ÉCOLE DE PHYSIQUE
des HOUCHES

31 March 2015

Intro to Pressure-Anisotropy-Driven Instabilities

Part II: Nonlinear Results

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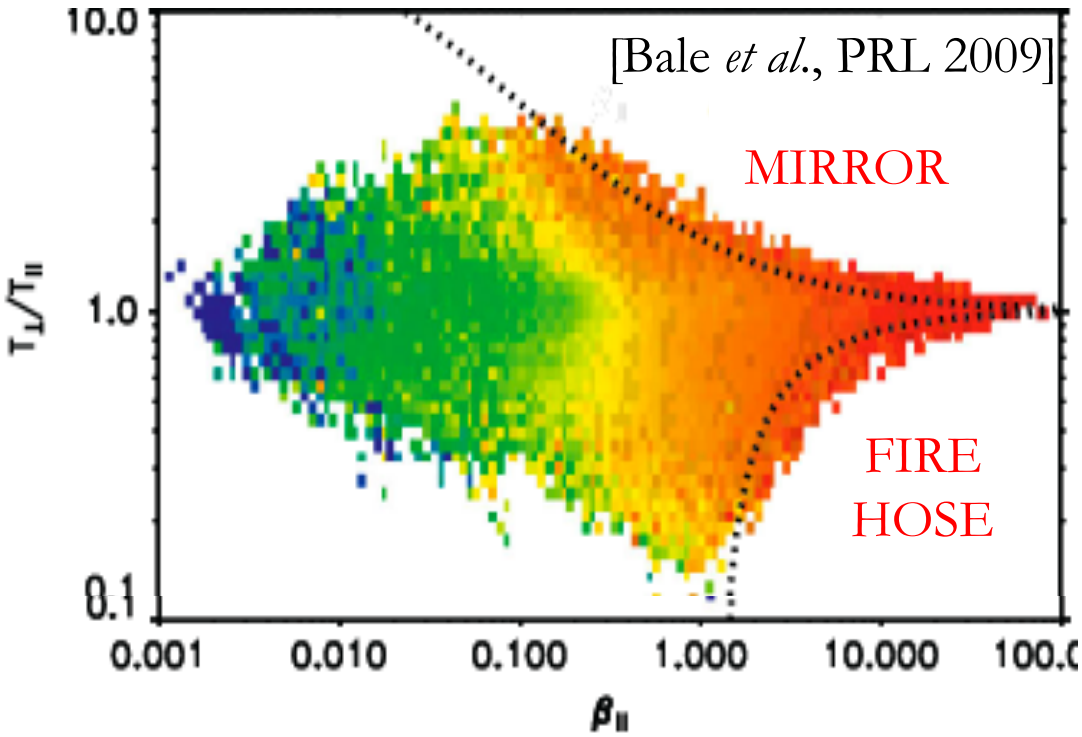
with debts and thanks to
S. Cowley (UKAEA), S. Melville (Oxford),
F. Mogavero (ENS Paris), F. Rincon (Toulouse),
M. Rosin (Pratt Inst.)

http://www-thphys.physics.ox.ac.uk/people/AlexanderSchekochihin/notes/LESHOUCHES15

Kunz, AAS & Stone, *PRL* **112**, 205003 (2014) [arXiv:1402.0010]

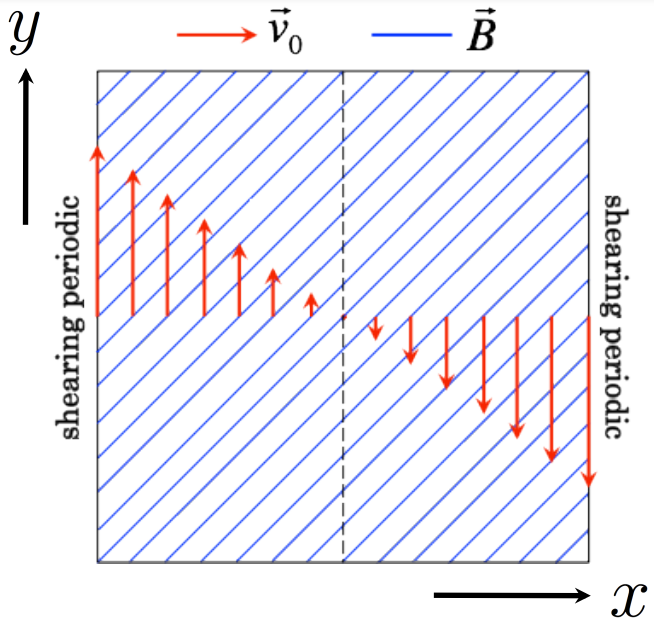
Mirror and Firehose Regulate Plasma?

