Mirror Instability: 1. Physical Mechanism of Linear Instability

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The mirror instability is prevalent in planetary and cometary magnetosheaths and other high beta environments. We review the physics of the linear instability. Although the instability was originally derived from magnetohydrodynamic fluid theory, later work showed that there were significant differences between the fluid theory and a more rigorous kinetic approach. Here we point out that the instability mechanism hinges on the special behavior of particles with small velocity along the field. We call such particles resonant particles by analogy with other uses of the term, but there are significant differences between the behavior of the resonant particles in this instability and in other instabilities driven by resonant particles. We comment on the implications of these results for our understanding of the observations of mirror instability-generated signals in space.

INTRODUCTION

The mirror instability has long been of interest in space physics. It was identified theoretically [Rudakov and Sagdeev, 1961; Thompson, 1964] as one of the two magnetohydrodynamic instabilities that occur in the presence of extreme velocity space (pitch angle) anisotropy in a uniform plasma, the other instability being the fire hose. In 1967, Tajiri [1967] derived a kinetic description which, as we discuss further below, shows that the actual instability is not really correctly described as a fluid instability. In 1969, Hasegawa [1969] put forward a further development of the theory for a medium in which the plasma is nonuniform. He proposed that large storm time magnetic field and particle flux oscillations seen on the ATS1 spacecraft at geosynchronous orbit within the Earth's magnetosphere were due to the inhomogeneous form of the instability which he called the drift-mirror instability. Subsequent work by Southwood [1976] and Chen and Hasegawa [1988] amongst others has led to a revision of Hasegawa's original suggestion. However, since that time the mirror mode has garnered increasing interest following its identification in spacecraft data from the terrestrial magnetosheath [Kaufmann et al., 1970, 1971; Tsurutani et al., 1982; Tsurutani et al., 1984] and the vicinity of comets [Russell et al., 1987] and other planetary magnetosheaths [Balogh et al., 1992]. Further numerical work on the instability has been done by Price et al. [1986] and McKean et al. [1992]. Recently, the mode appears to have been identified in the solar wind in the events resulting from the large solar events of March 1991 [Tsurutani et al., 1992].

The instability occurs when β (the ratio of plasma to magnetic pressure) is large (the ratio of the perpendicular component of plasma pressure to the magnetic pressure $\beta \perp$ must necessarily be greater than unity). The anistropy required for instability is inversely dependent on β . In planetary magnetosheaths the source

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Paper number 92JA02837. 0148-0227/93/92JA-02837\$05.00 of anisotropy is likely to be the planetary bow shock and in cometary environments the ion pickup process is a natural source of anisotropy. Both types of environment tend to have relatively large values of β .

Recently, Gary [1992] has raised an interesting question regarding the occurrence of the mirror instability. There is a second instability that occurs at frequencies below the ion gyrofrequency in the presence of ion pitch angle anisotropy, the ion cyclotron instability. Gary [1992] presents the results of a numerical evaluation of the full kinetic dispersion relation. Both ion cyclotron and mirror instability are covered by the calculations and Gary points out that the former mode generally has the higher linear growth rate of the two, suggesting that it could suppress the anisotropy needed to produce the mirror instability. Price et al. [1986] have shown that heavy ions can reduce the linear growth of the resonant ion cyclotron instability, while leaving the mirror instability unaffected. Although we shall not address quantitatively the question of relative growth rates of the two instabilities, we note that the structure of the mirror instability itself may reduce the growth rate of the ion cyclotron instability.

The development of the ion cyclotron wave is usually described by the standard quasi-linear theory in which the unstable distribution equilibrates through a spatially smooth process of velocity space diffusion. Growth rate calculations assume that the waves propagate through a nearly uniform plasma and that the resonant ions have approximately the same parallel velocity everywhere. However, the mirror instability in its linear development phase depends crucially on the spatially structured nature of the field disturbance even though the instability occurs in the limit of long parallel wavelength. Any nonlinear saturation mechanism of the mirror instability is likely to leave the plasma spatially structured, as is also strongly suggested by the many observations. In practice, in almost any experimental detection of a plasma instability, the wave fields and the plasma population will have evolved to some quasi-steady condition that represents a nonlinear saturated state of the instability. We reserve a detailed discussion of the nonlinear development of the instability for another paper. Here we merely point out that in the spatially structured magnetic field associated with the mirror instability in both its linear and nonlinear phase, different parts of the ion distribution will resonate with an ion cyclotron wave as the wave propagates along the inhomogeneous field. This effect is likely to inhibit the growth of the ion cyclotron mode and may provide yet another answer to

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Gary's question about why the mirror instability is favored over the ion cyclotron instability in certain circumstances, especially in planetary magnetosheaths.

