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QUASILINEAR THEORY OF INSTABILITY OF A PLASMA WITH AN ANISOTROPIC ION VELOCITY DISTRIBUTION

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Equations are obtained for the changes in the longitudinal and transverse ion thermal energies and also the energy of an electromagnetic field in an instability due to anisotropy of the distribution function. Conditions are obtained for the applicability of the equations, and reduce to the requirement of smallness of the deviation of the initial plasma parameters from the critical values at which instability sets in. The equations are employed to determine the state which the plasma finally assumes as a result of development of the instability.

COLLECTIVE motion connected with the ion oscillation mode [1] is excited in a collision-free plasma placed in a magnetic field when the anisotropy of the temperatures (longitudinal and transverse) is sufficiently large. The excitation of the collective motions is accompanied by an increase in the longitudinal thermal energy in the plasma at the expense of the transverse one (or vice versa), until the plasma reaches a state which is stable relative to the collective interaction. In the present paper we analyze this process in the quasilinear approximation.

A very important feature of the instabilities under consideration is the fact that they are aperiodic (the real part of the frequency ω^r vanishes). Usually (see for example, [2]) the quasilinear theory is used when the condition that the increment be small compared with the frequency ($\gamma_k \ll \omega_k^r$) is satisfied. However, as shown in the present paper, the quasilinear theory can be applied also to aperiodic instabilities, provided the condition $\gamma_k \ll k_z v_{T\parallel}$ is satisfied ($v_{T\parallel}$ — longitudinal thermal velocity of the ions). The latter condition, as applied to the instabilities connected with the anisotropy of the distribution function, signifies that in the initial state the plasma parameters deviate little from the critical values at which instability sets in. Under this assumption, we obtain here the plasma temperature that sets in during the saturation stage and the collective-interaction energy of the magnetic field generated in the plasma.

1. According to [3], the dispersion equation for the ion oscillation mode takes when $|\omega| < \omega_{Hi}$ the form

$$\cos^2\varphi n^4 - (\varepsilon_{11} + \varepsilon_{22}\cos^2\varphi) n^2 + \varepsilon_{11}\varepsilon_{22} + \varepsilon_{12}^2 = 0, \quad (1)$$

$n = ck/\omega_k$, φ — angle between k and H_0, and $k_y = 0$.

We confine ourselves henceforth to excitation of the low-frequency part of the spectrum $|\omega| \ll \omega_{Hi}$, when $\epsilon_{12} \sim (|\omega|/\omega_{Hi})\epsilon_{22} \ll \epsilon_{22}$, and consider collective motions in which the electric field is perpendicular to the plane containing k and H_0. In this case we have from (1) Slow-ω-polarized!

$$c^2k^2 = \omega_k^2 \varepsilon_{22}. \quad (2)$$

Substituting ϵ_{22} from [3] and assuming the following conditions to be satisfied

$$k_z^2 v_{T\parallel}^2 \ll \omega_{Hi}^2, \qquad k_\perp^2 v_{T\perp}^2 \ll \omega_{Hi}^2, \quad (3)$$

we reduce (2) to the form

$$c^2k^2 - \omega_k^2 - \frac{\omega_{0i}^2}{N\omega_{Hi}^2}\left\{\int d\mathbf{v} f_0\left[\omega_k^2 + k_z^2\left(v_z^2 - \frac{1}{2}v_\perp^2\right) - k_\perp^2 v_\perp^2\right]\right.$$
$$\left. - \frac{1}{4}\int d\mathbf{v} k_\perp^2 v_\perp^4 \frac{k_z \partial f_0/\partial v_z}{k_z v_z - \omega_k}\right\} = 0. \quad (4)$$

Following [1], we consider separately two cases.

(a) $k_\perp = 0$. We have for ω_k from (4)

$$\omega_k^2 = -\frac{k_z^2}{1 + H_0^2/4\pi N M c^2}\frac{T_\parallel - T_\perp - H_0^2/4\pi N}{M}$$
$$= -k_z^2\frac{T_\parallel}{M(1 + H_0^2/4\pi N M c^2)} Y. \quad (5)$$

Here

$$T_\perp = \frac{M}{2N}\int v_\perp^2 f_0 d\mathbf{v}, \qquad T_\parallel = \frac{M}{N}\int v_z^2 f_0\, d\mathbf{v},$$

$$Y = \frac{T_\parallel - T_\perp - H_0^2/4\pi N}{T_\parallel}.$$

When $Y \ll 1$ (we henceforth confine ourselves to this case) we have $|\omega_k|^2 \ll k_z^2 v_{T\parallel}^2$. The condition for the occurrence of instability ($\omega_k^2 < 0$) reduces to $T_{\|} > T_\perp + H_0^2/4\pi N$.

(b) $k_\perp \neq 0$. In this case it is sufficient to retain in the dispersion equation only the terms that are linear in ω_k. Calculating the integral with respect to v_z in (4) under the assumption that $|\omega_k| \ll k_z v_{T_{\|}}$, we obtain

$$\omega_k = ik_z\Big[\frac{M}{8N}\int\frac{v_\perp^4}{v_z}\frac{\partial f_0}{\partial v_z}\,d\mathbf{v} + T_\perp + \frac{H_0^2}{8\pi N} - \frac{k_z^2}{2k_\perp^2}\Big(T_\perp + \frac{H_0^2}{4\pi N} - T_{\|}\Big)\Big] \times \Big[\frac{\pi M}{8N}\int v_\perp^4\frac{\partial^2 f_0}{\partial v_z^2}(v_\perp, 0)\,dv_\perp\Big]^{-1}. \quad (6)$$

Since for any function f_0 which has a maximum at $v_z = 0$ we have

$$\int v_\perp^4\frac{\partial^2 f_0}{\partial v_z^2}(v_\perp, 0)\,dv_\perp < 0,$$

the condition for the occurrence of instability in this case reduces to

$$-\frac{M}{8N}\int\frac{v_\perp^4}{v_z}\frac{\partial f_0}{\partial v_z}\,d\mathbf{v} > T_\perp + \frac{H_0^2}{8\pi N}.$$

The instability arises thus, unlike in case (a), only when the transverse temperature is sufficiently large.