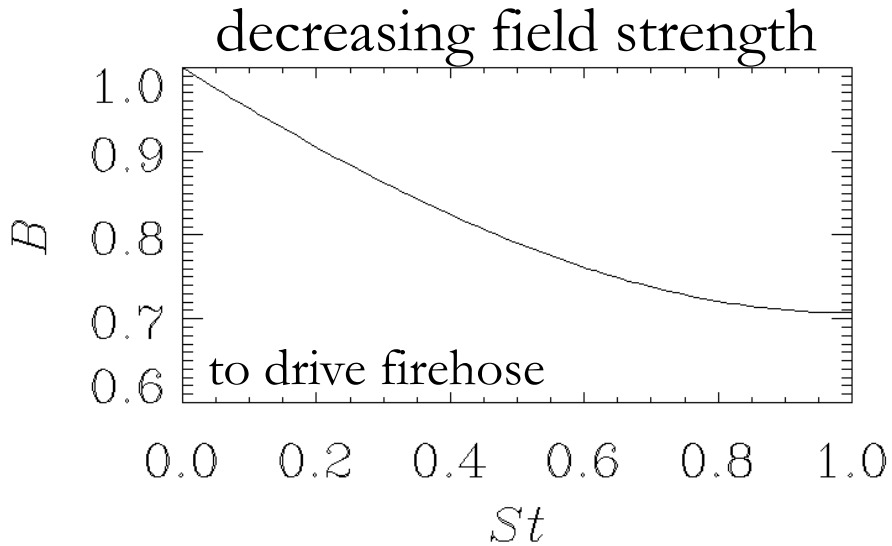
In the **solar wind**:

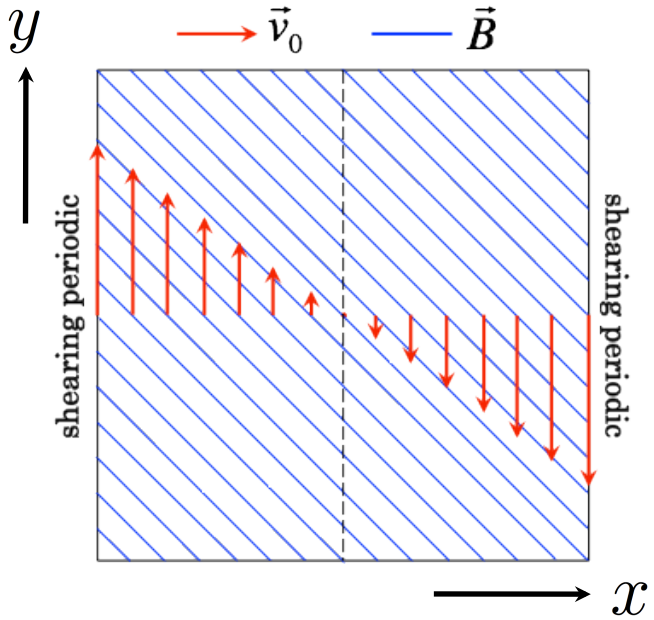


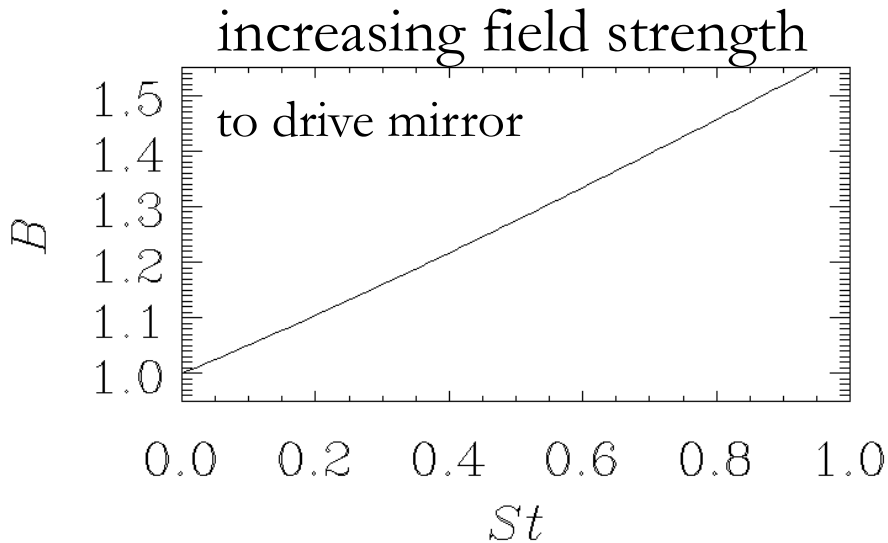
$$\Delta \equiv \frac{p_\perp - p_\parallel}{p} \sim \frac{1}{\nu}\frac{1}{B}\frac{\mathrm{d}B}{\mathrm{d}t} = \frac{\gamma}{\nu} \in \left[-\frac{2}{\beta}, \frac{1}{\beta}\right]$$

Instabilities in a Box (M. Kunz)