Gary also remarks that the two instabilities have a different nature. The ion cyclotron instability is a resonant instability in which the energy for the instability is fed from a subset of the particle population that are in gyroresonance with the unstable wave. In contrast, the mirror instability is referred to as a "fluid" instability, alluding to the fact that the phase space (pitch angle) anisotropy of the bulk of the hot plasma distribution serves as the source of energy. It is at best a partial truth to regard the mirror instability as a fluid instability, as we show in this paper. The instability grows because of a subtle coupling between a group of particles with small velocity parallel to the field and the rest of the population. The mirror instability has zero parallel phase velocity in the plasma frame of reference. It follows, by analogy with other uses of the term that we can call particles with near zero parallel velocity resonant. However, although we may show here that the mirror instability is resonant in the sense that its physics depends critically on resonant particle behavior, there are significant differences between the behavior of the resonant particles in this instability and in other resonant instabilities (such as, for example, the ion cyclotron mode discussed by Gary [1992]).

TRADITIONAL PICTURE OF THE MIRROR INSTABILITY

Thompson [1964] and Hasegawa [1969] give (slightly different) descriptions of the mirror instability mechanism which correspond to the traditional view of the instability as a fluid magnetohydrodynamic instability. The instability results from the fact that at very low frequencies the perpendicular pressure δp_{\downarrow} responds to a compressional change in the magnetic field strength δB in antiphase, namely,

$$\delta p_{4} = 2p_{4} \left(1 - \frac{T_{4}}{T_{11}} \right) \frac{\delta B}{B} \tag{1}$$

(for a bi-Maxwellian distribution with parallel temperature T_{\parallel} and perpendicular temperature T_{\downarrow}) [*Hasegawa*, 1969]. The formula shows that whenever the perpendicular pressure exceeds the parallel, the pressure decreases as the field strength and the field pressure increase.

The perturbed pressure in (1) is proportional to the unperturbed pressure and, in particular, if the unperturbed plasma pressure is large enough, the sense of the total pressure produced by the field change may be opposite to the change in magnetic pressure produced by the field change. For a bi-Maxwellian, the condition for this is

$$\delta p_{\downarrow} + \frac{B\delta B}{\mu_{0}} < 0 \tag{2}$$

On substituting from (1) one finds that when the following condition holds

$$1 + \beta_{\rm f} \left(1 - \frac{T_{\rm i}}{T_{\rm ii}} \right) < 0 \tag{3}$$

the force exerted by the total pressure (plasma + magnetic) in the direction transverse to the field decreases/increases with increasing/decreasing δB . Inequality (3) is the instability condition (in the short perpendicular, large parallel wavelength limit).

Let us examine how the instability is decribed in a fluid approximation. The pressure force associated with the total (magnetic plus plasma) pressure accelerates the plasma and would give rise to a plasma displacement, ξ . Assuming that the field strength perturbation δB varies in time and space as exp ($\gamma t + i \mathbf{k} \cdot \mathbf{r}$), we can write the equation of motion perpendicular to the field in the form

$$\rho \gamma^{2} \xi = -i k_{\downarrow} \left(\delta p_{\downarrow} + B \, \delta B / \mu_{o} \right) \tag{4}$$

