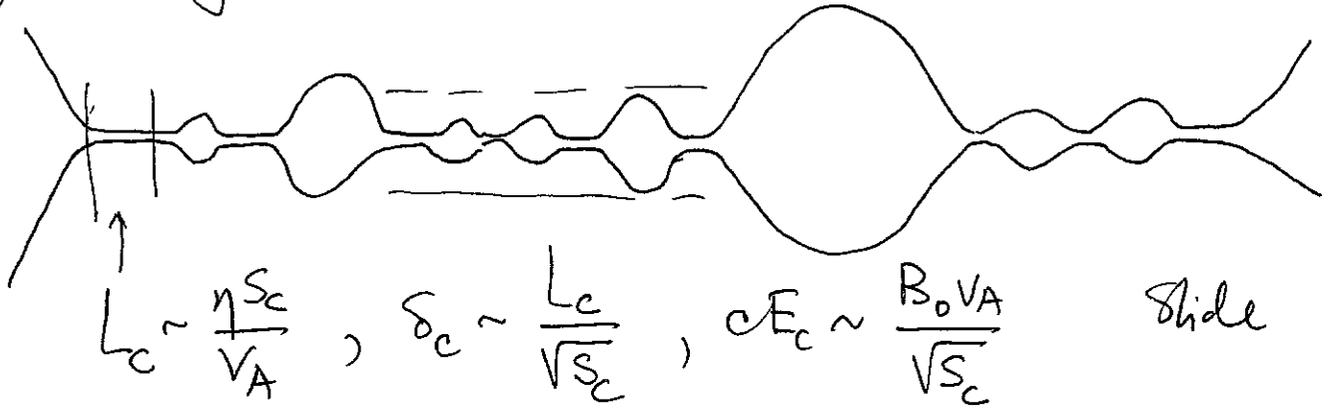
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Empirically we know that a current sheet undergoing reconnection with plasmoïds is a very stochastic and irregular affair with plasmoïds of different sizes, merging, swallowing each other, etc. This makes sense because once 2 plasmoïds appear next to each other, there is an X point in between them, which will then collapse into a sheet, which will become unstable etc. So let us consider a stochastic plasmoïd chain as a st. st. state. We will make some basic plausible assumption about it and see what those

imply. ~~Representative~~ What is the typical connecting layer between 2 plasmoïds: just stable ("critical layer"):  $S_c = \frac{V_A L_c}{\eta} \Rightarrow L_c \sim \frac{\eta S_c}{V_A}$

Assumption I. X-pt collapses and layer instabilities are suff. fast so any 2 adjacent plasmooids are separated by a critical layer.



Assumption II. Upstream (reconnecting) field in each <sup>in the hierarchy</sup> layer is the global rec. field  $B_0 \sim V_A$ . This means that locally the outflow from each sheet is Alfvénic. But this then also means that if we coarse-grain some bit of the global chain and consider it as a sheet between 2 large plasmooids, the <sup>mean</sup> outflow is again Alfvénic. This means that <sup>in</sup> each sublayer, the <sup>mean</sup> outflow is linear: Slide

$$v_y \sim V_A \frac{y}{L} \quad (\text{"Kusble flow"} \Rightarrow \text{Kusble-Subble reconnection})$$