Kunz, AAS & Stone, *PRL* **112**, 205003 (2014) [arXiv:1402.0010]

Instabilities in a Box (M. Kunz)

Hybrid kinetic system solved by PEGASUS code:

$$\left(\frac{\partial}{\partial t} - Sx\frac{\partial}{\partial y}\right) f_\mathrm{i} + \boldsymbol{v}\cdot\boldsymbol{\nabla} f_\mathrm{i} + \left[\frac{Ze}{m_\mathrm{i}}\left(\boldsymbol{E} + \frac{\boldsymbol{v}}{c}\times\boldsymbol{B}\right) + Sv_x\hat{\boldsymbol{y}}\right]\cdot\frac{\partial f_\mathrm{i}}{\partial \boldsymbol{v}} = 0$$

$$\left(\frac{\partial}{\partial t} - Sx\frac{\partial}{\partial y}\right)\boldsymbol{B} = -c\boldsymbol{\nabla}\times\boldsymbol{E} - SB_x\hat{\boldsymbol{y}}$$

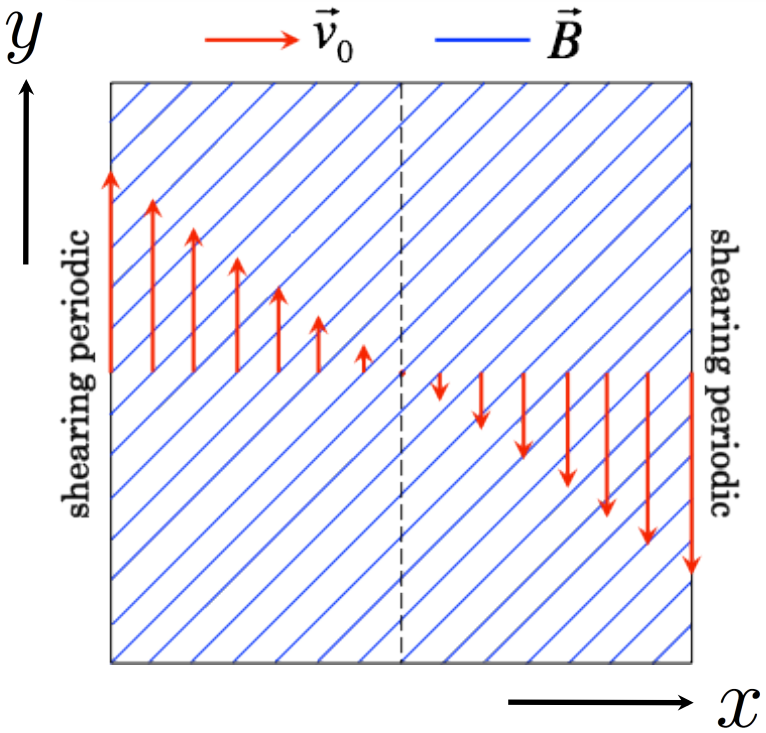
$$\boldsymbol{E} = -\frac{\boldsymbol{u}_\mathrm{i}\times\boldsymbol{B}}{c} + \frac{(\boldsymbol{\nabla}\times\boldsymbol{B})\times\boldsymbol{B}}{4\pi Zen_\mathrm{i}} - \frac{T_\mathrm{e}\boldsymbol{\nabla}n_\mathrm{i}}{en_\mathrm{i}}$$

…in a shearing sheet $\mathbf{u} = -Sx\hat{\mathbf{y}}$

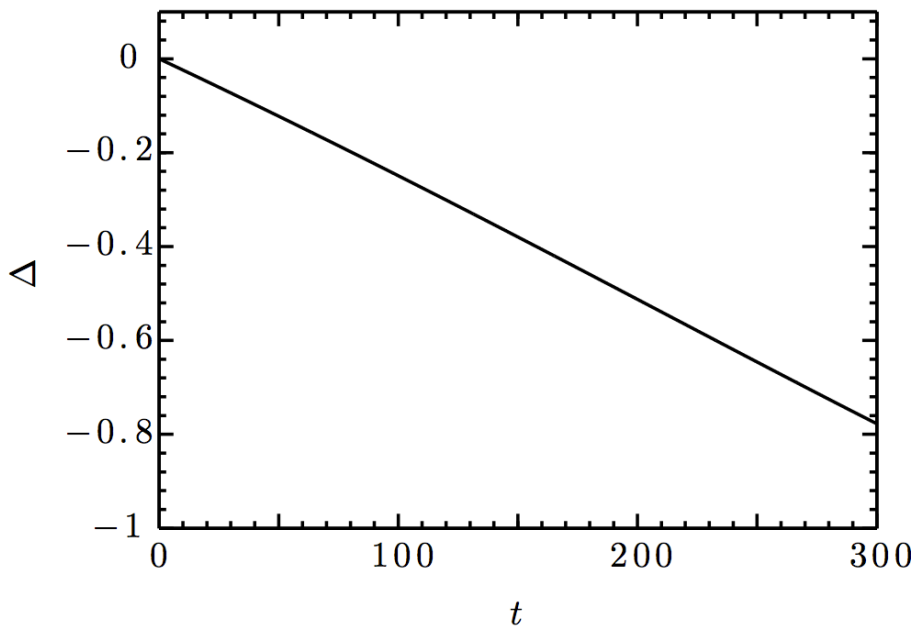
Kunz, Stone & Bai, *JCP* **259**, 154 (2014)

