## §3. ~~[illegible]~~ Magnetic Diffusion.

Let us study the induction equation (next couple of lectures – $\vec{u}$ given)

$$\boxed{\frac{\partial \vec{B}}{\partial t} = \underbrace{\nabla \times (\vec{u} \times \vec{B})}_{\text{advection}} + \underbrace{\eta \nabla^2 \vec{B}}_{\text{diffusion}}}$$

This term is simply <u>diffusion</u>. If the first term is unimportant (e.g. $\vec{u}=0$), we get, for each comp. of the m. field, a simple diffusion equation:

$$\frac{\partial B_i}{\partial t} = \eta \nabla^2 B_i$$

So gradients in magnetic field diffuse away.

If $\vec{B} \sim \vec{B}_k(t) e^{i\vec{k}\cdot\vec{x}}$ (Fourier transform), we get

$$\vec{B}_k(t) = \vec{B}_k(0) e^{-\eta k^2 t} \equiv \vec{B}_k(0) e^{-t/\tau_{diff}(k)}$$

So, for m. fields varying at scale $\ell$, the char. diff. time is $\tau_{diff} \sim \frac{\ell^2}{\eta}$

Now compare with the first term: <u>advection</u>.

The associated characteristic time is

$$\tau_{adv} \sim (\nabla u)^{-1} \sim \left(\frac{u}{\ell}\right)^{-1}$$

Introduce <u>magnetic Reynolds #</u>:

$$Rm \sim \frac{\tau_{diff}}{\tau_{adv}} \sim \left[\frac{|\eta \nabla^2 \vec{B}|}{|\nabla \times (\vec{u} \times \vec{B})|}\right]^{-1} \sim \frac{u\ell}{\eta}$$

– measures relative importance of advection & diffusion.

Typical values:

| | |
|---|---|
| Liquid metals in eng. applications (industrial processes) | $Rm \sim 10^{-3} \ldots 10^{-1}$ |
| Laboratory dynamos | $Rm \sim 60 \nearrow 120$<br>VKS, Cadarache; Madison |
| Planetary dynamos | $Rm \sim 100 \ldots 300$ |
| Solar convective zone (upper, photosphere) | $Rm \sim 10^6$ |
| Solar convective zone (base) | $Rm \sim 10^9$ |
| Galaxy (warm ISM) | $Rm \sim 10^{18}$ |
| Galaxy clusters (cores) | $Rm \sim 10^{29}$ |

So, Rm is very large in most astro contexts.

low $\eta$, large distances

Therefore, it is interesting to study the advection term / induction equation with $\eta = 0$ - <u>ideal MHD</u>