The frozen in field condition relates δB and ξ

$$\delta B = \frac{B}{B} \cdot i \boldsymbol{k} \times (\boldsymbol{\xi} \times \boldsymbol{B}) = -i (\boldsymbol{k} \cdot \boldsymbol{\xi}) \ B = -i \ k_4 \boldsymbol{\xi} \ \mathbf{B}$$
(5)

where the wave vector \mathbf{k} has been taken to have components of magnitude k_{\parallel} , k_{\perp} parallel and perpendicular to the field. Combining (4) and (5) with (1) and eliminating ξ yields

$$\rho \gamma^2 \frac{\delta B}{B} = -k_{\rm t}^2 2p_{\rm t} \left(1 - \frac{T_{\rm t}}{T_{\rm H}}\right) \frac{\delta B}{B} - k_{\rm t}^2 \frac{B \,\delta B}{\mu_o} \tag{6}$$

where ρ is the mass density. Eliminating δB and rearranging yields

$$\gamma^{2} = -k_{\rm L}^{2} A^{2} \left[1 + \beta_{\rm L} \left(1 - \frac{T_{\rm L}}{T_{\rm H}} \right) \right] \qquad (7)^{3}$$

where $A = (B^2/\mu_o \rho)^{\frac{1}{2}}$ is the Alfvén velocity. Equation (7) not only illustrates the fluid instability condition (3) once again, it also shows that when the fluid instability condition is not met, the fluid equations predict oscillations ($\gamma^2 < 0$).

DESCRIPTION OF INSTABILITY IN THE FLUID APPROXIMATION

By identifying the terms in equations (6) and (7) with their physical counterparts, we can give the following physical description of the instability [*Thompson*, 1964]. The fluid instability results when the pressure anisotropy is large enough that an increase in magnetic field produces a local decrease in total pressure which in turn causes the field lines to move closer together. The latter effect causes the field to continue to increase, hence driving the instability.

The pressure response described by equation (1) is central to the instability. The antiphase response of particle pressure to field increase is achieved despite the perpendicular particle energy rising in phase with the field as the first magnetic moment invariant is conserved. There is a balancing decrease in parallel energy which ensures that the total energy of the particle does not change.

The exchange of energy between parallel and perpendicular degrees of freedom in a spatially varying field when the magnetic moment invariant is conserved is often described by introducing the notion of the magnetic mirror force. The link to the mirror force is worth discussing at this point. Thompson refers to its role and Hasegawa's [1969] description of the physics of the instability relies heavily on it. In the mirror instability, the force can be thought of as squeezing the plasma out of the high field regions and into the weak field regions. When the magnetic moment and energy are conserved, a particle's motion into a weaker field leads to conversion of perpendicular (gyration) energy into parallel energy and so to motion along the field. It appears as if a force is acclerating the particle along the field. Viewing the motion parallel to the field in isolation from the perpendicular (gyro) motion, the changes in parallel energy can be regarded as providing a pseudopotential along the field which produces a force

directed away from the regions where the field is high. The corresponding (pseudo) force is the mirror force, but it is not a force in the normal sense as it does not change the total particle energy. Indeed we shall see that the pressure response (1) corresponds to the case where particle energy is strictly conserved. We shall also see that the instability cannot occcur without some particles' energy changing.

MIRROR INSTABILITY IN THE KINETIC LIMIT: RESONANT PARTICLES

The mathematical derivation leading up to equation (7) is not correct (as *Hasegawa* [1969] notes). The instability condition is not wrong, but the implication of equation (7) that the stable solution of the dispersion relation corresponds to oscillatory motion is. Furthermore, any physical description based on the equation is false. The instability, which is fundamentally a hot plasma instability, has to be treated in the kinetic limit rather than assuming the purely fluid response given by the equations leading up to (7).

Tajiri [1967] produced a kinetic treatment of the mirror instability as part of a thorough study of low-frequency wave behavior in a collision-free plasma. As Hasegawa [1969] notes, his result differed from earlier treatments in that it identified a fourth mode in the low-frequency range in addition to the familiar MHD wave modes; it is this mode that causes the mirror instability. The mode is nonoscillatory even when $T_{\perp}/T_{\parallel} < (1 + \beta_{\rm t})/\beta_{\rm t}$ The latter point contradicts the prediction of fluid theory (discussed in the previous section). In the kinetic theory, which should provide a more accurate description of collision-free plasma behavior, the mirror mode when stable simply represents a damped but otherwise time-stationary perturbation. As we shall show the major difference introduced by the kinetic approach is that not all of the plasma particles respond to the field in the same way. The pressure variation given by (1) is valid for particles with pitch angle sufficiently far from 90°, but as particles with small parallel velocity are not affected by the mirror force to the same degree, they respond differently. Their energy is not conserved as the field changes and their perpendicular pressure response is in phase with the field pressure change. Such particles with very small parallel velocities can be regarded as resonant with the time stationary signal.

