Nonlinear Growth of Firehose and Mirror Fluctuations in Astrophysical Plasmas

A. A. Schekochihin,[1,*] S. C. Cowley,[1,2] R. M. Kulsrud,[3] M. S. Rosin,[4] and T. Heinemann[4]
[1]Plasma Physics, Blackett Laboratory, Imperial College, London SW7 2AZ, United Kingdom
[2]Department of Physics and Astronomy, UCLA, Los Angeles, California 90095-1547, USA
[3]Princeton University Observatory, Princeton, New Jersey 08544, USA
[4]DAMTP, University of Cambridge, Cambridge CB3 0WA, United Kingdom
(Received 13 September 2007; published 29 February 2008)

In turbulent high-beta astrophysical plasmas (exemplified by the galaxy cluster plasmas), pressure-anisotropy-driven firehose and mirror fluctuations grow nonlinearly to large amplitudes, $\delta B/B \sim 1$, on a time scale comparable to the turnover time of the turbulent motions. The principle of their nonlinear evolution is to generate secularly growing small-scale magnetic fluctuations that on average cancel the temporal change in the large-scale magnetic field responsible for the pressure anisotropies. The presence of small-scale magnetic fluctuations may dramatically affect the transport properties and, thereby, the large-scale dynamics of the high-beta astrophysical plasmas.

DOI: 10.1103/PhysRevLett.100.081301 PACS numbers: 98.65.Hb, 52.35.Py, 95.30.Qd

Introduction.—Many astrophysical plasmas are magnetized and weakly collisional, i.e., the cyclotron frequency Ω_i is much larger than the collision frequency ν_{ii} and the Larmor radius ρ_i is smaller than the mean free path $\lambda_{\rm mfp}$. In such plasmas, all transport properties, most importantly the viscosity and thermal conductivity, become anisotropic with respect to the local direction of the magnetic field [1] — even if the field is dynamically weak.

A typical example where just such a physical situation is present is galaxy clusters [2,3]. While parameters vary significantly both within each cluster and between clusters, the weakly collisional magnetized nature of the intracluster medium (ICM) is well illustrated by the core of the Hydra A cluster, where $\Omega_i \sim 10^{-2}\ {\rm s}^{-1}$, $\nu_{ii} \sim 10^{-12}\ {\rm s}^{-1}$ and $\rho_i \sim 10^5$ km, $\lambda_{\rm mfp} \sim 10^{15}$ km [4]. Modeling global properties of clusters and physical processes inside them, such as shocks, fronts, radiobubbles, or the heating of the ICM [5], can only be successful if the viscosity and thermal conductivity of the ICM are understood [6]. Another fundamental problem is the origin, the spatial structure, and the global dynamical role of the magnetic fields in clusters. Turbulent dynamo models again require knowledge of the ICM viscosity [3,4,7], which itself depends on the field structure, so the problem is highly nonlinear and is as yet unsolved.

An additional complication is that in a turbulent plasma, pressure anisotropies develop in a spontaneous way [2,3,8]. In high-beta plasmas, they trigger a number of instabilities, most interestingly, firehose and mirror [9,10]. The instabilities are very fast compared to the motions of the ICM and give rise to magnetic fluctuations at scales as small as ρ_i. The spatial structure and the saturated amplitude of these fluctuations must be understood before quantitative models of transport can be constructed. In this Letter, we demonstrate how the nonlinear kinetic theory of these fluctuations can be constructed, elucidate the basic physical principle behind their nonlinear evolution, and show that they do not saturate at small quasilinear levels [11], but grow nonlinearly to large amplitudes ($\delta B/B \sim 1$).