Kunz, AAS & Stone, *PRL* **112**, 205003 (2014) [arXiv:1402.0010]

Firehose Instability



$$\frac{\mathrm{d}B}{\mathrm{d}t} < 0 \quad \Rightarrow \quad \Delta = \frac{p_\perp - p_\parallel}{p} < 0$$

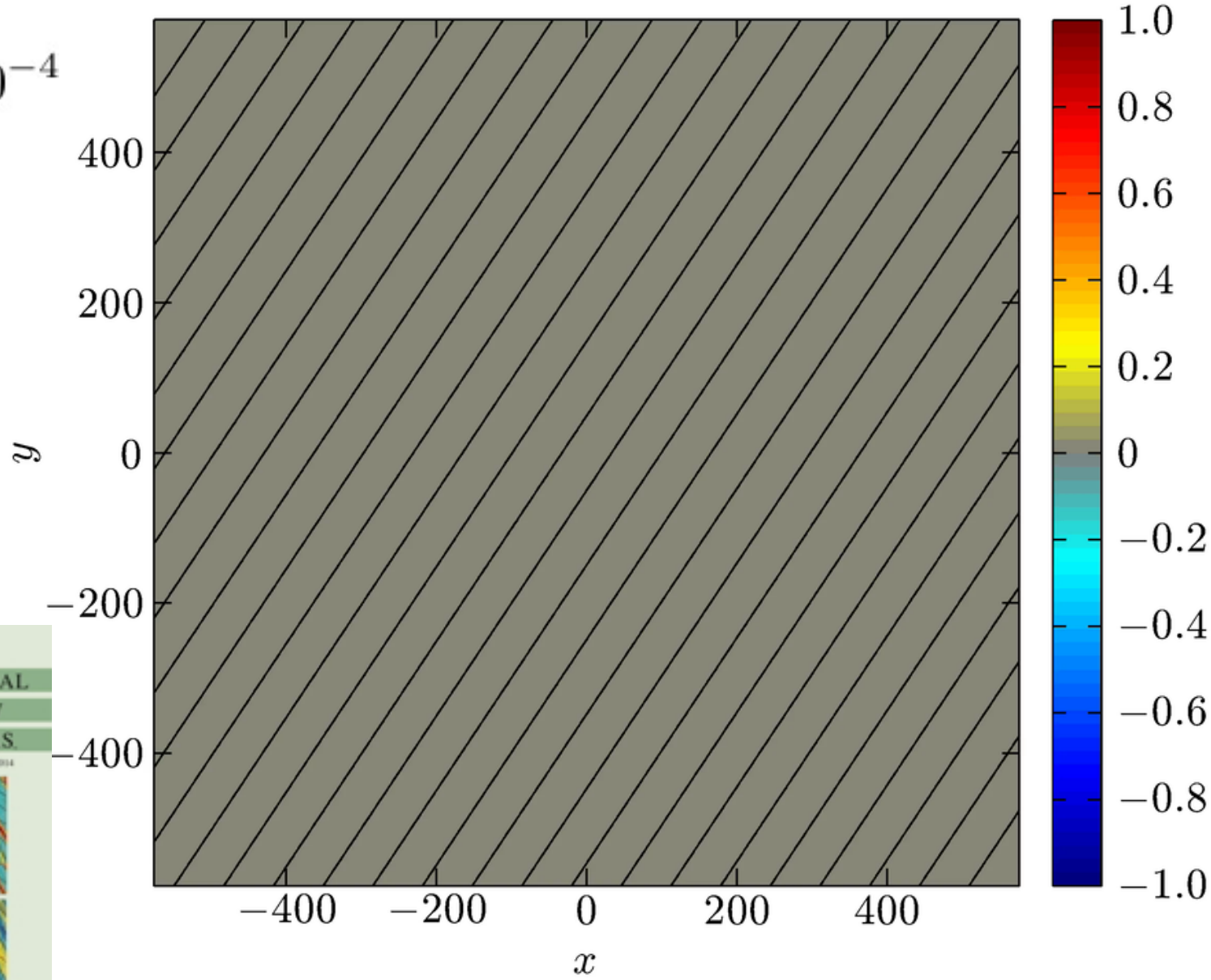


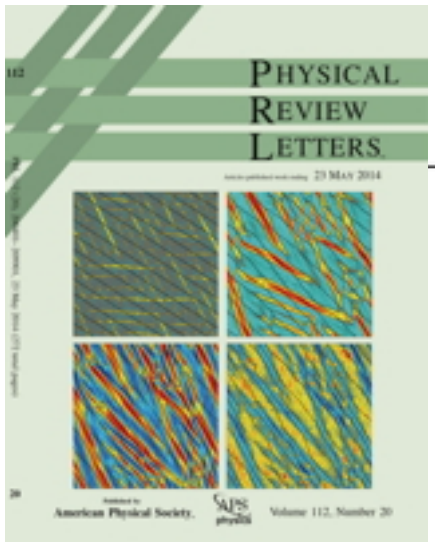
Kunz, AAS & Stone, *PRL* **112**, 205003 (2014) [arXiv:1402.0010]