Before demonstrating the role of resonant particles, let us show that the kinetic dispersion relation produces a nonoscillatory mode. The general form in a uniform plasma is given by

$$\frac{c^2k^2}{\omega^2} = \varepsilon_{yy} \tag{8}$$

where ε_{yy} is the appropriate element of the plasma dielectric tensor, y being the direction transverse to both **B** and **k**. The disturbance is now assumed to vary as exp $(-i\omega t)$ where ω is generally complex. In the low-frequency limit, ε_{yy} is given by [Hasegawa, 1975]

$$\varepsilon_{yy} = \sum_{j} \left(\frac{c^{2} \mu_{o} \rho_{j}}{B^{2}} - \frac{c^{2}}{\omega^{2}} \left[\frac{k_{\parallel}^{2}}{2} \left(\beta_{\downarrow j} - \beta_{\parallel j} \right) + k_{\downarrow}^{2} \beta_{\downarrow j} \left(1 + \frac{T_{\downarrow j}}{2T_{\parallel j}} Z' \left(\frac{\omega}{\sqrt{2} k_{\parallel} v_{T \parallel j}} \right) \right) \right] \right)$$
(9)

where Z' is the derivative of the plasma dispersion function [*Fried* and Conte, 1961], $v_{Tl} = (2kT_{l}/m)^{\frac{1}{2}}$ is the parallel thermal velocity, and the summation is taken over all species present (denoted by subscript *j*); the plasma distributions have been assumed to be bi-Maxwellian.

Substituting from (9) into (8) one finds (after some simple algebra)

$$\frac{\omega^{2}}{A^{2}} = k^{2} + \sum_{j} \left[k_{II}^{2} \frac{\beta_{\perp j} - \beta_{\parallel j}}{2} + k_{\downarrow}^{2} \beta_{\perp j} \left(1 + \frac{T_{\downarrow j}}{2T_{\parallel j}} Z' \left(\frac{\omega}{\sqrt{2} k_{\parallel} v_{T \parallel j}} \right) \right) \right]$$
(10)

where, as before, A is the Alfvén speed. We drop the summations over species and assume that $|\omega|^2 << k^2 A^2$ and that $|\omega|^2 << k_{\parallel}^2 v_{\parallel}^2 T$ to obtain

$$k^{2} + k_{\parallel}^{2} \frac{\beta_{\perp} - \beta_{\parallel}}{2} + k_{\perp}^{2} \beta_{\perp} \left[1 - \frac{T_{\perp}}{T_{\parallel}} \left(1 + i \frac{\omega}{k_{\parallel} \mathbf{v}_{T \parallel}} \left(\frac{\pi}{2} \right)^{1/2} \right) \right] = 0$$
(11)

where we have used the small argument approximation for $Z'(-Z'(z) \approx 2 + i2z)$.

In the limit, $k_{\parallel}^2 \ll k_{\perp}^2$ one recovers *Tajiri*'s [1967] result [see Hasegawa, 1969]

$$\beta_{4} \frac{\gamma}{k_{||} v_{T||}} \left(\frac{\pi}{2}\right)^{1/2} = -\beta_{4} \frac{i\omega}{k_{||} v_{T||}} \left(\frac{\pi}{2}\right)^{1/2} = -\left[1 + \beta_{4} \left(1 - \frac{T_{4}}{T_{||}}\right)\right] \frac{T_{||}}{T_{4}}$$
(12)

where $\gamma = -i\omega$. One sees that there is a single root for ω , which is purely imaginary whether the plasma is stable or not.