The physical origin of pressure anisotropies.—A fundamental property of a magnetized plasma is the conservation of the first adiabatic invariant for each particle, $\mu = v_\perp^2/2B$ (on time scales $\gg \Omega_i^{-1}$). This implies that any change in the field strength must be accompanied by a corresponding change in the perpendicular pressure, $p_\perp/B \sim$ const. In a heuristic way, we may write [2]

$$\frac{1}{p_\perp}\frac{dp_\perp}{dt} \sim \frac{1}{B}\frac{dB}{dt} - \nu_{ii}\frac{p_\perp - p_\parallel}{p_\perp}, \qquad (1)$$

where the last term represents collisions relaxing the pressure anisotropy. On the other hand, the magnetic field is frozen into the plasma flow velocity $\mathbf{u}$, and the field strength obeys [12]

$$\frac{1}{B}\frac{dB}{dt} = \hat{\mathbf{b}}\,\hat{\mathbf{b}}\,{:}\boldsymbol{\nabla}\mathbf{u} \sim \gamma_0, \qquad (2)$$

where $d/dt = \partial/\partial t + \mathbf{u}\cdot\boldsymbol{\nabla}$, $\hat{\mathbf{b}} = \mathbf{B}/B$, and γ_0 is the turnover rate of the turbulent motions. Taking the two terms in the right-hand side of Eq. (1) to be comparable and using Eq. (2), we get $\Delta \equiv (p_\perp - p_\parallel)/p_\perp \sim \gamma_0/\nu_{ii}$. This is the anisotropy persistently driven by the turbulent motions, which are excited at the large (system-size) scales by various macroscopic mechanisms [7].

If the turbulence is Kolmogorov, the dominant contribution to the turbulent stretching and, therefore, to the pressure anisotropy, comes from the viscous scale $l_\nu \sim {\rm Re}^{-3/4}L$, where L is the outer scale. The viscous-scale motions have the characteristic velocity $u \sim {\rm Re}^{-1/4}U$, where U is the characteristic velocity at the outer scale. The Reynolds number ${\rm Re} = UL/\nu$ is calculated using the viscosity of an unmagnetized plasma $\nu \sim v_{{\rm th}i}\lambda_{\rm mfp}$ ($v_{{\rm th}i}$ is the ion thermal speed) because for the motions that change the field strength, the viscosity is not reduced by the magnetic field [1,3]. We now introduce a small parameter

$\epsilon \sim M\mathrm{Re}^{-1/4}$, where $M = U/v_{\mathrm{th}i}$ is the Mach number [2]. Then we can order $u/v_{\mathrm{th}i} \sim \lambda_{\mathrm{mfp}}/l_\nu \sim \epsilon$, whence $\Delta \sim \gamma_0/\nu_{ii} \sim u\lambda_{\mathrm{mfp}}/l_\nu v_{\mathrm{th}i} \sim \epsilon^2$. Using again our fiducial parameters for the Hydra A cluster core, $U \sim 250$ km/s, $v_{\mathrm{th}i} \sim 700$ km/s, $L \sim 10^{17}$ km [4], we have $\epsilon \sim 0.1$. The relatively small resulting typical anisotrophy due to turbulence will have a dramatic effect on the magnetic field.

Qualitative derivation.—Consider first the firehose instability. It is activated when, or in the regions where, $\Delta < 0$ [9], i.e., the magnetic-field strength is decreasing. Such events/regions will always exist in a turbulent plasma. The growing fluctuations are polarized as Alfvén waves, with magnetic perturbations perpendicular to the original field: $\mathbf{B} = \mathbf{B}_0 + \delta\mathbf{B}_\perp$. Using Eq. (1), we estimate

$$\Delta \sim -\frac{|\gamma_0|}{\nu_{ii}} + \frac{\gamma}{\gamma + \nu_{ii}} \frac{\overline{\delta B_\perp^2}}{B_0^2}, \tag{3}$$