Firehose Instability

$$\frac{S}{\Omega_i} = 3 \times 10^{-4}$$

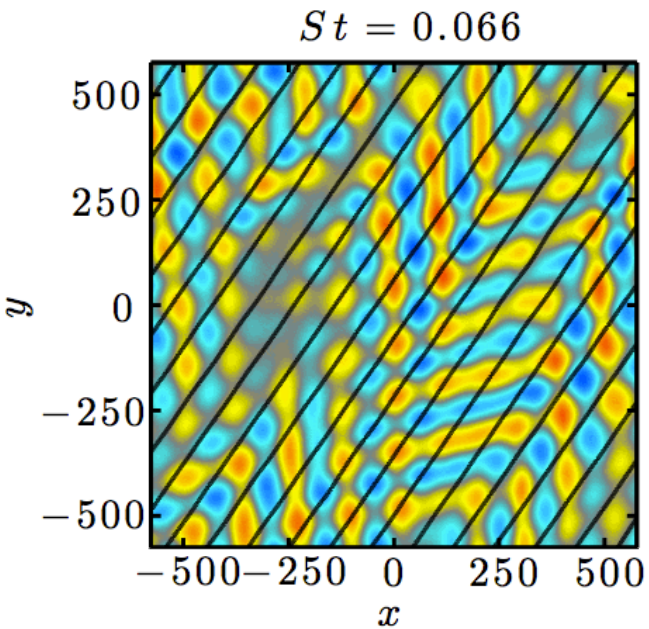
$$\beta_i = 200$$

