Lecture 13 21.02.05

§15. Magnetic Reconnection

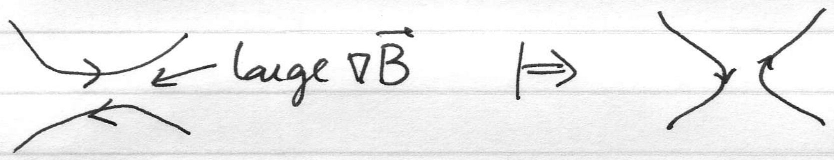
We now abandon the ideal-MHD assumption and move to Resistive MHD

$$\frac{\partial \vec{B}}{\partial t} = \nabla \times (\vec{u} \times \vec{B}) + \underbrace{\eta \nabla^2 \vec{B}}$$

← not neglected anymore

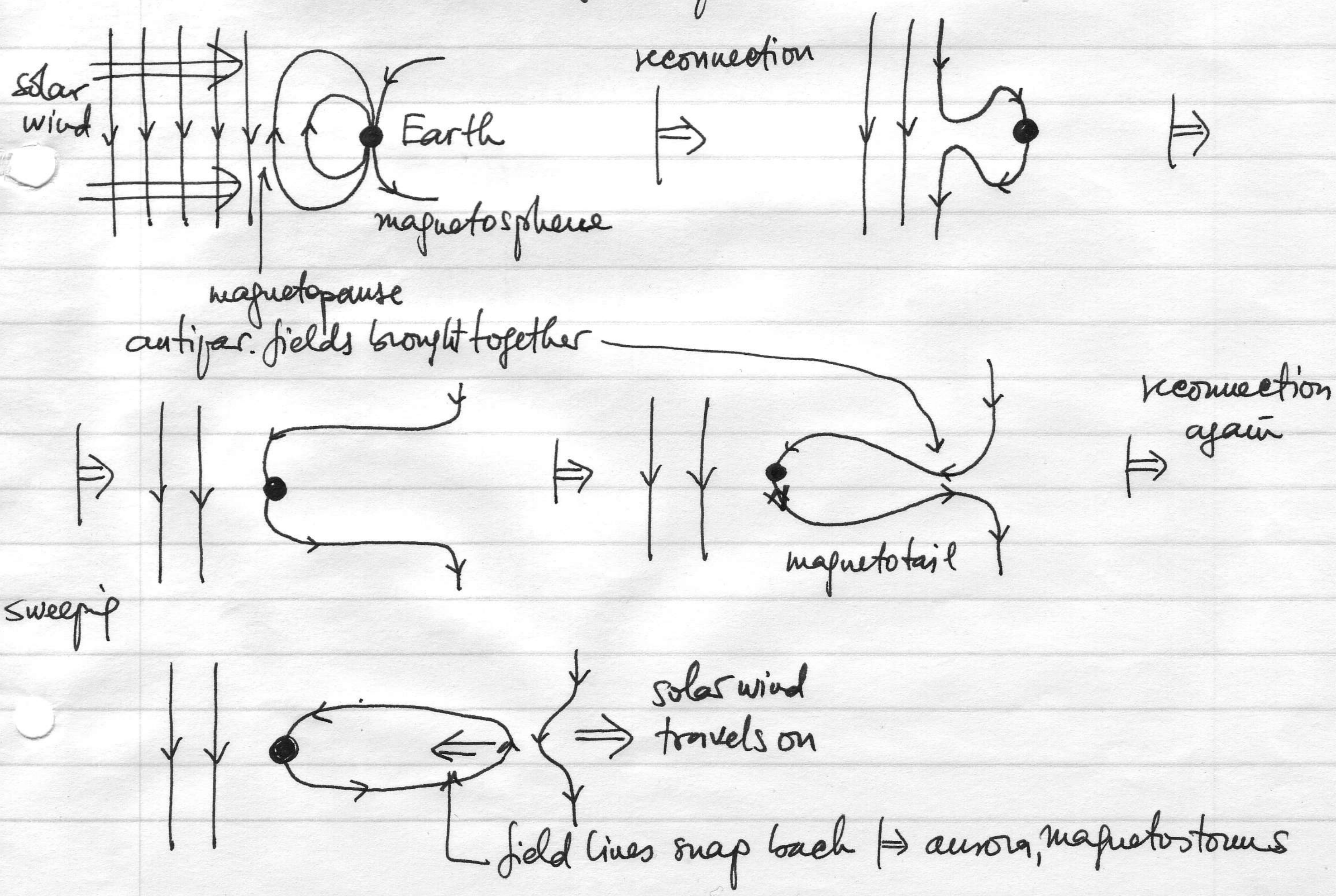
The resistive term is important when very large gradients in the m. field appear.