where $\gamma_0 = (1/B_0)dB_0/dt < 0$, the instability growth rate is $\gamma = (|\Delta| - 2/\beta_i)^{1/2} k_\| v_{\mathrm{th}i} \gg \gamma_0$ (for $k\rho_i \ll 1$) [2,9], $\beta_i = 4\pi m_i n v_{\mathrm{th}i}^2/B_0^2$, and the overbar denotes averaging over the fluctuation scales. Intuitively, the fluctuations are averaged because the particles streaming along the field lines traverse the field fluctuations faster than the fluctuations grow ($k_\| v_{\mathrm{th}i} \gg \gamma$). Initially, $\gamma \gg \nu_{ii}$; as $\delta B_\perp$ grows, the instability is quenched because the negative anisotropy associated with the large-scale turbulence is compensated by a positive anisotropy due to the small-scale fluctuations. The amplitude at which the quenching occurs is $\delta B_\perp/B_0 \sim (|\gamma_0|/\nu_{ii})^{1/2} \sim \epsilon$. This estimate can also be obtained via a formal quasilinear calculation [11]. However, it does not, in fact, describe a steady state. Indeed, if $\delta B_\perp$ stops changing while the unperturbed field B_0 continues to decrease, the resulting negative pressure anisotropy is again uncompensated, and the firehose instability will be reignited. Since the anisotropy is reduced in the nonlinear regime, the growth of the fluctuations eventually slows down so that $\gamma \ll \nu_{ii}$. Then Eq. (3) shows that the anisotropy stays at the marginal level if $(1/B_0^2)d\overline{\delta B_\perp^2}/dt \sim |\gamma_0|$, whence $\delta B_\perp/B_0 \sim (|\gamma_0|t)^{1/2}$. The physical principle of this nonlinear evolution is that the total average field strength does not change: $d\overline{(B_0^2 + \delta B_\perp^2)}/dt = 0$.

Thus, after an initial burst of exponential growth, the firehose fluctuations grow secularly until the anisotropy-driving fluid motion decorrelates. As this happens on the time scale $\sim|\gamma_0|^{-1}$, the fluctuations will have time to become large, $\delta B_\perp/B_0 \sim 1$. For Hydra A parameters used above, the time needed for that is $|\gamma_0|^{-1} \sim 10^6$ yrs.

Kinetic theory.— We now derive these results in a systematic way. Although finite ion Larmor radius (FLR) effects are important for the quantitative theory of the firehose instability [14,15], the limit $k\rho_i \ll 1$ provides the simplest possible analytical framework for elucidating the key elements of the nonlinear physics, which persist with FLR [16]. We start with the Kinetic MHD equations [17], valid for $k\rho_i \ll 1$ and $\omega \ll \Omega_i$:

$$m_i n \frac{d\mathbf{u}}{dt} = -\nabla\left(p_\perp + \frac{B^2}{8\pi}\right) + \nabla\cdot\left[\hat{\mathbf{b}}\,\hat{\mathbf{b}}\left(p_\perp - p_\| + \frac{B^2}{4\pi}\right)\right], \tag{4}$$

$$\frac{d\mathbf{B}}{dt} = \mathbf{B}\cdot\nabla\mathbf{u}. \tag{5}$$

We set $n = \mathrm{const}$ and $\nabla\cdot\mathbf{u} = 0$. This can be obtained self-consistently, but to reduce the amount of formal derivations, we simply assume incompressibility at all scales (the motions are subsonic). The pressure anisotropy is $p_\perp - p_\| = \int d^3\mathbf{v} m_i(v_\perp^2/2 - v_\|^2) f(t, \mathbf{r}, \mathbf{v})$, where f is the ion distribution function and $\mathbf{v}$ the ion velocity in the frame moving with the mean velocity $\mathbf{u}$. The electron contribution to $p_\perp - p_\|$ is smaller by $(m_e/m_i)^{1/2}$. The ion distribution function satisfies [17]