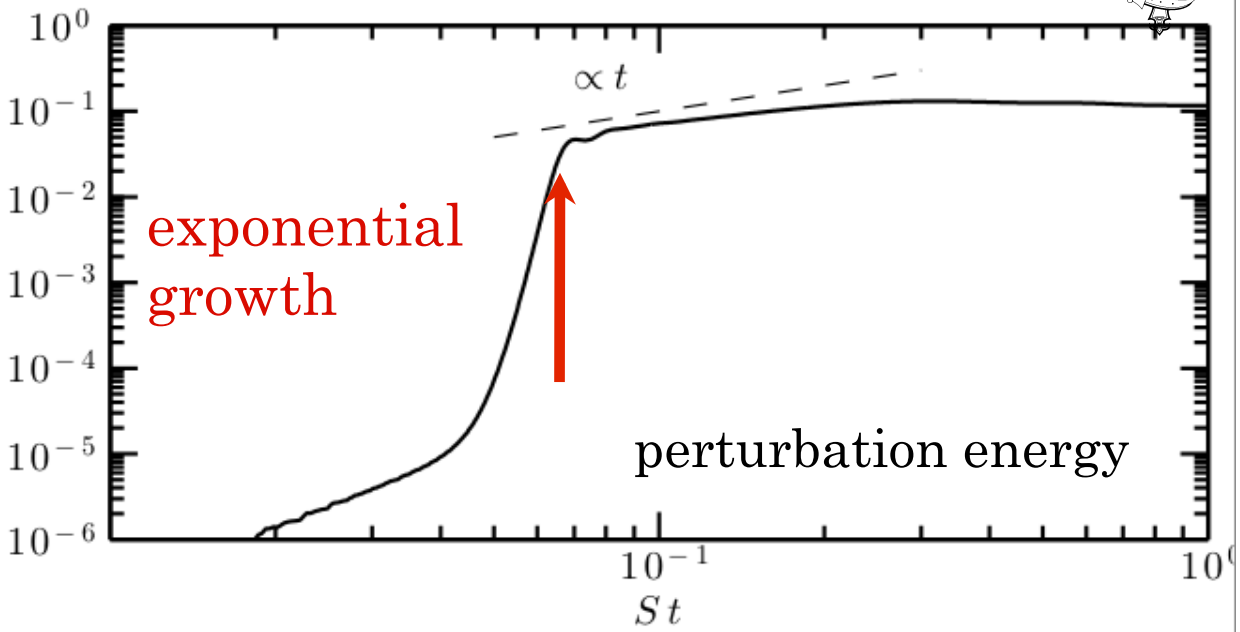


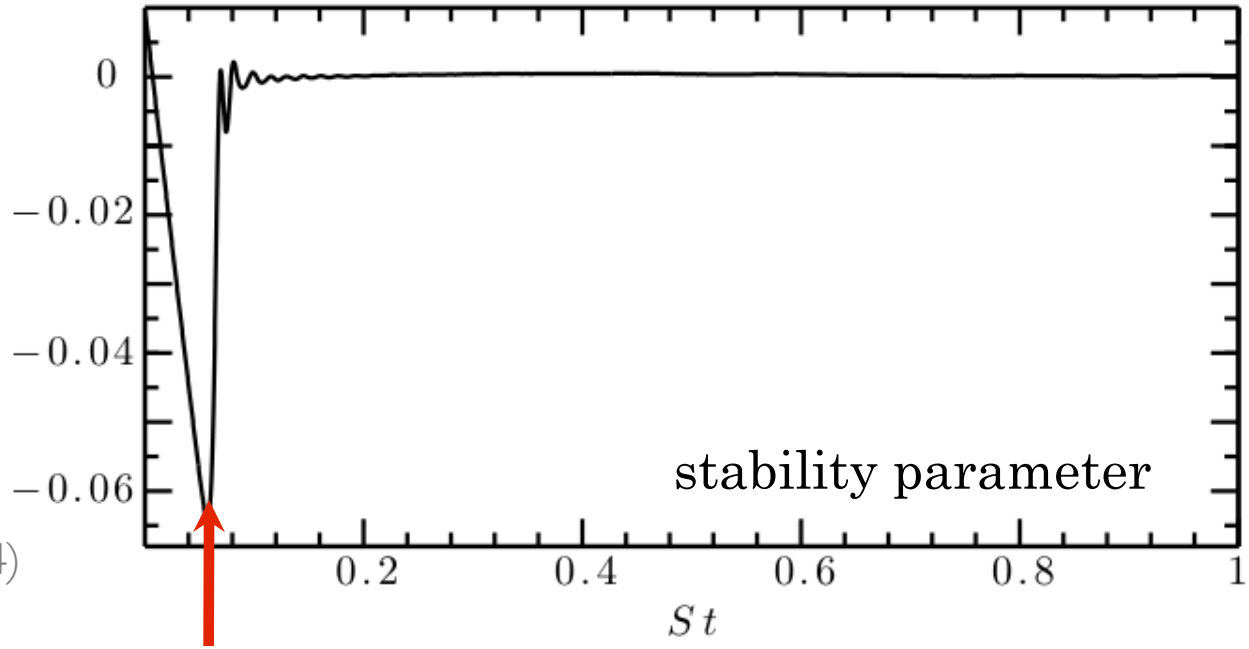


Kunz, AAS & Stone, *PRL* **112**, 205003 (2014) [arXiv:1402.0010]