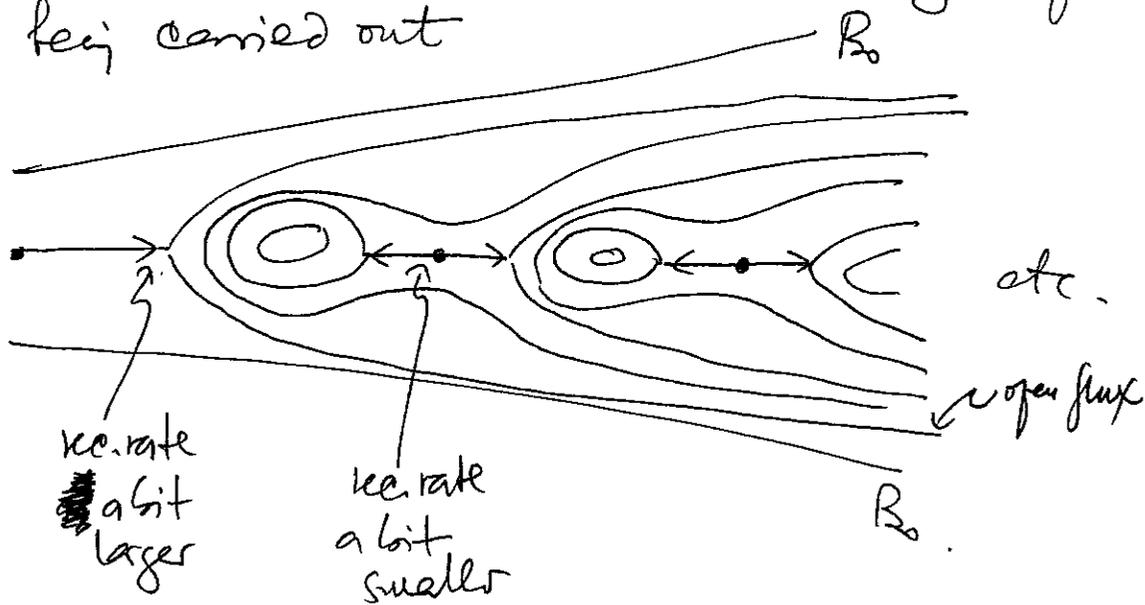
Assumption III. Plasmooids do not saturate <sup>didn't make it into PRL</sup> before they are ejected into larger plasmooids into which they are embedded.

[This will be verified a fortiori]

Reconnection Rate.

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As we look from the centre of the global sheet outwards, the local re. rate decreases slowly because the upstream field decreases; so actually, open flux is being carried out



The total flux carried out is the sum of all open flux parcels (a fully closed field line carries out ~~no~~ net  $B_x$  because it cancels). Summing them all, we see that only the middle contributes. This is true at any level of the plasmoid/current sheet hierarchy:

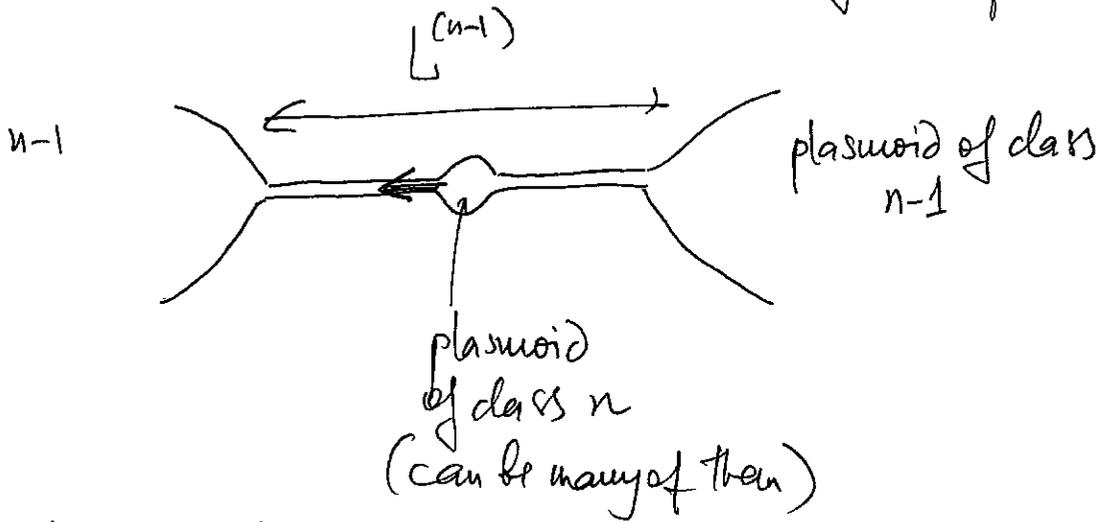
$$cE_{\text{global}} = cE^{(n)} \sim cE_c \sim \frac{v_A B_0}{\sqrt{S_c}} \sim 0.01 v_A B_0.$$

Thus, re. rate is fast and only down by a factor of  $\sqrt{S_c}$  from the Alfvénic value.