$$\frac{df}{dt} + \xi v\hat{\mathbf{b}}\cdot\nabla f - \left(\frac{\partial f}{\partial t}\right)_{\mathrm{c}} = \frac{\hat{\mathbf{b}}\cdot\nabla B}{B} v\frac{1-\xi^2}{2}\frac{\partial f}{\partial \xi} + \left(\hat{\mathbf{b}}\cdot\frac{d\mathbf{u}}{dt} - \frac{e}{m_i}E_\|\right)\left(\xi\frac{\partial f}{\partial v} + \frac{1-\xi^2}{v}\frac{\partial f}{\partial \xi}\right) - \hat{\mathbf{b}}\,\hat{\mathbf{b}}:\nabla\mathbf{u}\left[\frac{1-3\xi^2}{2}v\frac{\partial f}{\partial v} - \frac{3}{2}(1-\xi^2)\xi\frac{\partial f}{\partial \xi}\right], \tag{6}$$

where $v = |\mathbf{v}|$, $\xi = v_\|/v$, and the last term on the left-hand side is the collision operator.

We take $\mathbf{B} = \mathbf{B}_0 + \delta\mathbf{B}_\perp$, $\mathbf{u} = \mathbf{u}_0 + \delta\mathbf{u}_\perp$, and $E_\| = 0$, where the *slow* fields $\mathbf{B}_0$, $\mathbf{u}_0$ (the background turbulence) vary at the rate γ_0 on the scale l_ν of the viscous motions (or larger) and the *fast* perturbations $\delta\mathbf{B}_\perp$, $\delta\mathbf{u}_\perp$ have the growth rate γ and wave number k. We formally order all scales and amplitudes with respect to the small parameter ϵ introduced above. As we see from Eq. (3), it is sensible to let the fluctuation growth rate be (at least) the same order as the collision rate: $\gamma \sim \epsilon k v_{\mathrm{th}i} \sim \nu_{ii}$, whence $k_\| \sim (\epsilon\lambda_{\mathrm{mfp}})^{-1}$ [18]. For the fluid motions, $u_0/v_{\mathrm{th}i} \sim \epsilon$, $\gamma_0 \sim \epsilon^3 k v_{\mathrm{th}i}$, and $l_\nu^{-1} \sim \epsilon^2 k$. The expected fluctuation level at which the instability starts being nonlinearly quenched tells us to order $\delta B_\perp/B_0 \sim \epsilon$ and, using Eq. (5), $\delta u_\perp/v_{\mathrm{th}i} \sim \epsilon^2$. From Eq. (4), we see that the pressure anisotropy is destabilizing only if it is not overwhelmed by the magnetic tension, so we order $1/\beta_i \sim \Delta \sim \epsilon^2$.

We seek the distribution function $f = f_0 + \delta f_1 + \delta f_2 + \cdots$, where f_0 only has slow variation in space and time. To order ϵ (the lowest nontrivial order), Eq. (6) becomes $\xi v\hat{\mathbf{b}}_0\cdot\nabla\delta f_1 - (\partial f_0/\partial t)_{\mathrm{c}} = 0$. Averaging along the magnetic field, we get $(\partial f_0/\partial t)_{\mathrm{c}} = 0$, whence f_0 is a Maxwellian: $f_0 = n_0\exp(-v^2/v_{\mathrm{th}i}^2)/(\pi v_{\mathrm{th}i}^2)^{3/2}$. Then $\xi v\hat{\mathbf{b}}_0\cdot\nabla\delta f_1 = 0$, i.e., δf_1 has no fast variation along the magnetic field. To order ϵ^2, we learn, in a similar fashion, that δf_1 converges to a Maxwellian on the collision time scale (so it can be absorbed into f_0) and that δf_2 has no fast variation along the magnetic field. Finally, to order ϵ^3, the kinetic equation averaged along $\hat{\mathbf{b}}_0$ is

$$\frac{\partial \overline{\delta f_2}}{\partial t} - \left(\frac{\partial \overline{\delta f_2}}{\partial t}\right)_{\rm c} = \overline{\hat{\mathbf{b}}\,\hat{\mathbf{b}}:\nabla\mathbf{u}}(1-3\xi^2)\frac{v^2}{v_{\rm th\mathit{i}}^2}f_0, \quad (7)$$