Now it is clear that equation (12) gives the same instability condition as the fluid treatment (see equation (7)). However, the equation no longer is quadratic. Were one to include terms to order ω^2 in (12) the ordering would require the addition of terms in $(\omega/k_{\parallel} v_{T\parallel})^2$ from the expansion of Z' that normally would be larger than the term in ω^2 (or γ^2) which controls the fluid expression (7). It follows that the physics of the instability cannot be the familiar fluid picture that is generally used. The physics of the actual mirror instability must involve nonfluid aspects which we will examine in the next section.

PHYSICS OF LINEAR KINETIC INSTABILITY

The new terms in the Tajiri treatment of the mirror instability emerge from the explicit retention of the collision-free velocity distribution function in the kinetic treatment. In the discussion so far, the distribution function has been assumed bi-Maxwellian and has appeared in the calculation through the presence of the plasma dispersion function, Z. Let us now reexamine what happens in the instability if one retains more explicitly the plasma velocity distribution function F.

We can regard the distribution as gyrotropic and thus can write the velocity distribution function as a function of two quantities; for example, W_{\parallel} and W_{\perp} , the parallel and perpendicular energy are useful if we wish to express the result in terms of parallel and perpendicular temperature. At low frequencies the magnetic moment invariant μ is conserved (the basis of the mirror effect) and the perpendicular energy at any point is directly linked to the local magnetic field strength, $W_{\perp} = \mu B$. Introducing the total energy W we can write the changes in W_{\parallel}, W_{\perp} , $\delta W_{\parallel}, \delta W_{\perp}$ as

$$\delta W_{\parallel} = \delta W - \mu \,\delta B$$
 and $\delta W_{\perp} = \mu \delta B$ (13)

We can calculate the change in distribution associated with changes in field strength and energy by using the fact that the value of F for any particle remains constant as the particle moves (the Liouville theorem). The change in distribution function is then given by [*Kivelson and Southwood*, 1985]

$$\delta F = -\delta W_{\rm H} \frac{\partial F}{\partial W_{\rm H}} - \delta W_{\rm H} \frac{\partial F}{\partial W_{\rm H}} \tag{14}$$

Thus

$$\delta F = -\delta W \frac{\partial F}{\partial W_{||}} - \mu \delta B \left(\frac{\partial F}{\partial W_{1}} - \frac{\partial F}{\partial W_{||}} \right)$$
(15)

and

$$\delta F = \left[\frac{\delta W}{T_{||}} + \frac{\mu \delta B}{T_{\perp}} \left(1 - \frac{T_{\perp}}{T_{||}}\right)\right] F \tag{16}$$

The latter term on the right-hand side of (16) corresponds to the mirror term as the factor proportional to the anistropy $(T_{\parallel}/T_{\parallel}-1)$ shows. The change in energy due to a change in field strength is given in the low-frequency limit by the adiabatic expression [*Northrop*, 1963]

$$\frac{dW}{dt} = \mu \frac{\partial B}{\partial t} \tag{17}$$

We derive an expression for δW by integrating (17) for a disturbance varying as exp ($ik \cdot r + \gamma t$), to find

$$\delta W = \frac{\gamma}{\gamma + ik_{||\mathbf{V}||}} \,\mu \,\delta B \tag{18}$$

(where for low frequencies perpendicular velocity averages to 0), and thus we have the distribution function in the low-frequency limit

$$\delta F = \left[\frac{\mu \delta B}{T_{1}} \left(1 - \frac{T_{1}}{T_{||}}\right)\right] F + \left(\frac{\gamma \mu \, \delta B}{\gamma + ik_{||}v_{||}}\right) \frac{F}{T_{||}}$$
(19)

The last term on the right-hand side is an additional contribution that arises from the kinetic approach. Note that if γ is small as one expects, at least for marginal stability, then the term is negligible except for particles with $v_{\parallel} = 0$. However, for these particles the term is of the same order and potentially of larger magnitude than the preceding mirror term. The term shows that the particles with small v_{\parallel} behave radically differently to the rest of the distribution. These particles can be identified as resonant with the (zero frequency) wave. Thus it is the presence of resonant particles that causes the difference in dispersion relation between kinetic and fluid mirror instability treatments. Correctly treated, the mirror instability is a resonant instability, albeit with further some unusual properties.