The condition $\gamma_k \ll k_z v_{T_{\|}}$, as follows from (6), is satisfied when

$$Y^* = \Big[\frac{M}{8N}\int\frac{v_\perp^4}{v_z}\frac{\partial f_0}{\partial v_z}\,d\mathbf{v} + T_\perp + \frac{H_0^2}{8\pi N}\Big]\Big/\frac{M}{8N}\int\frac{v_\perp^4}{v_z}\frac{\partial f_0}{\partial v_z}\,d\mathbf{v} \ll 1.$$

It also follows from (6) that when $Y^* \ll 1$, the greatest growth occurs in the oscillations with $k_z \ll k_\perp$ ($k_z^2/k_\perp^2 \sim Y^*$). In the case when the distribution function f_0 is Maxwellian with two different temperatures

$$f_0 = N\frac{M^{3/2}}{(2\pi)^{3/2}T_\perp T_{\|}^{1/2}}\exp\Big[-\frac{Mv_\perp^2}{2T_\perp} - \frac{Mv_z^2}{2T_{\|}}\Big], \quad (7)$$

the increments obtained from (5) and (6) coincide with those obtained in [1]. We retain in the dispersion equation only the terms of lowest order $k_\perp v_{T_\perp}/\omega_{Hi}$. Account of the finite Larmor radius

NB shows [4,5] that when $k_\perp^2 v_{T_\perp}^2/\omega_{Hi}^2 \gtrsim Y^*$ the investigated instability becomes stabilized. Thus, in our case, when $Y^* \ll 1$, we have $k_\perp^2 v_{T_\perp}^2/\omega_{Hi}^2 \ll 1$ for the entire unstable part of the spectrum.

2. We now turn to the derivation of the equation of the quasilinear approximation. We break up the ion velocity distribution function, which is a solution of the nonlinear Boltzmann-Vlasov equation, into two parts—one oscillating in space and characterizing the collective motions of the plasma excited during the instability, and one homogeneous and monotonically time-varying, characterizing the "background" against which these motions occur:

$$f = f_0(t, \mathbf{v}) + f_1(t, \mathbf{r}, \mathbf{v}).$$

If all the quantities characterizing the collective motions of the plasma are represented in the form of a superposition of Fourier harmonics

$$f_1 = \tfrac{1}{2}\sum_k f_k e^{i\mathbf{kr}} + \text{c.c.}, \qquad \mathbf{E} = \tfrac{1}{2}\sum_k \mathbf{E}_k e^{i\mathbf{kr}} + \text{c.c.},$$

$$\frac{\partial \mathbf{E}_k}{\partial t} = -i\omega_k\mathbf{E}_k, \qquad \mathbf{H}_k = \frac{c}{\omega_k}[\mathbf{kE}_k].$$

By averaging the Boltzmann-Vlasov equation over distances that are large compared with the oscillation wavelength we obtain for $f_0 = \langle f\rangle$ the following equation (in the averaging it is necessary to take it into account that $\langle \mathbf{E}\rangle = 0$, $\langle \mathbf{H}\rangle = \mathbf{H}_0 \parallel Oz$, and that f_0 does not depend on ϑ—the azimuthal angle in velocity space):

$$\frac{\partial f_0}{\partial t} = \frac{e}{2M}\sum_k\Big\{\mathbf{E}_k^*\Big(1 - \frac{\mathbf{kv}}{\omega_k^*}\Big) + \frac{\mathbf{k}}{\omega_k^*}(\mathbf{vE}_k^*)\Big\}\frac{\partial f_k}{\partial \mathbf{v}} + \text{c.c.} \quad (8)$$

In this equation it is convenient to go over to the polar coordinates $v_\perp$, v_z and θ; $\theta = \vartheta - \Phi$; ϑ and Φ are the azimuth angles of the vectors $\mathbf{v}_\perp$ and $\mathbf{k}_\perp$. In this case, when $\mathbf{E}_k$ does not depend on Φ, integration with respect to Φ in the right half of (8) is equivalent to averaging over θ. The dependence on ϑ in the diffusion coefficients of (8) then drops out, and the axial symmetry is time-invariant. Equation (8) is reduced to the form

$$\frac{\partial f_0}{\partial t} = \Big(\frac{\partial}{\partial v_\perp} + \frac{1}{v_\perp}\Big)\sum_k(J_1\cos\theta + J_2\sin\theta) + \frac{\partial}{\partial v_z}\sum_k J_3.$$

$$J_1 = \frac{e}{2M}\Big(\frac{k_\perp v_\perp}{\omega_k^*}E_k^* f_k\sin\theta + \text{c.c.}\Big),$$

$$J_2 = -\frac{e}{2M}\Big[\Big(1 - \frac{k_z v_z}{\omega_k^*} - \frac{k_\perp v_\perp}{\omega_k^*}\cos\theta\Big)E_k^* f_k + \text{c.c.}\Big],$$

$$J_3 = -\frac{e}{2M}\Big(\frac{k_z v_\perp}{\omega_k^*}E_k^* f_k\sin\theta + \text{c.c.}\Big). \quad (9)$$

We have used here the fact that $\mathbf{E}_k$ is perpendicular to $\mathbf{k}$ and $\mathbf{H}_0$; f_k in (9) depends on the "background" distribution function f_0. To find the specific form of this dependence we use two assumptions that are customary in the quasilinear theory.

A. We assume that the variation of f_0 is adiabatic:

$$\left|\frac{1}{(\omega - k_z v_z + n\omega_H) f_0} \frac{\partial f_0}{\partial t}\right| \ll 1, \qquad n = 0, \pm 1 \ldots,$$

$$\omega = \omega^r + i\gamma$$

(see [6]). In the case under consideration, when the instability is aperiodic ($\omega^r = 0$) and the thermal scatter is the small ($k_z v_{T\|} \ll \omega_{Hi}$) we obtain the strongest condition when $n = 0$. Inasmuch as $|f_0^{-1} \partial f_0/\partial t| \lesssim \gamma$, this condition can be written in the form [1)] $\gamma \ll k_z v_{T\|}$. From (5) and (6) it follows that this condition is satisfied when Y or Y* $\ll 1$, i.e., when the deviation from the critical plasma parameters at which the instability sets in is small.

B. We also assume that we can confine ourselves to the linear approximation in the description of the plasma collective motions. In Sec. 5 we shall clarify the condition under which we can neglect the nonlinear interaction between the different harmonics of the collective motions.