Firehose Instability: Linear





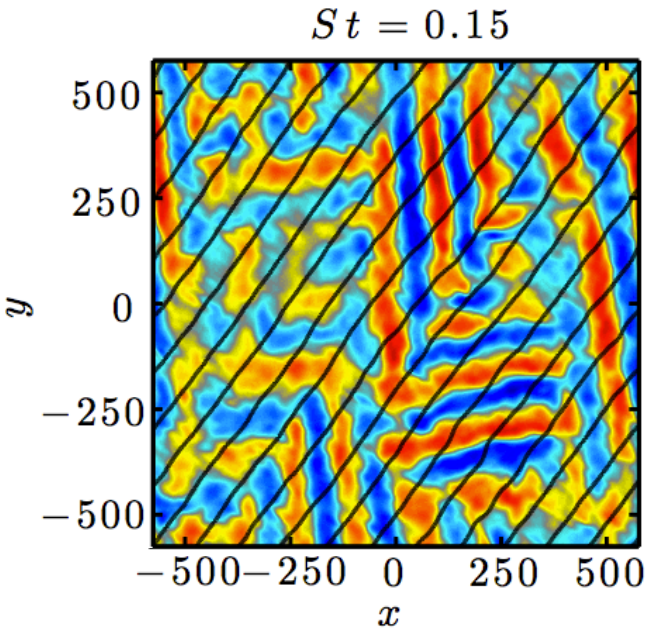


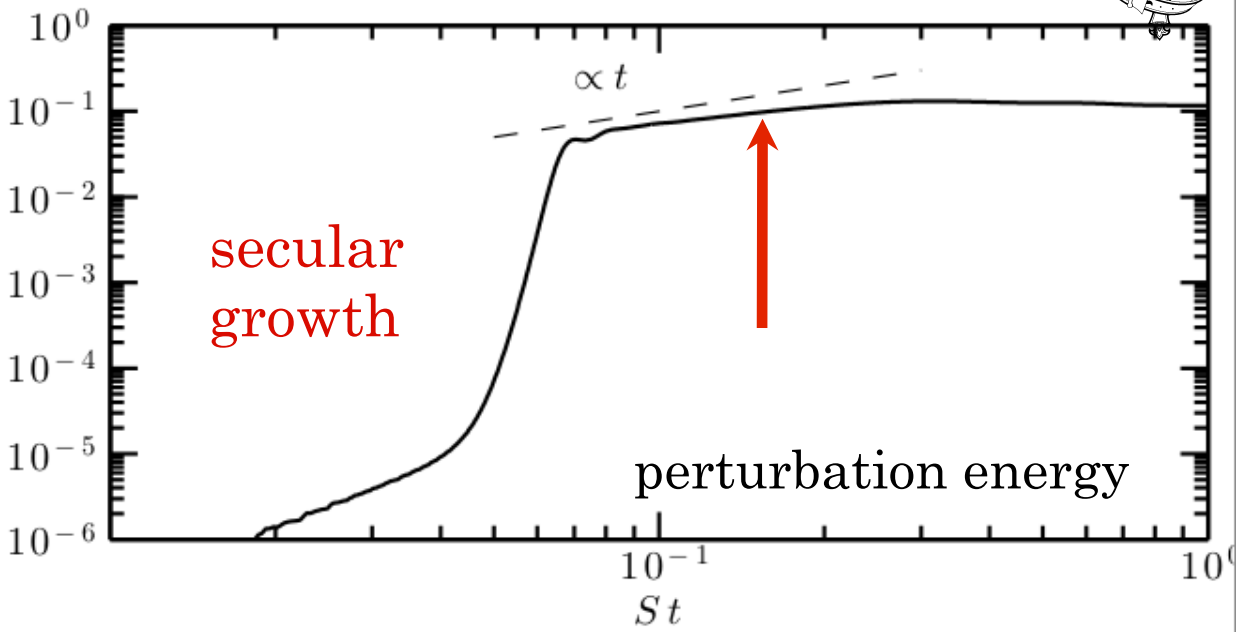
oblique modes

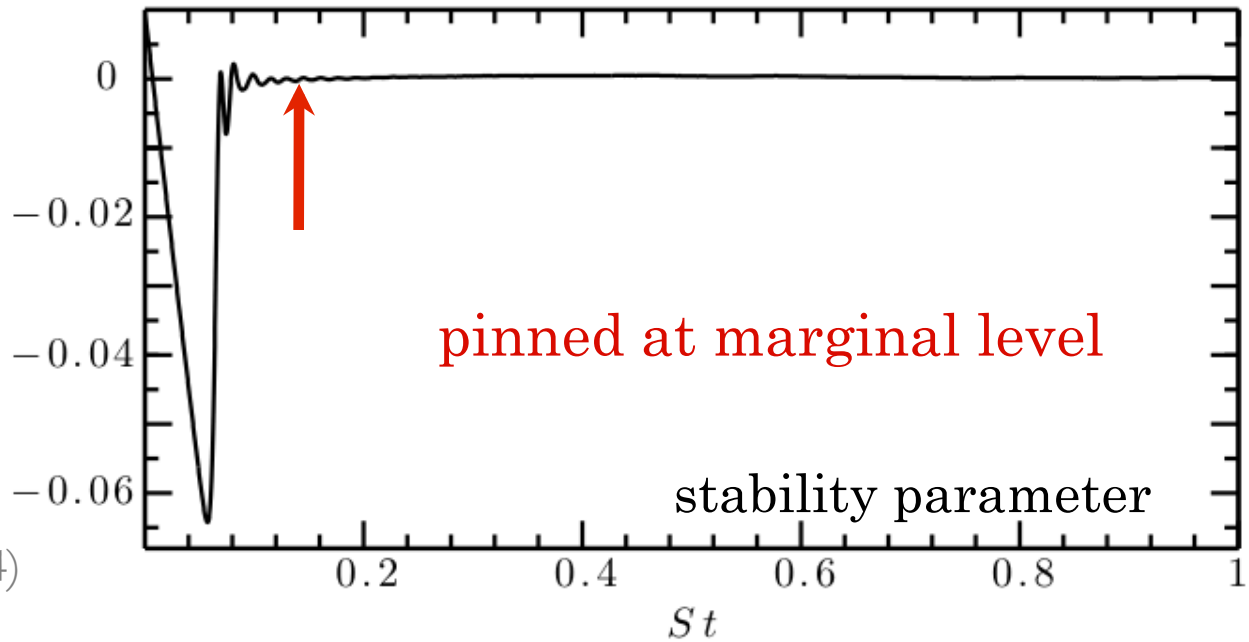
$$k_{\parallel}\rho_i \approx k_{\perp}\rho_i \approx 0.4$$

Kunz et al., *PRL* **112**, 205003 (2014)
[arXiv:1402.0010]