A particularly interesting and relevant process of this kind is magnetic reconnection:

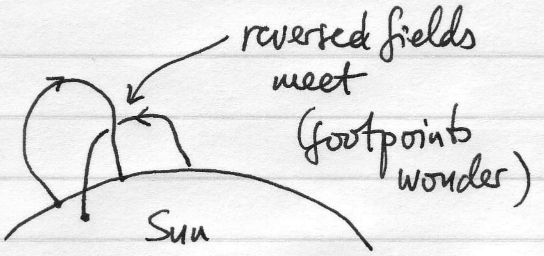


Two astrophysical examples:

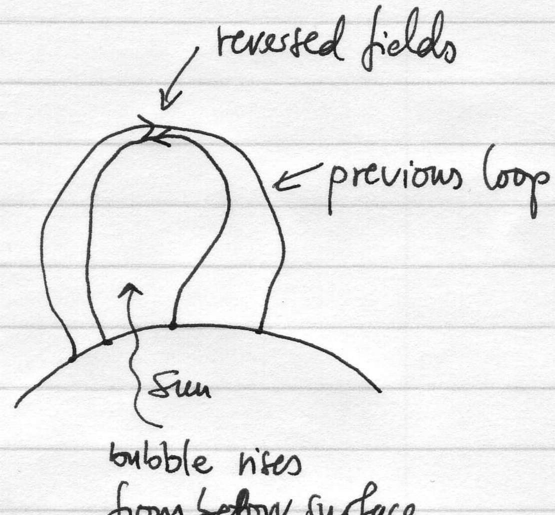
1) Solar wind + magnetosphere



2) Solar Flares



or

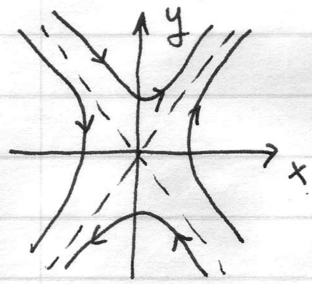


Reconnection releases energy into heat ⟹ flare

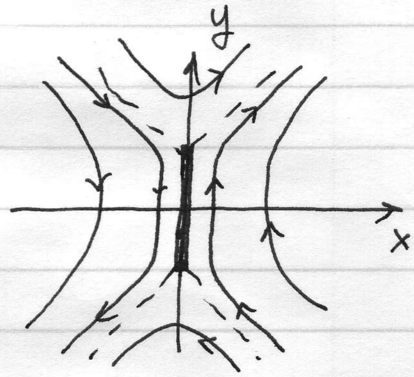
one can argue that the situation is locally 2D around a neutral line → see Syrovatskii review

Theoretical Considerations in 2D

- X-point collapse



tends to collapse to a current sheet (of finite length)



(i.e. ideal MHD wants to have an equilibrium that has a singular line — where ideal assumption breaks down and resistive theory needs to be done).