Under these assumptions we have for $f_{\mathbf{k}}$ the usual formula of the linear theory

$$f_{\mathbf{k}} = -\frac{e}{M} \sum_{n, n'} J_{n'}(\lambda_2) J'_n(\lambda_2) \frac{\exp i(n-n')\theta}{n\omega_{Hi} - k_z v_z + \omega_{\mathbf{k}}} E_{\mathbf{k}} \left[\frac{\partial f_0}{\partial v_\perp} - \frac{k_z}{\omega_{\mathbf{k}}}\left(v_z \frac{\partial f_0}{\partial v_\perp} - v_\perp \frac{\partial f_0}{\partial v_z}\right)\right]. \tag{10}$$

Substituting (10) in (9), averaging over θ, and simplifying the obtained expression by using the fact that $\omega^r = 0$ and conditions (3) are satisfied, we reduce (9) to the form [2)]

$$\frac{\partial f_0}{\partial t} = \frac{e^2}{2M^2}\left\{\frac{1}{v_\perp}\frac{\partial}{\partial v_\perp}\left[v_\perp \sum_k \frac{|H_{\mathbf{k}}|^2}{c^2k^2} \times \left(\frac{\gamma_{\mathbf{k}}}{\omega_H^2}\left(k_z^2 v_z^2 + \gamma_{\mathbf{k}}^2 + \frac{k_\perp^2 v_\perp^2}{2}\right)\frac{\partial f_0}{\partial v_\perp} - \frac{k_\perp^2 v_\perp^3}{\omega_H^2}\frac{\gamma_{\mathbf{k}}}{k_z^2 v_z^2 + \gamma_{\mathbf{k}}^2} k_z^2 v_z \frac{\partial f_0}{\partial v_z}\right)\right] + \frac{\partial}{\partial v_z}\sum_k \frac{|H_{\mathbf{k}}|^2}{c^2k^2} \right. \tag{11}$$

$$\left. \times\left[-\frac{2\gamma_{\mathbf{k}}}{\omega_H^2} k_z^2 v_z v_\perp \frac{\partial f_0}{\partial v_\perp} + \left(k_z^2 v_\perp^2 \frac{\gamma_{\mathbf{k}}}{\omega_H^2} + \frac{k_\perp^2 k_z^2 v_\perp^4}{\omega_H^2}\frac{\gamma_{\mathbf{k}}}{k_z^2 v_z^2 + \gamma_{\mathbf{k}}^2}\right)\frac{\partial f_0}{\partial v_z}\right]\right\}.$$

3. To solve (11) we consider separately, as in the solution of the dispersion equation, the two cases $k_\perp = 0$; $k_\perp \gg k_z$.

When $k_\perp = 0$ the instability is connected with all the plasma particles and not with some preferred group of resonant frequencies: the increment is determined by the average quantities $T_\|$ and $T_\perp$ [see (5)] while the diffusion coefficients in (11) are smooth functions of v_z and $v_\perp$ when $k_\perp = 0$. For this reason it is sufficient to confine the analysis of this case to the change in the quantities averaged over the velocities, i.e., the moments of the distribution functions relative to the velocities. From (11) we have

$$\int \frac{\partial f_0}{\partial t} d\mathbf{v} = 0, \qquad \int v_z \frac{\partial f_0}{\partial t} d\mathbf{v} = 0,$$

i.e., as expected f_0 remains in time an even function of v_z.

For the second-order moments we have from (11) for $k_\perp = 0$ the following equations

$$N\frac{dT_\perp}{dt} = \int \frac{Mv_\perp^2}{2}\frac{\partial f_0}{\partial t} d\mathbf{v} = \frac{1}{4\pi}\frac{\omega_{0i}^2}{\omega_H^2}\sum_k \gamma_k |H_k|^2 \frac{k^2 T_\| / M + \gamma_k^2}{c^2k^2}$$

$$= N\sum_k \gamma_k \frac{|H_k|^2}{H_0^2}\left(2T_\| - T_\perp - \frac{H_0^2}{4\pi N}\right); \tag{12}$$

$$N\frac{dT_\|}{dt} = \int Mv_z^2 \frac{\partial f_0}{\partial t} d\mathbf{v} = -\frac{1}{\pi}\frac{\omega_{0i}^2}{\omega_H^2}\sum_k \gamma_k |H_k|^2 \frac{T_\| - T_\perp/2}{Mc^2}$$

$$= -4N\sum_k \gamma_k \frac{|H_k|^2}{H_0^2}\left(T_\| - \frac{T_\perp}{2}\right). \tag{12'}$$

The change in the energy of the electromagnetic field is determined from

$$\frac{d}{dt}\sum_k \frac{|H_k|^2 + |E_k|^2}{8\pi} = \frac{1}{4\pi}\sum_k \gamma_k |H_k|^2\left(1 + \frac{\gamma_k^2}{k^2c^2}\right). \tag{12''}$$

From (12)–(12″) we get the law of energy conservation

$$N\frac{dT_\perp}{dt} + \frac{N}{2}\frac{dT_\|}{dt} + \frac{1}{4\pi}\sum_k \gamma_k(|H_k|^2 + |E_k|^2)$$

$$= \frac{1}{4\pi}\frac{\omega_{0i}^2}{\omega_{Hi}^2}\sum_k \frac{\gamma_k |H_k|^2}{k^2c^2}\left[\gamma_k^2\left(1 + \frac{H_0^2}{4\pi NMc^2}\right) + k^2\frac{T_\perp - T_\| + H_0^2/4\pi N}{M}\right] = 0. \tag{13}$$

The electrons make no contribution in this problem to the energy conservation law, because the

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[1)]It is assumed here that the essential region of the quantities averaged over the velocities is $v_z \sim v_{T_\|}$. This is satisfied for all the averages with the exception of

$$\int \frac{v_\perp^4}{v_z}\frac{\partial \delta f}{\partial v_z} d\mathbf{v},$$

in which the region $v_z \sim v_T (W^\infty)^{1/4}$ is significant as $t \to \infty$ [see (28)]. However, as can be readily seen, when $t \to \infty$ the condition $v_z \gg \gamma_k/k_z$ is satisifed also for these v_z.

[2)]In the derivation of (11) we have neglected the resonant particles with velocities $v_{res} \approx (\omega + \omega_H)/k_z \gg v_{T_\|}$. An account of these particles in the dispersion equation leads to a new type of anisotropic plasma instability, considered in [7]. However, since the number of resonant particles is small ($v_{res} \gg v_{T_\|}$), these instabilities cannot lead to any appreciable change in the plasma parameters.

maximum electron current is in a direction perpendicular to the electric field (and in this direction the electron current cancels the ion current). The electron current in the direction of the electric field is $\mu = M_e/M_i$ times smaller than the ion current.[3)] Therefore the energy absorbed by the electron in the case of instability is smaller by a factor μ than the variation of the ion energy.