where the overbar denotes spatial averaging along the field line. In order to solve this equation, we assume, as a simple model, a pitch-angle-scattering collision operator with a constant collision rate: $(\partial\overline{\delta f_2}/\partial t)_{\rm c} = (\nu_{ii}/2)(\partial/\partial\xi)\times(1-\xi^2)\partial\overline{\delta f_2}/\partial\xi$. While this is not quantitatively correct, it is sufficient for our purposes. Solving for $\overline{\delta f_2}$ and calculating the pressure anisotropy, we find

$$\begin{aligned}\Delta(t) \equiv \frac{p_\perp - p_\parallel}{p_0} &= 3\int_0^t dt' e^{-3\nu_{ii}(t-t')}\overline{\hat{\mathbf{b}}\,\hat{\mathbf{b}}:\nabla\mathbf{u}}(t') \\ &= -\frac{|\gamma_0|}{\nu_{ii}}(1-e^{-3\nu_{ii}t}) + \frac{3}{2}\int_0^t dt' e^{-3\nu_{ii}(t-t')}\frac{d}{dt'}\frac{\overline{\delta B_\perp^2(t')}}{B_0^2},\end{aligned} \quad (8)$$

where $\gamma_0 = (1/B_0)dB_0/dt < 0$, $p_0 = n_0 m_i v_{\rm th\mathit{i}}^2/2$, and we have used Eq. (2). Equation (8) is the quantitative form of Eq. (3). Note that it generalizes the Braginskii [1] formula $p_\perp - p_\parallel = (p_0/\nu_{ii})\hat{\mathbf{b}}\,\hat{\mathbf{b}}:\nabla\mathbf{u}$, which is only valid for fields varying slowly in space in time (cf. [2]).

Applying our ordering to Eqs. (4) and (5), we get

$$\frac{\partial^2 \delta\mathbf{B}_\perp}{\partial t^2} = \frac{1}{2}v_{\rm th\mathit{i}}^2\left[\Delta(t) + \frac{2}{\beta_i}\right]\nabla_\parallel^2\,\delta\mathbf{B}_\perp, \quad (9)$$

where $\beta_i = 8\pi p_0/B_0^2$ and $\nabla_\parallel$ is the gradient along $\mathbf{B}_0$.

Equations (9) and (8) describe the evolution of firehose perturbations both in the linear and nonlinear regimes. Consider the evolution of a single Fourier mode (Fig. 1). When $\delta B_\perp/B_0 \ll \epsilon$, the first (linear) term in Eq. (8) dominates and the perturbations grow exponentially with the firehose growth rate $\gamma = (|\gamma_0|/\nu_{ii} - 2/\beta_i)^{1/2}k_\parallel v_{\rm th\mathit{i}}$. Once the nonlinearity becomes significant (at $\delta B_\perp/B_0 \sim \epsilon$), the anisotropy is gradually suppressed and, for $t\nu_{ii} \gg 1$, $\delta B_\perp^2(t)/B_0^2 \simeq \Lambda t - 2\nu_{ii}/k_\parallel^2 v_{\rm th\mathit{i}}^2 t + c_1\sqrt{\Lambda t}\int^t dt' e^{-3\nu_{ii}t'/2}t'^{-1/4}\times\sin(\sqrt{2\Lambda/3}k_\parallel v_{\rm th\mathit{i}}t'^{3/2} + c_2)$, where $\Lambda = 2(|\gamma_0| - 2\nu_{ii}/\beta_i)$ and c_1 and c_2 are integration constants. The dominant behavior (the first term) is the secular growth we already derived qualitatively above. The second term is the long-time subdominant correction, and the third is an oscillatory transient, which decays on the collision time scale.

Considering the nonlinear evolution from arbitrary initial conditions involving many Fourier modes requires inclusion of the FLR terms that set the wave number of maximum growth. While the spatial structure of the fluctuations becomes more complex and a power-law energy spectrum emerges [16], the key physical result derived above persists: the fluctuation energy grows secularly with time until finite amplitudes are reached.