Taking the second moment of δF given in (19) and imposing the pressure balance condition yields

$$\frac{B\delta B}{\mu_o} + 2p_1 \left(1 - \frac{T_1}{T_{||}}\right) \frac{\delta B}{B} + 2\left(\int d\mathbf{v}_{||} \frac{\gamma^2}{\gamma^2 + k_{||}^2 \mathbf{v}_{||}^2} F_{||}\right) \frac{T_1^2}{T_{||}} \frac{\delta B}{B} = 0$$
(20)

where the notation F_{\parallel} is used for the distribution of paralle, velocities after the integral over the perpendicular velocities has been carried out. F_{\parallel} has been assumed a symmetric function of v_{\parallel} .

In the limit of γ goes to zero, the integral may be carried out using the approximation

$$\lim_{x \to 0} \frac{\gamma}{\gamma^2 + x^2} = \pi \,\delta(x)$$

where $\delta(x)$ is the Dirac delta function. One finds

$$\frac{B\delta B}{\mu_o} + 2p_4 \left(1 - \frac{T_1}{T_{||}}\right) \frac{\delta B}{B} + 2\frac{\gamma}{k_{||}} \left(\int d\mathbf{v}_{||} \,\pi \,\delta(\mathbf{v}_{||}) \,F_{||}\right) \frac{T_1^2}{T_{||}} \frac{\delta B}{B} = 0$$
(21)

Substituting a Maxwellian parallel velocity distribution into (21) reproduces the $k_{\parallel}^2 \ll k_{\pm}^2$ limit of the *Tajiri* [1965] result given in equation (11).

$$-\frac{\gamma}{k_{||}v_{T||}} = \frac{1 + \beta_{\rm L} \left(1 - \frac{I_{\rm L}}{T_{||}}\right)}{\beta_{\rm L} \left(T_{\rm L} / T_{||}\right)} \left(\frac{2}{\pi}\right)^{1/2}$$
(22)

Now the third term in equation (21) contains the resonant contribution; indeed, it is possible to rewrite the expression for an distribution function

$$\frac{\gamma}{k_{||}} \int d\mathbf{v}_{||} \pi \,\delta(\mathbf{v}_{||}) \,F(\mathbf{v}_{||}) = \frac{\gamma}{k_{||}} \pi \,F(0) = \frac{\gamma}{k_{||}} \pi \,F_{\mathsf{res}}$$

and so

$$-\frac{\gamma}{k_{\rm H}} = \frac{B^2}{\mu_o} \frac{1 + \beta_{\rm L} (1 - \frac{T_{\rm H}}{T_{\rm H}})}{2 \pi (T_{\rm H}^2 / T_{\rm H}) F_{\rm res}}$$
(23)

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It is immediately apparent that the resonant particles play a role somewhat different from that met in other resonant instabilities like the ion cyclotron [e.g., Gary, 1992]. Whereas the linear growth rate is usually proportional to the number of resonant particles, here it is inversely proportional to the number present or to the resonant pressure. The reason will become apparent in our physical description of the instability.

PHYSICAL MECHANISM OF THE LINEAR INSTABILITY

Equation (22) is a useful vehicle to describe what happens in the instability. The equation itself represents total pressure balance. The first term on the left hand side is the magnetic pressure perturbation, the second is the (mirrorlike) response of the bulk of the plasma and the third term represents the resonant response of the particles with close to zero parallel velocity (90° pitch angle). Figure 1 is designed to illustrate the disparate behavior of the bulk of the plasma population and the resonant particles which arise solely from the difference in parallel motion.

The resonant pressure response is proportional to the growth rate. Equations (18) and (19) show that the dependence on growth rate is due to the particle undergoing betatron acceleration according to equation (17). Unlike what occurs for the bulk of the plasma where energy is simply exchanged between perpendicular and parallel degrees of freedom, the energy of the resonant particles does change as the instability develops. Being clustered about zero parallel velocity, the resonant particles do not move a significant distance along the field in the instability growth time $(1/\gamma)$. Thus the change in field that a resonant particle detects is simply due to the local temporal increase or decrease in field. In contrast,



Fig. 1. The sketch illustrates the difference between resonant and nonresonant particle behavior in the perturbed magnetic field of the mirror instability. The resonant particles have close to zero parallel velocity and are shown by simple solid circles. A resonant particle in the weak field region detects a continually decreasing field. A resonant particle in the region of increasing field detects a continually increasing field. The former set of resonant particles cool; the latter set heat as the instability develops. The bulk of the particle population are nonresonant and, as the arrows attached to the open ovals indicate, they travel through the mirror field structure as the instability develops. They experience both increasing and decreasing field as they move through the mirror structure; their energy does not change significantly but it does shift back and forth between parallel and perpendicular (gyrational) degrees of freedom. The resonant particles move little along the field and change energy as the field magnitude changes locally.