From (12)–(12″) we can easily find the variations of $T_\parallel$, $T_\perp$, and

$$W = \tfrac{1}{8}\pi^{-1}\sum_k H_0^{-2}|H_k|^2$$

during the instability. Dividing (12) by (12′) and replacing $T_\perp$ and $T_\parallel$ by their initial values $T_\perp^0$ and $T_\parallel^0$ in the right halves of the obtained equations, which is equivalent to neglecting terms $\sim Y$, we obtain

$$dT_\perp/dT_\parallel \approx -T_\parallel^0/2(2T_\parallel^0 - T_\perp^0).$$

From this we have a relation between the changes of the longitudinal and transverse temperatures in the instability:

$$\delta T_\perp = -\delta T_\parallel \frac{T_\parallel^0}{2(2T_\parallel^0 - T_\perp^0)}, \tag{14}$$

On the other hand, for saturation, when $\gamma_k \to 0$, we have

$$T_\parallel^\infty = T_\perp^\infty + H_0^2/4\pi N. \tag{15}$$

From this we get one more relation between the total changes in the longitudinal and transverse temperatures:

$$\delta T_\parallel^\infty = \delta T_\perp^\infty - YT_\parallel^0. \tag{15′}$$

From (14) and (15′) we have for $\delta T_\parallel^\infty$ and $\delta T_\perp^\infty$

$$\delta T_\perp^\infty = Y\frac{T_\parallel^{02}}{5T_\parallel^0 - 2T_\perp^0}; \qquad \delta T_\parallel^\infty = -Y\frac{2T_\parallel^0(2T_\parallel^0 - T_\perp^0)}{5T_\parallel^0 - 2T_\perp^0}. \tag{16}$$

Analogously as $t \to \infty$, we have from (12″) for W,

$$W^\infty = \frac{Y}{4\pi}\frac{T_\parallel^0}{5T_\parallel^0 - 2T_\perp^0} \tag{17}$$

($|E_k|^2 = \gamma_k^2/k^2c^2|H_k|^2 \to 0$ as $t \to \infty$ when $\gamma_k \to 0$).

As already noted, the quasilinear theory does not hold when $Y \sim 1$. However, in view of the lack of a more rigorous theory, it is of interest to extrapolate the solutions of (12) and (12′) to the region $Y \sim 1$. In this case we have from (12) and (12′)

$$\frac{dT_\parallel}{dT_\perp} = -2\,\frac{2T_\parallel - T_\perp}{2T_\parallel - T_\perp - H_0^2/4\pi N}. \tag{18}$$

[3)]It is assumed that the electron velocity distribution is isotropic.

Solving (18) with the conditions $T_\perp = T_\perp^0$ when $T_\parallel = T_\parallel^0$ and using the fact that (15) is satisfied when $t \to \infty$, we obtain for the transverse temperature during the saturation stage the following equation:

$$T_\perp^\infty - \frac{H_0^2}{30\pi N}\ln\frac{T_\perp^\infty + 9H_0^2/20\pi N}{2T_\parallel^0 - T_\perp^0 - H_0^2/20\pi N} = \frac{1}{3}(T_\parallel^0 + 2T_\perp^0 - H_0^2/4\pi N). \tag{19}$$

When $Y \ll 1$ we obtain from (19) formula (16) for $\delta T_\perp^\infty$, but we must note that (16) and (19) gives results that are quite close when $Y \sim 1$. Thus, for example, for $T_\perp^0 = 0.5\,T_\parallel^0$ and $Y = 0.4$ the value of $\delta T_\perp^\infty$ determined from (19) is $0.12\,T_\parallel^0$ and that determined from (16) is $0.1\,T_\parallel^0$.

4. We now consider an instability with $k_\perp \neq 0$, which arises when the transverse temperature is sufficiently high. Confining ourselves in the diffusion coefficients of (11) to the contribution from the most unstable part of the spectrum, for which $k_z \ll k_\perp \sim k$ [$k_z^2 \sim k_\perp^2 Y^*$, see (6)], we obtain an equation for the time variation of f_0:

$$\frac{\partial f_0}{\partial t} = \frac{1}{2}\sum_k \frac{|H_k|^2}{H_0^2}\left\{\frac{1}{v_\perp}\frac{\partial}{\partial v_\perp}\left[v_\perp\left(\gamma_k\frac{v_\perp^2}{2}\frac{\partial f_0}{\partial v_\perp} - v_\perp^3\frac{\gamma_k k_z^2 v_z}{k_z^2v_z^2+\gamma_k^2}\frac{\partial f_0}{\partial v_z}\right)\right] + \frac{\partial}{\partial v_z}\left(\frac{k_z^2 v_\perp^4}{2}\frac{\gamma_k}{k_z^2v_z^2+\gamma_k^2}\frac{\partial f_0}{\partial v_z}\right)\right\}. \tag{20}$$

From this we get for the change in the ion transverse temperature

$$N\frac{dT_\perp}{dt} = \int\frac{Mv_\perp^2}{2}\frac{\partial f_0}{\partial t}dv = \sum_k\gamma_k\frac{|H_k|^2}{H_0^2}\left(\int Mv_\perp^2 f_0\,dv + \frac{1}{2}\int Mv_\perp^4\frac{k_z^2v_z}{k_z^2v_z^2+\gamma_k^2}\frac{\partial f_0}{\partial v_z}dv\right). \tag{21}$$

The last integral in (21) is equal to

$$\int Mv_\perp^4\frac{k_z^2v_z}{k_z^2v_z^2+\gamma_k^2}\frac{\partial f_0}{\partial v_z}dv = \int\frac{Mv_\perp^4}{v_z}\frac{\partial f_0}{\partial v_z}dv - \gamma_k^2\int Mv_\perp^4\frac{1/v_z}{k_z^2v_z^2+\gamma_k^2}\frac{\partial f_0}{\partial v_z}dv$$

$$= \int\frac{Mv_\perp^4}{v_z}\frac{\partial f_0}{\partial v_z}dv - \pi\frac{\gamma_k}{k_z}\int Mv_\perp^4\frac{\partial^2 f_0}{\partial v_z^2}(v_\perp, 0)\,dv_\perp.$$