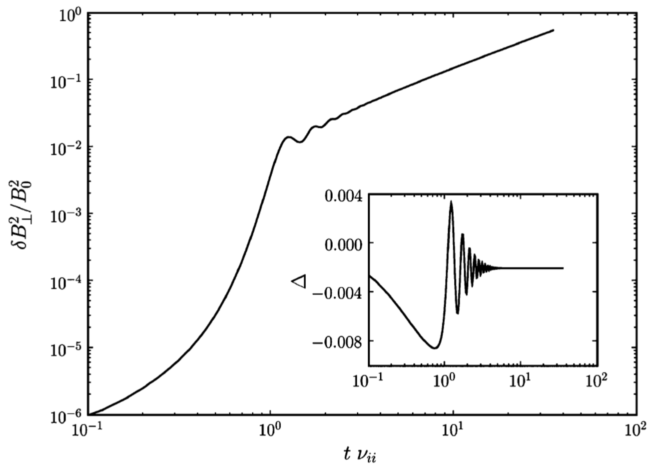

FIG. 1. Evolution of $\delta B_\perp^2(t)/B_0^2$ and $\Delta(t)$ (inset) obtained by numerically solving Eqs. (9) and (8) for a single Fourier mode. Here $\gamma_0/\nu_{ii} = 0.01$, $\beta_i = 1000$, and $k_\parallel v_{\rm th\mathit{i}}/\sqrt{2}\nu_{ii} = 10$.

The mirror instability.—The nonlinear evolution of the mirror instability shares some of the features of the firehose, but the full kinetic calculation is much more complicated. Here we only present a qualitative discussion.

The mirror instability is triggered for $\Delta > 0$ (increasing B), has the growth rate $\gamma \sim \Delta k_\parallel v_{\rm th\mathit{i}}$ for $k\rho_i \ll 1$, and gives rise to growing perturbations of the magnetic-field strength, $\delta B_\parallel$ [2,10]. The pressure anisotropy is, as before, determined by the changing field strength seen on the average by parallel-streaming particles:

$$\Delta \sim \frac{\gamma_0}{\nu_{ii}} + \frac{1}{\gamma + \nu_{ii}}\frac{d}{dt}\frac{\overline{\delta B_\parallel}}{B_0}. \quad (10)$$

For particles traveling the full length of the field line, $\overline{\delta B_\parallel} = 0$; the particles for which $\xi < \xi_{\rm tr} \sim |\delta B_\parallel/B_0|^{1/2}$ are trapped by the fluctuations (“mirrors”) and play a key role in the nonlinear dynamics [19]. Trapping becomes important when the bounce frequency approaches the instability growth rate: $\omega_{\rm b} \sim k_\parallel v_{\rm th\mathit{i}}\xi_{\rm tr} \sim \gamma \sim (\gamma_0/\nu_{ii})k_\parallel v_{\rm th\mathit{i}}$, or $\delta B_\parallel/B_0 \sim (\gamma_0/\nu_{ii})^2 \sim \epsilon^4$. For amplitudes above this level, $\overline{\delta B_\parallel}/B_0 \sim \xi_{\rm tr}\delta B_\parallel/B_0 \sim -|\delta B_\parallel/B_0|^{3/2}$ (negative because particles are trapped in the regions of weaker field). We substitute this estimate into Eq. (10), assume slow evolution ($\gamma \ll \nu_{ii}$), and find that the marginal state is achieved for $\delta B_\parallel/B_0 \sim (\gamma_0 t)^{2/3}$. This secular growth continues until the turbulent motion responsible for the pressure anisotropy decorrelates, by which time $\delta B_\parallel/B_0 \sim 1$. The FLR effects, while important [10,20], are ignored in this qualitative argument, but are unlikely to change the main result (secular growth).