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for the bulk of the plasma, the predominant change in field experienced by particles is through the spatial variation of the field perturbation and is experienced due to the particle motion through the field.

The linear instability thus progresses in the following manner. An increase/decrease in field leads to a pressure decrease/increase in the bulk of the plasma that causes a net local pressure deficit/surplus. The pressure is balanced by the resonant particle pressure in which the resonant particles respond being accelerated/decelerated by the field in the increasing/decreasing field regions thus responding in antiphase to the bulk plasma. Although some motion along the field is implicit in the field changes, this response plays no dynamic role.

Note that the issue of any pressure imbalance causing the field lines to move together or apart does not enter the simple descrip-

tion. The term describing field line acceleration is dropped in the limit appropriate to the kinetic instability. (The term is of order $|\omega|^2/k_L^2 A^2$ and was dropped between equation (10) and (11).)

In Figure 2, we show a sketch illustrating the effect of the instability during its linear phase on a contour of the plasma distribution function in the $(v_{\parallel}, v_{\perp})$ plane. A bi-Maxwellian has elliptical contours in this plane. The unperturbed contour is elongated in the v_{\perp} direction because of the distribution anisotropy. We have indicated the regime $v_{\parallel} < \gamma/k_{\parallel}$ in which the particles are resonant with the disturbance. The energy of the nonresonant particles does not change as they move through the disturbance; only their pitch angle changes. When the field is increasing, the pitch angle increases and the nonresonant part of the contour is adiabatic (conserving μ), particles with $v_{\perp} = 0$ are unaffected and therefore the contours remain at constant levels along the v_{\parallel} axis.



Fig. 2. Contours of the distribution function in the (v_{lb}, v_4) plane indicating what happens in the linear phase of the instability at different locations along the background field. The middle distribution contour shows the distribution at a location where there is no field perturbation. This distribution is repeated for reference in the other two panels. The top and bottom distribution contours apply to positions where the field strength has increased or decreased, respectively. All particles conserve the magnetic moment μ and thus the change in perpendicular energy is proportional to the change in field strength and μ . The perpendicular energy change of the nonresonant (bulk) of the distribution is simply balanced by a change in parallel energy so as to conserve total energy. The parallel energy increases where the perpendicular energy decreases and vice versa. The resonant particles (which have small parallel velocities, $v_{\parallel} < \gamma/k_{\parallel}$) experience changes predominantly in their perpendicular energy and their total energy increases (or decreases) wherever the field increases (or decreases). The range of v_{\parallel} that contains resonant particles is shaded.

In the same region at low parallel energy, where the resonant particles are found, the rising field causes the total particle energy and the perpendicular energy to change proportional to B; the contour is thus moved away from the origin in this regime. Thus in the resonant regime the distribution contour develops a bulge extending outside the original contour. Where the field is decreasing ($\delta B < 0$), the nonresonant part of the contour moves further from the origin as pitch angles decrease. In this part of the disturbance, the resonant particles lose energy and the contours develop an indentation toward lower energy near the W_{\perp} axis which will extend inside the position of the original contour.

The pressure and number density variations for the nonresonant part of the plasma are well known [e.g., *Hasegawa*, 1969] and are implicit in the changes shown in the contours. However, the description of the special role of the resonant particles is critical to understanding the properties of the linear waves.