We thus obtain ultimately for $T_\perp$ the equation

$$N\frac{dT_\perp}{dt} = \sum_k\gamma_k\frac{|H_k|^2}{H_0^2}\left(2NT_\perp + \frac{1}{2}\int\frac{Mv_\perp^4}{v_z}\frac{\partial f_0}{\partial v_z}dv - \frac{\pi}{2}\frac{\gamma_k}{k_z}\int Mv_\perp^4\frac{\partial^2 f_0}{\partial v_z^2}(v_\perp, 0)\,dv_\perp\right). \tag{22}$$

Analogously, for the time variation of the longitudinal temperature we have from (20)

$$N\frac{dT_\parallel}{dt}=-\frac{1}{2}\sum_k\gamma_k\frac{|H_k|^2}{H_0^2}\left(\int\frac{Mv_\perp^4}{v_z}\frac{\partial f_0}{\partial v_z}dv\right.$$

$$\left.-\pi\frac{\gamma_k}{k_z}\int Mv_\perp^4\frac{\partial^2 f_0}{\partial v_z^2}(v_\perp,0)\,dv_\perp\right). \quad (23)$$

From (22) and (23) we obtain the law of energy conservation—the change in the ion thermal energy is equal to the energy of the magnetic field generated during the instability:

$$N\frac{dT_\perp}{dt}+\frac{N}{2}\frac{dT_\parallel}{dt}+\frac{d}{dt}\sum_k\frac{|H_k|^2}{8\pi}$$

$$=N\sum_k\gamma_k\frac{|H_k|^2}{H_0^2}\left(2T_\perp+\frac{1}{4}\int\frac{Mv_\perp^4}{v_z}\frac{\partial f_0}{\partial v_z}dv+\frac{H_0^2}{4\pi N}\right.$$

$$\left.-\frac{\pi}{4}\frac{\gamma_k}{k_z}\int Mv_\perp^4\frac{\partial^2 f_0}{\partial v_z^2}(v_\perp,0)\,dv_\perp\right). \quad (24)$$

Substituting in (24) γ_k from (6) and neglecting the terms[4] proportional to $k_z^2/k_\perp^2$ we find that the integral in the right half of (24) vanishes. Neglect of the change in the energy of the electric field

$$\sum|E_k|^2\approx\sum\gamma_k^2k^{-2}c^{-2}|H_k|^2$$

in (24) is equivalent to neglecting in the dispersion equation the terms quadratic in γ_k [see (16)].

We now proceed to solve (20). When $v_z \sim v_{T\parallel}$ we can solve (20) by successive approximations. Substituting f_0 in the form $f_0(v)+\delta f_0(t,v)$, where $f_0^0(v)=f_0(0,v)$ is determined by (7), and assuming $|\delta f_0/f_0|\ll 1$ we obtain for δf_0 the relation

$$\delta f_0=2\pi W\left[\frac{1}{v_\perp}\frac{\partial}{\partial v_\perp}\left(\frac{1}{T_\parallel^0}-\frac{1}{2T_\perp^0}\right)-\frac{1}{2T_\parallel^0}\frac{\partial}{\partial v_z}\frac{1}{v_z}\right]Mv_\perp^4f_0^0. \quad (25)$$

In the case considered, when $Y^*\ll 1$, $W\ll 1$, and

$$v_z\sim v_{T\parallel}\gg\gamma_k/k_z$$

we have

$$\delta f_0/f_0\ll 1.$$

When $v_z \ll v_{T\parallel}$ the last term $\sim 1/v_z^2$ in (25) becomes large and this method cannot be used to solve (20), because the third term of (20) has a sharp maximum when $v_z \ll v_{T\parallel}$, so that in the region of small v_z the value of f_0 can change appreciably, although the values of $T_\parallel$ and $T_\perp$, averaged over the velocities, vary little when $Y^* \ll 1$. In the region of small v_z (20) must be solved exactly with only the highest-order third term retained. In this case we have for δf_0 the equation[5]

$$\frac{\partial\delta f_0}{\partial t}-\frac{1}{4}\sum_k\gamma_k\frac{|H_k|^2}{H_0^2}\frac{\partial}{\partial v_z}\left(\frac{k_z^2v_\perp^4}{k_z^2v_z^2+\gamma_k^2}\frac{\partial\delta f_0}{\partial v_z}\right)$$

$$=-\frac{1}{4}\sum_k\gamma_k\frac{|H_k|^2}{H_0^2}\frac{\partial}{\partial v_z}\left(\frac{k_z^2v_z}{k_z^2v_z^0+\gamma_k^2}\right)\frac{Mv_\perp^4}{T_\parallel^0}f_0^0(v_\perp,0). \quad (26)$$

We shall henceforth be interested in the solution of (26) as $t\to\infty$ when $\gamma_k\to 0$. Substituting δf_0 as $t\to\infty$ in the form

$$\delta f_0=\sum_k|H_k|^2H_0^{-2}F(v_\perp,v_z),$$

we obtain for $F(v_\perp, v_z)$ the equation

$$\frac{\partial^2F}{\partial v_z^2}-\frac{2}{v_z}\frac{\partial F}{\partial v_z}-\frac{1}{\pi W^\infty}\frac{v_z^2}{v_\perp^4}F=-\frac{M}{8\pi W^\infty T_\parallel^0}f_0^0(v_\perp,0). \quad (27)$$

From (27) we have

$$\delta f_0=\frac{Mv_z^{3/2}}{T_\parallel^0}\frac{\pi/4}{\sin(\pi/4)}f_0^0(v_\perp,0)\left\{I_{-3/4}\left(\sqrt{\frac{1}{4\pi W^\infty}}\frac{v_z^2}{v_\perp^2}\right)\Phi_+(v_\perp,v_z)\right.$$

$$\left.-I_{3/4}\left(\sqrt{\frac{1}{4\pi W^\infty}}\frac{v_z}{v_\perp^2}\right)\Phi_-(v_\perp,v_z)\right\},$$

$$\Phi_\pm(v_\perp,v_z)=\int_{v^*}^{v_z}I_{\pm 3/4}\left(\sqrt{\frac{1}{4\pi W^\infty}}\frac{v_z'}{v_\perp^2}\right)\frac{dv_z'}{v_z'^{1/2}}$$

$$+\frac{1}{2}v_\perp(4\pi W^\infty)^{1/8}\Psi_\pm(\alpha). \quad (28)$$

Here $I_n(z)$ is the modified Bessel function, $v^* = \sqrt{\alpha}\,v_\perp\sqrt{4\pi W^\infty}$, and α an arbitrary large constant; for $\alpha\gg 1$ we have for $\Psi_+(\alpha)$ and $\Psi_-(\alpha)$ the asymptotic expansions

$$\Psi_\pm(\alpha)\approx\frac{1}{\alpha^{3/4}}\left(I_{\mp 1/4}(\alpha)\sum_0^\infty\frac{(2n)!}{\alpha^{2n}}+I_{\pm 1/4}(\alpha)\sum_0^\infty\frac{(2n+1)!}{\alpha^{2n+1}}\right).$$