How do we describe plasmod reconnection?  
 $f(\Phi, W_x)$

Plasmod Fluxes

How are fluxes inside plasmods distributed?  
 (a measurable characteristic of the plasmod chain)



Ejection time/lifetime:

$$t_{ej}^{(n)} \sim \frac{L^{(n-1)}}{V_{out}^{(n)}} \sim \frac{L^{(n-1)}}{V_A} \quad (\text{As. II - Alfvénic outflows})$$

It does not saturate (As. III) and its flux is growing at eff. rate  $cE_c$ :

$$\Phi^{(n)} \sim cE_c \cdot t_{ej}^{(n)} \sim \frac{cE_c}{V_A} L^{(n-1)}$$

more generally (see on p. 9)  
 $\Phi^{(n)} \sim cE_c^{(n+1)} t_{ej}^{(n)} \sim \frac{cE_c^{(n+1)} L^{(n-1)}}{V_{out}^{(n)}}$

But  $L^{(n-1)} \sim \frac{L}{N(\Phi^{(n)})}$  ← global layer length

↖ # of plasmods with fluxes  $> \Phi^{(n)}$

$$\boxed{\Phi N(\Phi) \sim \frac{cE_c L}{V_A} \sim \frac{B_0 L}{\sqrt{S_c}}}$$

quantitative prediction

Distribution:

$$f(\Phi) = -\frac{dN}{d\Phi} \sim \frac{cE_c L}{V_A} \frac{1}{\Phi^2}$$

(upper classes have larger fluxes)

Alternative derivation:

pick a plasmod with flux  $\Phi_0$ .

its age:  $\tau_{\text{past}} \sim \frac{\Phi_0}{cE_c}$

its life expectancy:  $\tau_{\text{future}} \sim \frac{\Delta y (\Phi > \Phi_0)}{v_A} \sim \frac{L}{N(\Phi_0)} \frac{1}{v_A}$

But the plasmod was picked randomly,

So  $\tau_{\text{past}} \sim \tau_{\text{future}} \Rightarrow \Phi_0 N(\Phi_0) \sim \frac{cE_c L}{v_A}$

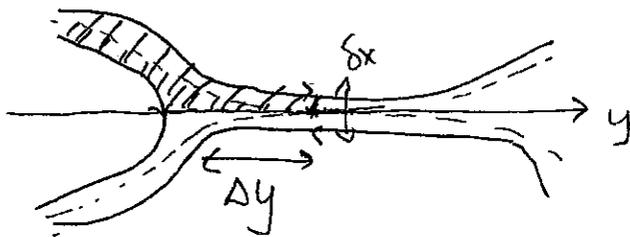
J.R. Gott (Nature 363, 315, 1993)

same result.

Copernicus Principle

length of sheet it's embedded in

Plasmod Sizes.



Roughly, when

$\delta\Phi \approx \delta x B_0$

is reconnected, we get change in area

$\delta A \sim \Delta y \delta x \sim \Delta y \frac{\delta\Phi}{B_0}$

Therefore  $A \sim \Delta y \frac{\Phi}{B_0}$  for plasmons of each class.

$A \sim w_x w_y$  [no saturation]

As long as  $w_x \ll w_y$ ,  $w_y \sim L - \Delta y \sim$  const  $\sim L$  and

$w_x \sim \frac{A}{w_y} \sim \frac{\Phi}{B_0}$

$\Rightarrow$  Distribution of  $w_x$  is the same as of  $\Phi$

Note that in this context, we can provide the full alternative derivation of the reo. rate (à la Shibata-Tanuma). Let the reo. rate at level  $n$  be the SP rate associated with the current sheet of length  $L^{(n+1)}$ , outflow  $V_{out}^{(n)}$  and the width  $w_x^{(n)} \sim \frac{\psi^{(n)}}{B_0^{(n)}}$

Then

$$cE^{(n)} \sim V_{in}^{(n)} B_0^{(n)} \sim \frac{w_x^{(n)}}{L^{(n+1)}} V_{out}^{(n)} B_0^{(n)} \sim \frac{\psi^{(n)} V_{out}^{(n)}}{L^{(n+1)}} \sim \frac{cE^{(n+1)} \cancel{L^{(n+1)}} \cancel{V_{out}^{(n)}}}{\cancel{L^{(n+1)}}} \sim cE^{(n+1)} = \text{const!}$$

← off. width of layer

Another remark is that plasroids indeed do not saturate. (Verification of AS. III):

Saturation happens if at any level

$$w_x \sim \Delta y(\psi) \sim \frac{L}{N(\psi)}$$

$$\frac{w_x}{\Delta y(\psi)} \sim \frac{\psi/B_0}{L/N(\psi)} \sim \frac{\psi N(\psi)}{L B_0} \sim \frac{1}{\sqrt{S_c}} \ll 1 \text{ OK!}$$

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Importance of coherence

Note: if plasroids did not devour each other, this would not work. Then we would get saturation at

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$$w_x \sim L_c \rightarrow \psi_{sat} \sim L_c B_0 \rightarrow t_{sat} \sim \frac{\psi_{sat}}{cE_c} \sim \frac{L_c}{V_A} \sqrt{S_c}$$

$$t_{sat} < \frac{L}{V_A} \text{ if } S_c > S_c^{3/2} \sim 10^6 \quad \left( \frac{L}{L_c} \sim \frac{S}{S_c} \right)$$

(global ej. time)

# Plasmoid Fluxes and Sizes - A More Careful look.

Consider a sheet at some level in the hierarchy,  $L$ .