[see Syr. review]

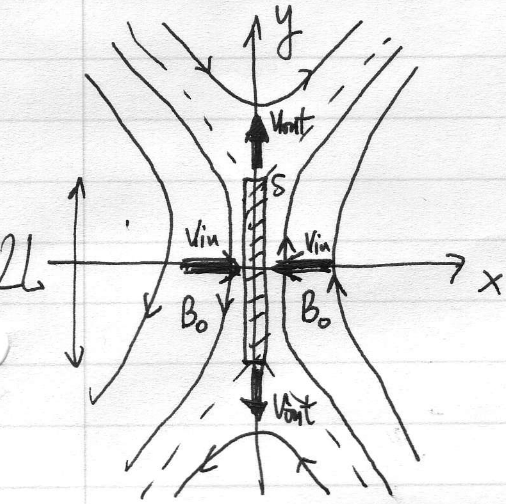
— Syrovatskii solution : compressible case
singularity reached in finite time (explosive collapse)

— Chapman-Kendall solution: incompressible case
exponential-time collapse.

[E.D. Chapman & P.C. Kendall, Proc. Roy. Soc. A271, 435 (1963)]

- Sweet-Parker Theory.

Suppose we have a developed current sheet. Antiparallel field lines meet and reconnect.



~~[illegible]~~ — Total pressure outside:
$$p_0 + \frac{B_0^2}{8\pi}$$

— Total pressure inside (at the centre):
$$p_{centre} \qquad (B=0)$$

— Pressure balance:
$$p_{centre} = p_0 + \frac{B_0^2}{8\pi}$$

— Pressure gradient along the sheet drives outflows:

Along $x=0$: $\rho\Big(\underbrace{\frac{\partial u_y}{\partial t}}_{\text{steady}} + \underbrace{u_y}_{u_x=0}\frac{\partial u_y}{\partial y}\Big) = -\frac{\partial}{\partial y}p$

integrate $\int_0^L dy$

$$\frac{\partial}{\partial y}\frac{u_y^2}{2} = -\frac{\partial}{\partial y}p \Rightarrow \rho\frac{u_{out}^2}{2} = -\underbrace{p_{endpt}}_{=p_0} + \underbrace{p_{centre}}_{=p_0+\frac{B_0^2}{8\pi}} = \frac{B_0^2}{8\pi}$$

Therefore $\boxed{u_{out} = V_A = \frac{B_0}{\sqrt{4\pi\rho}}}$ outflows are Alfvénic

— Plasma heats up because of Ohmic term inside the layer:
$$\frac{\partial p}{\partial t} \sim \eta j^2 \frac{4\pi}{c^2} \sim \eta|\nabla\times\vec{B}|^2 \sim \frac{\eta B_0^2}{\delta^2}$$

So Δp ~~[illegible]~~ $\sim \frac{\eta B_0^2}{\delta^2}t \sim \frac{\eta B_0^2}{\delta^2}\frac{L}{V_A} \sim p_{centre} - p_0 \sim \frac{B_0^2}{8\pi}$

~~[illegible]~~ amount of heating up compared to outside

time plasma spends inside layer $\sim \frac{L}{V_A}$

NB: $L \sim$ system scale

Therefore $\delta \sim \left(\frac{\eta L}{V_A}\right)^{1/2} \sim \left(\frac{\eta}{V_A L}\right)^{1/2} L \sim \frac{L}{\sqrt{S}}$

where $S = \frac{V_A L}{\eta}$ is the Lundquist number.

NB: $S = \frac{V_A}{L} \frac{L^2}{\eta} = \frac{\tau_{res}}{\tau_A}$

– Finally, use conservation of mass:

$$\underset{\text{going in}}{\rho u_{in} L} \sim \underset{\text{coming out}}{\rho u_{out} \delta} \sim \rho V_A \frac{L}{\sqrt{S}}$$

$$\Downarrow$$

$$\boxed{u_{in} \sim \frac{V_A}{\sqrt{S}}}$$

The main SP result.