The constants in the solution (28) are chosen such that for sufficiently large v_z [$v_z \gg v_\perp(W^\infty)^{1/4}$], the value of δf_0 given by (28) goes over into the highest-order term in (25), equal to $\pi W^\infty(Mv_\perp^4/T_\parallel^0v_z^2)f_0^0(v_\perp, 0)$. This can be readily verified by noting that when $z\gg 1$

$$\int_\alpha^z I_{\pm 3/4}(z')\frac{dz'}{z'^{3/4}}=\Psi_\pm(z)-\Psi_\pm(\alpha),$$

and by using for $I_{\pm 3/4}(z)$ or $I_{\pm 1/4}(z)$ with large z the known asymptotic expansions [see, for example, [8]]:

$$I_n(z)=\frac{1}{\sqrt{2\pi z}}\left(e^z+e^{-\pi i(n+1/2)}e^{-z}\right)\left(1+O\left(\frac{1}{z}\right)\right).$$

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[4] An account of these terms would correspond to an account of small corrections $\sim k_z^2/k_\perp^2\sim Y^*$ in the equations for $T_\perp$ and $T_\parallel$.

[5] Equation (26) can be used for small $v_z\ll v_{T\parallel}$, and we have therefore replaced $f_0^0(v_\perp, v_z)$ in the right half of this equation by $f_0^0(v_\perp, 0)$.

All the remaining terms in (25) remain small also when $v_z \to 0$, provided $W^\infty \ll 1$. Therefore, combining (25) and (28), we obtain for δf_0 a relation valid for all $v_z (0 < v_z \lesssim v_T)$ with $W^\infty \ll 1$:

$$\delta f_0 = \Big\{2\pi W^\infty \frac{1}{v_\perp}\frac{\partial}{\partial v_\perp}\left(\frac{1}{T^0_\parallel} - \frac{1}{2T^0_\perp}\right) M v_\perp^4 + \frac{\pi}{4}\sqrt{2}\,\frac{M v_z^{3/2}}{T^0_\parallel}$$
$$\times \Big[I_{-3/4}\left(\sqrt{\frac{1}{4\pi W^\infty}}\,\frac{v_z^2}{v_\perp^2}\right)\Phi_+(v_\perp, v_z)$$
$$+ I_{3/4}\left(\sqrt{\frac{1}{4\pi W^\infty}}\,\frac{v_z^2}{v_\perp^2}\right)\Phi_-(v_\perp, v_z)\Big]\Big\} f_0^0(v). \qquad (29)$$

Now, having a solution of (20), we can determine the variation of $T_\perp$, $T_\parallel$, and W during the instability. Replacing in the right half of (22) f_0 by f_0^0, which corresponds to limitation to the highest-order terms in Y^*, and discarding the last term $\sim \gamma_k$, we get

$$\frac{dT_\perp}{dt} = 2\sum_k \gamma_k |H_k|^2 H_0^{-2} T^0_\perp (1 - 2T^0_\perp/T^0_\parallel). \qquad (30)$$

For the time variation of W we have the equation

$$\frac{dW}{dt} = \frac{1}{4\pi}\sum_k \gamma_k |H_k|^2 H_0^{-2}. \qquad (30')$$

Dividing (30) by (30′) we obtain a relation between the variation of the transverse temperature and the magnetic field energy in the instability:

$$dT_\perp/dW = 8\pi T^0_\perp (1 - 2T^0_\perp/T^0_\parallel), \text{ т. е. } \delta T_\perp = 8\pi W T^0_\perp (1 - 2T^0_\perp/T^0_\parallel). \qquad (31)$$

On the other hand, as $t \to \infty$, when $\gamma_k \to 0$, we have

$$\frac{M}{8}\int \frac{v_\perp^4}{v_z}\frac{\partial f_0^\infty}{\partial v_z}\,d\mathbf{v} + NT_\perp^\infty + H_0^2/8\pi = 0.$$

Substituting here $f_0^\infty = f_0^0 + \delta f_0$ and $T_\perp^\infty = T_\perp^0 + \delta T_\perp^\infty$, we obtain one more equation relating $\delta T_\perp^\infty$ and W^∞:

$$\delta T_\perp^\infty + \frac{M}{8N}\int \frac{v_\perp^4}{v_z}\frac{\partial \delta f_0}{\partial v_z}\,d\mathbf{v} = Y^*(T^0_\perp)^2/T^0_\parallel. \qquad (32)$$

Subsituting in (32) the value of $\delta T_\perp$ from (31) and δf_0 from (29), and going over in the integral with respect to v to the dimensionless variables

$$z = \frac{v_z^2}{v_\perp^2}\sqrt{\frac{1}{4\pi W^\infty}}, \qquad x = \sqrt{\frac{M}{2T^0_\perp}}\,v_\perp,$$

we obtain the following equation for W^∞:

$$8\pi W^\infty T^0_\perp (1 - 2T^0_\perp/T^0_\parallel)$$
$$+ \tfrac{1}{4}\sqrt{\pi}\,(T^0_\perp/T^0_\parallel)^{3/2} T^0_\perp (16\pi W^\infty)^{1/4} G = Y^*(T^0_\perp)^2/T^0_\parallel, \qquad (33)$$

where G is a constant equal to

$$G = \int_0^\infty x^6 e^{-x^2}dx \int_0^\infty \frac{dz}{\sqrt{z}}\frac{\partial}{\partial z}\left[I_{-3/4}(z)\,\Phi_+(z) - I_{3/4}(z)\,\Phi_-(z)\right] \approx 1,$$
$$\Phi_\pm(z) = \left[\int_\alpha^z I_{\pm 3/4}\frac{dz'}{z'^{3/4}} + \Psi_\pm(\alpha)\right] z^{3/4}.$$

We have used $\alpha = 3$ in the calculations.