DISCUSSION AND CONCLUSIONS

We have derived a physical description of the mirror instability appropriate for the linear theoretical treatment of the kinetic instability first given by Tajiri [1967]. The analysis reveals that the instability is resonant but that the resonant particle role is unusual. The instability results from pressure imbalance between the bulk of the plasma and the magnetic field. For this to occur, the bulk (nonresonant) pressure response must be in antiphase with the magnetic pressure as occurs at low frequencies when the magnetic moment and the particle energy are conserved. The response is illustrated by equation (1). The resonant particles produce a pressure perturbation however, in phase with the field pressure change. A corollary is that unlike the nonresonant particles, the resonant particles experience energy changes as the instability develops. The computer simulations of McKean et al. [1992] are consistent with our analysis and the earlier work of Hasegawa [1985] in showing that the mirror instability selectively affects a portion of the particle distribution with large pitch angles. However, we believe that the special way in which the different parts of the distribution affect wave growth has not been previously pointed out.

The resonant particle behavior is very unlike the resonant particle behavior in resonantly driven instabilities. *Melrose* [1986] describes several such instabilities. Commonly, the nonlinear effect of the instability is expected to be a diffusion of resonant particles in velocity space described by quasi-linear theory [*Melrose*, 1986, p. 49]. The diffusion is spatially uniform (the quasilinear approximation involves averaging the distribution in space). The resonant particles lose energy in the diffusion, and this energy loss can be equated to the energy gain of the unstable waves. The ion cyclotron instability falls into the class of such instabilities [*Melrose*, 1986, p. 233].

The linear growth rate of most resonant instabilities is proportional to the number of resonant particles. As discussed above, in the case of the mirror instability, equation (23) shows that the growth is inversely proportional to the number (and pressure contribution) of resonant particles. The reason for the anomalous result is clearly explained by our physical description. The fewer particles there are at small parallel velocity, the higher the growth rate needs to be to balance the pressure imbalance generated by the nonresonant distribution.

Unlike the usual resonant instabilities, the mirror instability is nonoscillatory so that the resonant particles have zero velocity along the field. It is clear from this fact alone that as the resonant particles themselves are not moving along the field, a spatial

averaging process would not be an appropriate way to analyze the nonlinear development. In fact, the resonant particles at the peak of the field perturbation accelerate and the resonant particles near the bottom of the magnetic wells decelerate. Thus the instability does lead to resonant particle heating but only in part of the disturbance; elsewhere, the resonant particles cool.

We reserve for a future paper a treatment of the nonlinear evolution of mirror unstable plasmas. Suffice it to say that the physical mechanism described here suggests two features of any likely nonlinear saturated state. First, the resonant interaction is expected to modify the distribution significantly in the vicinity of zero parallel velocity. The nonresonant particles are not likely to be as greatly affected so their anisotropy is not likely to change much. Putting the two notions together one expects that the instability is likely to lead to a nonuniform plasma in which the mirror field structure along the field is retained and pressure, number density, and plasma temperature vary along the field as well as across the field. We predict that at large pitch angles the resonant distribition will be strongly modified and, in particular, that the pitch angle anisotropy of the very large pitch angle particles will have changed. The simplest scenario would predict that the pitch angle anistropy would reverse near large pitch angle so that resonant particle pressure varies spatially in phase with the static nonuniform field, while the bulk of the plasma still retains the antiphase response to field variation given by (1).

The observations of large almost purely compressional field irregular oscillations attributed to mirror instability certainly are consistent with our speculation that a mirror unstable plasma evolves to a nonuniform state in which substantial pitch angle anistropy may remain in the distribution. One interprets the oscillatory nature of the disturbances as due to the spatial structure which may be time stationary in the plasma frame that sweeps by the spacecraft at the plasma flow velocity [e.g., *Tsurutani et al.*, 1982]. Our prediction that the pitch angle distribution near 90° could be radically different in order to balance pressure in a quasi-steady manner is well worth examining if appropriate data can be found.

We have not answered the problem raised by *Gary* [1992] concerning the discrepancy in growth rates between ion cyclotron and mirror modes. However, we have shown that although both rely on resonant processes, one expects very different nonlinear responses. The tendency of the mirror instability to create inhomogeneous field-aligned structures could reduce the resonance efficiency of the ion cyclotron mechanism if inhomogeneity in the background field causes the resonance condition to become position dependent. Further analysis is required to elucidate the competition between the two instabilities driven by ion pitch angle anistropy.

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