Conclusion.—We have shown that, in high-beta turbulent plasmas, small-scale magnetic fluctuations are continually generated by plasma instabilities and grow nonlinearly to large amplitudes, $\delta B/B \sim 1$, so strongly “wrinkled” magnetic structures emerge on the fluid time scales. The main difference between our theory and most others [11,15,19,20] is that they consider an *initial* pressure anisotropy gradually cancelled by fluctuations in a collisionless plasma, whereas in our calculation, the anisotropy is continually *driven* by the turbulent motions and relaxed by (weak) collisions; the evolution of the fluctuations is followed over times longer than the collision time, up to the fluid time scale. The underlying physical principle of the nonlinear evolution is the tendency for the growing fluctuations to compensate on the average the pressure anisotropies generated by the turbulence.

This mechanism of making small-scale magnetic fields is distinct from the fluctuation dynamo, which exponentiates the magnetic energy at the turbulent stretching rate ($\sim\gamma_0$, much slower than the plasma instabilities) and produces long filamentary folded structures, so the parallel correlation length of the field remains macroscopically large ($\sim$ outer scale) [13], in contrast to the instability-produced wrinkles with parallel scales possibly as small as the ion gyroscale. How the dynamo operates in the presence of the instabilities [3] is a subject of an ongoing investigation motivated by the fundamental problem of the origin of cosmic magnetism in general and of magnetic fields in galaxy clusters, in particular.

To illustrate the potentially dramatic effect of firehose and mirror fluctuations on the transport properties of magnetical turbulent plasmas, consider the ICM thermal conduction problem. The standard estimates of the electron thermal conductivity in a tangled magnetic field are [21] $\kappa_e \sim v_{\mathrm{th}e}\lambda_{\mathrm{mfp}}$ if $\lambda_{\mathrm{mfp}} \ll l_B$ (collisional), $\kappa_e \sim v_{\mathrm{th}e}\lambda_{\mathrm{mfp}} l_B/L_{\mathrm{RR}}$ if $l_B \ll \lambda_{\mathrm{mfp}} \ll L_{\mathrm{RR}}$ (semicollisional), and $\kappa_e \sim v_{\mathrm{th}e} l_B$ if $\lambda_{\mathrm{mfp}} \gg L_{\mathrm{RR}}$ (collisionless), where l_B is the (parallel) correlation length of the magnetic field and $L_{\mathrm{RR}} = l_B \ln(l_B/\rho_e)$ is the Rechester-Rosenbluth length. In most MHD models [21] (including the fluctuation dynamo [13]), l_B is macroscopic and all three estimates yield an effectively isothermal ICM (except at macroscopic scales). However, if magnetic wrinkles with $\delta B/B \sim 1$ develop at scales $\sim\rho_i$, we have $l_B \sim \rho_i$ and $L_{\mathrm{RR}} \sim \rho_i \ln(\rho_i/\rho_e) \ll \lambda_{\mathrm{mfp}}$, so $\kappa_e \sim v_{\mathrm{th}e}\rho_i$. For our fiducial Hydra A parameters, this is 10^{10} times smaller than the collisional value, so there is effectively no thermal conduction on macroscopic scales. The ICM viscosity is similarly reduced, from $v_{\mathrm{th}i}\lambda_{\mathrm{mfp}}$ to $v_{\mathrm{th}i}\rho_i$ because with $l_B \sim \rho_i$, the effective ion mean free path is $\sim\rho_i$. Curiously, in stronger-field regions where $2/\beta_i > \Delta$ and the instabilities are suppressed, the transport is more effective: the thermal conductivity and viscosity remain large (although highly anisotropic).

Because of spatial resolution constraints, the firehose and mirror structures are not directly detectable in clusters, but the huge changes in the transport coefficients that they may cause will have a potentially predictable effect on observable large-scale fields and flows [5,6]. More direct information is available from satellite measurements in space plasmas. Mirror structures with $\delta B/B \sim 1$ have, indeed, been found [22], and there is strong evidence that the directly measured temperature anisotropies match the firehose and mirror marginal stability conditions [23].

This work was supported by UK STFC (A. A. S., M. S. R., and T. H.), US DOE CMPD, and the Leverhulme Trust Network for Magnetized Plasma Turbulence.