It must be noted that in the integral with respect to v_z in (32), the most essential region is that of small v_z [$v_z \sim v_\perp (W^\infty)^{1/4}$], which makes a contribution $\sim (W^\infty)^{1/4}$. The region of large v_z ($v_z \sim v_T$) makes a small contribution $\sim W^\infty$, which we neglect. With the same accuracy we can neglect the first term in (33). We then obtain for W^∞ the formula

$$W^\infty = \frac{16}{\pi^3}Y^{*4}\left(\frac{T^0_\parallel}{T^0_\perp}\right)^2. \qquad (34)$$

Using (31), we obtain for the total variation of the transverse temperature

$$\delta T_\perp^\infty = -\frac{128}{\pi^2}Y^{*4}\frac{(T^0_\parallel)^2}{T^0_\perp}\left(\frac{2T^0_\perp}{T^0_\parallel} - 1\right). \qquad (34')$$

Analogously we have from (23) for the total variation of the longitudinal temperature

$$\delta T_\parallel^\infty = \frac{256}{\pi^2}Y^{*4}T^0_\parallel. \qquad (34'')$$

5. In order to justify the possibility of using the quasilinear approximation in the problem under consideration, we must demonstrate that the linear *interaction between the harmonics is insignificant* at the saturation amplitudes H_k^∞, determined by relations (17) and (34). This interaction will be considered in the present section. We start from the nonlinear kinetic equation for the electrons and ions (α = e, i) in the form

$$\frac{\partial f_k^\alpha}{\partial t} + i(k_\perp v_\perp \cos\theta + k_z v_z) f_k^\alpha - \omega_{H\alpha}\frac{\partial f_k^\alpha}{\partial \theta}$$
$$+ \frac{e^\alpha}{M^\alpha}E_k\left[\frac{\partial f_0^\alpha}{\partial v_\perp}\left(1 - \frac{k_z v_z}{\omega_k}\right) + \frac{k_z v_\perp}{\omega_k}\frac{\partial f_0^\alpha}{\partial v_z}\right]$$
$$= -\frac{e^\alpha}{M^\alpha}\Big\{\left(\frac{\partial}{\partial v_\perp} + \frac{1}{v_\perp}\right)\sum_{k'} E_{k-k'} f_{k'}^\alpha \sin\theta\left(1 - \frac{(k_z - k_z')v_z}{\omega_{k-k'}}\right)$$
$$+ \frac{1}{v_\perp}\frac{\partial}{\partial\theta}\sum_{k'} E_{k-k'} f_{k'}^\alpha\left[\cos\theta\left(1 - \frac{(k_z - k_z')v_z}{\omega_{k-k'}}\right)\right.$$
$$\left. - \frac{|k_\perp - k_\perp'|\,v_\perp}{\omega_{k-k'}}\right] + \frac{\partial}{\partial v_z}\sum_{k'} E_{k-k'} f_{k'}^\alpha \sin\theta\,\frac{(k_z - k_z')v_\perp}{\omega_{k-k'}}\Big\}. \qquad (35)$$

For simplicity we assume that k, k′ and H_0 lie in one plane [6]).

[6])Since our problem reduces to an order of magnitude estimate of the interaction between the harmonics, this assumption does not limit its generality.

In this case of aperiodic instability the interaction between the harmonics leads to a change in the time dependence of the amplitude $H_{\mathbf{k}}$ even in the second order of the oscillation amplitude. In order to find $H_{\mathbf{k}}(t)$ in this approximation, we substitute $f_{\mathbf{k}}^{\alpha}$ from (10) in the nonlinear terms of (35), which we transfer to the right side. Solving the resultant equation and simplifying the result under the assumption that $|\omega_{\mathbf{k}-\mathbf{k}'}|$, $|\omega_{\mathbf{k}'}| \ll \omega_{Hi}$ and that conditions (3) are satisfied for $\mathbf{k}$ and $\mathbf{k}'$, we obtain after straightforward but laborious calculations the following expression for the nonlinear addition to the ion distribution function $f_{\mathbf{k}}$:

$$f_{\mathbf{k}}^{(2)} = \frac{1}{4}\sum_{\mathbf{k}'}\frac{H_{\mathbf{k}-\mathbf{k}'}H_{\mathbf{k}'}}{H_0^2 k' |\mathbf{k}-\mathbf{k}'|}\left\{\sum_{\pm}\frac{e^{\pm i\theta}\omega_{Hi}}{k_z v_z \mp \omega_{Hi} - \omega_{\mathbf{k}-\mathbf{k}'} - \omega_{\mathbf{k}'}}\right.$$
$$\times\left[2|\mathbf{k}_\perp - \mathbf{k}'_\perp| \mp k'_\perp \frac{\partial}{\partial v_\perp} v_\perp \frac{\omega_{\mathbf{k}-\mathbf{k}'} - (k_z - k_{z'}) v_z}{\omega_{\mathbf{k}'} - k'_z v_z}\right.$$
$$\left.\mp \frac{\partial}{\partial v_z}\frac{(k_z - k'_z) k'_\perp v_\perp^2}{\omega_{\mathbf{k}'} - k'_z v_z}\right]$$
$$\left.\times\left[\frac{\partial f_0}{\partial v_\perp}(\omega_{\mathbf{k}'} - k'_z v_z) + k'_z v_\perp \frac{\partial f_0}{\partial v_z}\right] + \sum_{s=0,\pm 2} A_s e^{is\theta}\right\}. \quad (36)$$

We do not present here the very cumbersome expressions for A_s, since they make no contribution to j_x or j_y [7].

Using Maxwell's equations, we obtain the following equations for the determination of $H_{\mathbf{k}}$

$$\frac{1}{c^2}\frac{\partial^2 H_{\mathbf{k}}}{\partial t^2} + k^2 H_{\mathbf{k}} + \frac{4\pi}{c^2}\hat{\sigma}_{22}\frac{\partial H_{\mathbf{k}}}{\partial t} = -\frac{4\pi}{c} ik \sum_{\alpha} e^{\alpha}\int v_\perp \sin\theta f_{\mathbf{k}}^{\alpha}\, d\mathbf{v},$$
$$\frac{1}{c^2}\frac{\partial^2 \widetilde{H}_{\mathbf{k}}}{\partial t^2} + k_z^2 \widetilde{H}_{\mathbf{k}} + \frac{4\pi}{c^2}\hat{\sigma}_{11}\frac{\partial \widetilde{H}_{\mathbf{k}}}{\partial t} = \frac{4\pi}{c} ik_z \sum_{\alpha} e^{\alpha}\int v_\perp \cos\theta f_{\mathbf{k}}^{\alpha}\, d\mathbf{v}. \quad (37)$$

Here $H_{\mathbf{k}}$ is the amplitude of the magnetic field in a plane containing $\mathbf{k}$ and $\mathbf{H}_0$, arising during the instability, $\widetilde{H}_{\mathbf{k}}$ is the amplitude of the magnetic field perpendicular to this plane, resulting from the interaction of the harmonics, while $\hat{\sigma}_{11}$, and $\hat{\sigma}_{22}$ are the components of the electric conductivity tensor in the linear theory, in which $\omega_{\mathbf{k}}$ must be replaced by $i\,\partial/\partial t$.