Characteristic reconnection time:

$$\tau_{SP} \sim \frac{L}{u_{in}} \sim \tau_A \sqrt{S}, \text{ where } \tau_A \sim \frac{L}{V_A} \text{ Alfvén time}$$

Compare this to the ordinary diffusion time:

$$\tau_{res} \sim \tau_A S$$

So, we've achieved something: $\tau_{SP} \ll \tau_{res}$

But: solar flare — putting in numbers (Kulsrud §14.4) $S \sim 3 \cdot 10^{12}$

$\tau_A \sim 40$ sec $\quad \tau_{SP} \sim 7 \cdot 10^{7}$ sec (~ 2 yrs)

$\tau_{res} \sim 10^{14}$ sec ($\sim 10^6$ yrs)

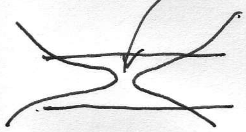
But the observed release time for flare energy is 15 min ÷ several hours
10^3 sec $\quad 10^4$ sec

(long current sheet is a bottleneck!)

SP reconnection 1000 times too slow!

(similar discrepancy for magnetosphere etc.)

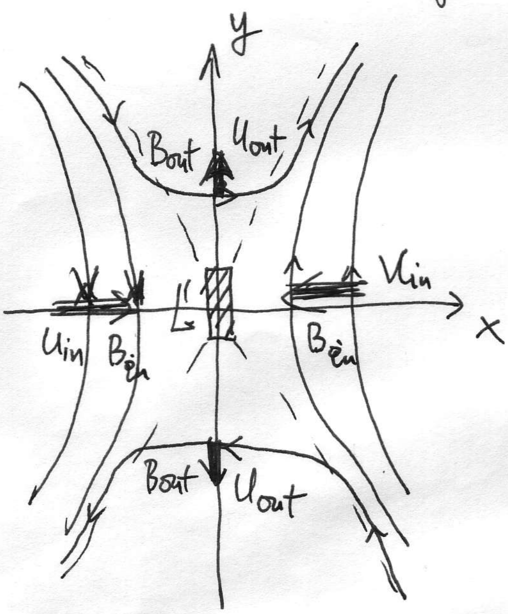
- <u>Petscheck Theory</u>.



Petschek realized that

1) The problem with SP reconnection (what makes it slow) is that plasma outflow has to go through very thin channel $\delta \sim \frac{L}{\sqrt{S}}$.

2) For reconnection, we only need to break the lines at the central point – resistivity not needed anywhere else. So he suggested that this might happen:



The current layer very short

$$L' \ll L \sim \text{system scale}$$

Then

$$\tau_{Petschek} \sim \sqrt{\frac{L'}{L}}\,\tau_{SP} \ll \tau_{SP}$$

reconnection much faster provided it is still true that

$$u_{out} \sim V_A$$

But in a short layer, the pressure gradient along the layer cannot accelerate plasma to V_A.

Petscheck suggested this is done by magnetic tension.

The answer from his theory is

$$\tau_{Petschek} \sim \tau_A \ln S \quad \sim 20\,\text{min for solar flare}$$

However, Petschek's model is a scenario, not a solution. He derived (the value of) B_{out} that was necessary for his model to work, but ~~[illegible]~~ did not explain where B_{out} comes from and how it is resupplied (considering it is being taken out at the speed v_A).

It is a subject of much ongoing research basically to come up with a self-consistent mechanism that would give Petschek-like reconnection.

~~[illegible]~~ Most such schemes involve going beyond MHD description

- Anomalous resistivity : $\eta = \eta(j)$ increases sharply when $j > j_{crit}$ [Kulsrud, Uzdensky...]
- Hall reconnection : it turns out the Hall term helps a lot [simulations by Drake et al.]
- FLR effects

changes the flux freezing conditions (ions slip)

These seem to work, but there is no definitive theory.

... At this point we touch upon an area of modern practical research and recoil back to academic certainties – but not for long!