*a.schekochihin@imperial.ac.uk

[1] S. I. Braginskii, Rev. Plasma Phys. **1**, 205 (1965).
[2] A. A. Schekochihin *et al.*, Astrophys. J. **629**, 139 (2005).
[3] A. A. Schekochihin and S. C. Cowley, Phys. Plasmas **13**, 056501 (2006).
[4] T. A. Enßlin and C. Vogt, Astron. Astrophys. **453**, 447 (2006).
[5] J. R. Peterson and A. C. Fabian, Phys. Rep. **427**, 1 (2006); M. Markevitch and A. Vikhlinin, Phys. Rep. **443**, 1 (2007).
[6] A. Fabian *et al.*, Mon. Not. R. Astron. Soc. **363**, 891 (2005); K. Dolag *et al.*, Mon. Not. R. Astron. Soc. **364**, 753 (2005); T. J. Dennis and B. D. G. Chandran, Astrophys. J. **622**, 205 (2005); M. Ruszkowski *et al.*, Mon. Not. R. Astron. Soc. **378**, 662 (2007).
[7] K. Subramanian *et al.*, Mon. Not. R. Astron. Soc. **366**, 1437 (2006).
[8] P. Sharma *et al.*, Astrophys. J. **637**, 952 (2006).
[9] M. N. Rosenbluth, LANL Report No. LA-2030, 1956 (unpublished); S. Chandrasekhar *et al.*, Proc. R. Soc. A **245**, 435 (1958); E. N. Parker, Phys. Rev. **109**, 1874 (1958); A. A. Vedenov and R. Z. Sagdeev, Sov. Phys. Dokl. **3**, 278 (1958); A. Barnes, Phys. Fluids **9**, 1483 (1966).
[10] A. Hasegawa, Phys. Fluids **12**, 2642 (1969).
[11] V. D. Shapiro and V. I. Shevchenko, Sov. Phys. JETP **18**, 1109 (1964).
[12] We consider magnetic fields at scales greater than ρ_i. In plasmas such as the ICM, the resistive scale is smaller than ρ_i [3], so we neglect the resistivity (this hinges on assuming that the characteristic scale of the magnetic field is *not* determined by the resistive cutoff, unlike in standard, collisionally dominated MHD [13]). We also assume $\nabla \cdot \mathbf{u} = 0$ because the plasma motions are subsonic.
[13] A. A. Schekochihin *et al.*, Astrophys. J. **612**, 276 (2004).
[14] P. H. Yoon *et al.*, Phys. Fluids B **5**, 1971 (1993); P. Hellinger and H. Matsumoto, J. Geophys. Res. **105**, 10519 (2000).
[15] W. Horton *et al.*, J. Geophys. Res. **109**, A09216 (2004).
[16] M. S. Rosin *et al.* (unpublished).
[17] R. M. Kulsrud, in *Handbook of Plasma Physics*, edited by A. A. Galeev and R. N. Sudan (North-Holland, Amsterdam, 1983), Vol. 1, p. 115.
[18] This ordering is a formal imposition that will help us proceed in a systematic fashion. In reality, the most linearly unstable wave number is determined by the FLR modification of the growth rate [14].
[19] M. G. Kivelson and D. S. Southwood, J. Geophys. Res. **101**, 17365 (1996).
[20] E. A. Kuznetsov *et al.*, Phys. Rev. Lett. **98**, 235003 (2007); P. Hellinger, Phys. Plasmas **14**, 082105 (2007).
[21] A. B. Rechester and M. N. Rosenbluth, Phys. Rev. Lett. **40**, 38 (1978); B. D. G. Chandran and S. C. Cowley, Phys. Rev. Lett. **80**, 3077 (1998).
[22] E. A. Lucek *et al.*, Ann. Geophys. **19**, 1421 (2001); F. Sahraoui *et al.*, Phys. Rev. Lett. **96**, 075002 (2006).
[23] P. Hellinger *et al.*, Geophys. Res. Lett. **33**, L09101 (2006).