Substituting $f_{\mathbf{k}}^{(2)}$ from (36), we calculate the integrals with respect to $\mathbf{v}$ in the right half of (37), under the assumption that $|\omega_{\mathbf{k}}| \ll k_z v_{T\parallel}$. We assume that the electron velocity distribution function is isotropic. Then the contribution to j_x^{nonl}, j_y^{nonl} is made only by the ions. It must also be noted that the terms proportional to $e^{\pm i\theta}$ vanish from $f_{\mathbf{k}}^{(2)}$ when $k_\perp = 0$. In this case only one component, $j^{\text{nonl}} - j_z^{\text{nonl}}$, differs from zero, i.e., when $k_\perp = 0$ the interaction between two transverse modes can lead only to the occurrence of longitudinal oscillations with $\mathbf{E} \parallel \mathbf{H}_0$ [8]. However, as already noted, in view of the fact that ϵ_{33} is large at low frequencies, the magnitude of the field in these oscillations is negligibly small. Thus, in the second approximation in the amplitude $H_{\mathbf{k}}$, interaction between harmonics actually appears only when $k_\perp \neq 0$, i.e., for the instability due to the high transverse temperature. In this case we have from (37)

$$\frac{\partial H_{\mathbf{k}}}{\partial t} = \gamma_{\mathbf{k}} H_{\mathbf{k}} - \frac{k_z^2}{\pi k_\perp}\sum_{\mathbf{k}'}\frac{k_z - k'_z}{|\mathbf{k}_\perp - \mathbf{k}'_\perp|} H_{\mathbf{k}'}\frac{H_{\mathbf{k}-\mathbf{k}'}}{H_0}\left(3T_\perp - T_\parallel\right.$$
$$\left.+ \frac{M}{4}\int \frac{v_\perp^4}{v_z}\frac{\partial f_0}{\partial v_z}\, d\mathbf{v}\right) \Big/ \frac{M}{4}\int v_\perp^4 \frac{\partial^2 f_0}{\partial v_z^2}(v_\perp, 0)\, d\mathbf{v}_\perp; \quad (38)$$

$$-\frac{\partial^2 \widetilde{H}_{\mathbf{k}}}{\partial t^2}\left(1 + \frac{H_0^2}{4\pi N M c^2}\right) + k_z^2 \frac{T_\parallel - T_\perp - H_0^2/4\pi N}{M}\widetilde{H}_{\mathbf{k}}$$
$$= ik_z^2 \sum_{\mathbf{k}'}\frac{k'_z}{k'_\perp} H_{\mathbf{k}'}\frac{H_{\mathbf{k}-\mathbf{k}'}}{H_0}\frac{T_\perp - T_\parallel}{M}. \quad (39)$$

The quantity $\gamma_{\mathbf{k}}$ in (38) is determined by (6). In the quasilinear theory account is taken only of the first term in the right half of (38), the magnitude of which is of the order of $k_z v_{T\parallel} Y^* H_{\mathbf{k}}$. The second term of the right half of (38) is connected with the interaction of the harmonics. For amplitudes $H_{\mathbf{k}}^{\infty}$ determined by relation (34) this term is small, of the order of

$$k_z v_{T\parallel} k_z^2 H_{\mathbf{k}}^2 / k_\perp^2 H_0 \sim k_z v_{T\parallel} Y^{*3} H_{\mathbf{k}}.$$

Thus, the interaction of the harmonics cannot be appreciable after a time $\sim 1/\gamma$ and can be neglected in the analysis of the establishment of the stationary amplitudes (34).

It follows from (39) that the interaction between the harmonics leads also to the occurrence of a magnetic field perpendicular to the $(\mathbf{k}, \mathbf{H}_0)$ plane, but the amplitude $H_{\mathbf{k}}$ of this field is small compared with $H_{\mathbf{k}}$:

$$\widetilde{H}_{\mathbf{k}} \sim k_z H_{\mathbf{k}}^2 / k_\perp H_0 \sim Y^{*5/2} H_{\mathbf{k}}.$$

Thus, when $k_\perp \neq 0$ we can neglect the interaction of the harmonics if $Y^* \ll 1$.

When $k_\perp = 0$ the interaction of the harmonics appears only in the third order in the amplitude $H_{\mathbf{k}}$. Carrying out calculations in accordance with the usual scheme (obtaining the nonlinear addition

[7]We note that the presence in (36) of a term with $s = 0$ denotes that the interaction between harmonics leads in the approximation under consideration to the occurrence of j_z, and consequently also $\mathbf{E} \| \mathbf{H}_0$. However, in view of the fact that ϵ_{33} is large at low frequencies, $E_\parallel$ is small.

[8]In accordance with the previously obtained results (see [9]).

to the distribution function $f_k^{(3)}$, determining with its aid the values of j_x^{nonl} and j_y^{nonl}, and substituting the result in Maxwell's equations), we find that in this case the nonlinear interaction of the harmonics is more appreciable than when $k_\perp \neq 0$. The reason is that although when $k_\perp = 0$ the interaction of the harmonics appears only in the third order and H_k, the small terms $\sim \gamma_k^2$ in the left half of (37) are significant in this case. The condition under which the nonlinearity of the collective motions in the plasma can be neglected when $k_\perp = 0$ reduces to $|((T_\parallel - T_\perp)/T_\parallel| \ll 1$. This is equivalent to the requirement $Y \ll 1$ only for small magnetic fields H_0, when $H_0^2/4\pi NT \ll 1$. The conditions for the applicability of the quasilinear theory of instability for $k_\perp = 0$ thus reduce to $Y \ll 1$ and $H_0^2/4\pi NT \ll 1$.

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