Consider a plasmoid born at location  $y_0$  in that sheet

this technically means having ascended to this level in the hierarchy.

$y_0 = 0$  means in the centre.

$$\text{Ejection time: } t_{ej} = \int_{y_0}^L \frac{dy}{v_y} = \frac{L}{v_A} \int_{y_0}^L \frac{dy}{y} = \tau_A \ln \frac{L}{y_0}$$

Corresponding flux:  $\Delta \sim c E_c \tau_A \ln \frac{L}{y_0}$  } this can be  $\gg 1$  if  $y_0$  close to the centre.

*interplasmoid distance* We argued on p. 8 that  $\delta A \sim \Delta y \delta x \sim \Delta y \frac{\delta^4}{B_0}$

$\tilde{E} B_0 L$ ,  $\tilde{E} \sim 0.01$  eff. re. rate non-dimensionalised

$$\text{So } \frac{dA}{dt} \sim \frac{\Delta y}{B_0} \frac{d\Delta}{dt} \sim \frac{\Delta y}{B_0} c E_c$$

$$\sim \tilde{E} \frac{L \Delta y(0)}{\tau_A} e^{t/\tau_A} \leftarrow \text{growth of interplasmoid distance due to bulk flow}$$

$$A(t) \sim \tilde{E} L \Delta y(0) (e^{t/\tau_A} - 1)$$

In general,  $A \sim w_x w_y$ . When plasmoids are born,

$w_x \ll w_y \sim \Delta y(0)$ , so, from above

$\leftarrow$  at ejection time

$$w_x(t) \sim \tilde{E} L (e^{t/\tau_A} - 1) \sim \tilde{E} L \left( \frac{L}{y_0} - 1 \right)$$

In order for  $w_x \ll w_y$  all the way, we must have

$$\tilde{E} L \left( \frac{L}{y_0} - 1 \right) < w_y \sim \Delta y(0) \quad \text{or} \quad y_0 \gtrsim \frac{\tilde{E} L^2}{\Delta y(0)}$$

Suppose  $y_0 \sim \Delta y(0)$  (makes sense?)

Then the condition is  $y_0 \gtrsim \sqrt{\tilde{E}} L \sim L S_c^{-1/4}$

The ones that do not satisfy this,  $y_0 \lesssim L S_c^{-1/4} \sim 0.1L$  will circularise at some point in their evolution, so, at ejection,

$$A \sim \tilde{E} L \underbrace{\Delta y(0)}_{y_0} \left( \frac{L}{y_0} - 1 \right) \sim \tilde{E} L^2$$

NB: This does not change  $f(w_x)$  because  $w_x \sim \sqrt{\tilde{E}} L$  either way (on p.10 became  $y_0 \sim \sqrt{\tilde{E}} L$ )

and  $A \sim w_x^2 \Rightarrow w_x \sim \sqrt{\tilde{E}} L \sim L S_c^{-1/4} \sim 0.1L$

If all of this happens at the global-level sheet,

this gives rise to macroscopic monster plasmoids  $\sim 10\%$  of the system size.

NB: Note that the max. non-circularised one was

$$w_x \sim \frac{A}{w_y} \sim \frac{\tilde{E} L^2}{\Delta y(0)} \sim \frac{\tilde{E} L^2}{y_0} \lesssim \sqrt{\tilde{E}} L \sim 0.1L \text{ same cutoff.}$$

NB: In order for such circular monsters to appear,

$y_0 \lesssim L S_c^{-1/4}$  must be consistent with  $y_0 \gtrsim L_c$  (basically precision with which any  $y$  position can be described)

so  $L > L_c S_c^{1/4} \sim \left( \frac{L_c}{L} \right) L S_c^{1/4} \Rightarrow S > S_c^{5/4} \sim 10^5$

considering the global level of hierarchy

critical  $L \#$  for monsters