Firehose Instability: Secular







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Firehose Instability: Secular

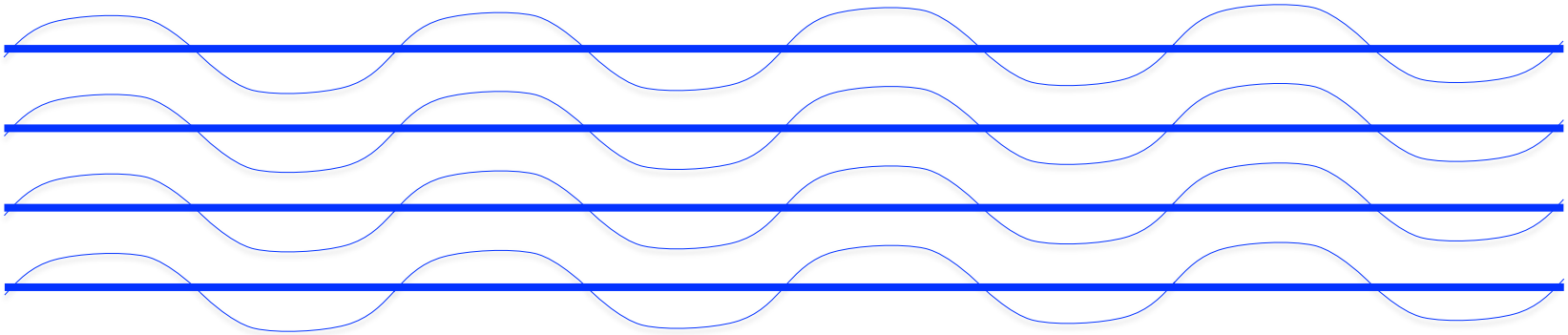
$$\overline{B^2} = \overline{|\mathbf{B}_0 + \delta\mathbf{B}_\perp|^2} = B_0^2 + \overline{|\delta\mathbf{B}_\perp|^2}$$

$$\Delta = 3\int^t \mathrm{d}t' \, \overline{\frac{\mathrm{d}\ln B}{\mathrm{d}t}} = \int^t \mathrm{d}t' \left(-3\left|\frac{\mathrm{d}\ln B_0}{\mathrm{d}t}\right| + \frac{3}{2}\frac{\mathrm{d}}{\mathrm{d}t}\frac{\overline{|\delta\mathbf{B}_\perp|^2}}{B_0^2} \right) \to -\frac{2}{\beta}$$

pressure anisotropy driven by shear

pressure anisotropy from firehose

marginal stability



AAS et al., *PRL* **100**, 081301 (2008) [arXiv:0709.3828]
Rosin et al., *MNRAS* **413**, 7 (2011) [arXiv:1002.4017]
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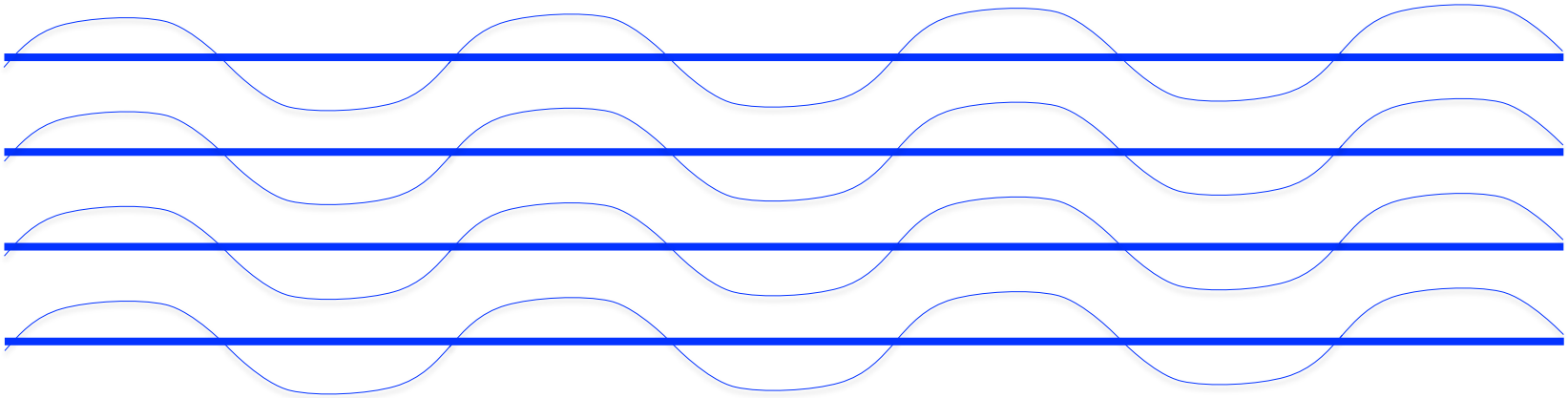

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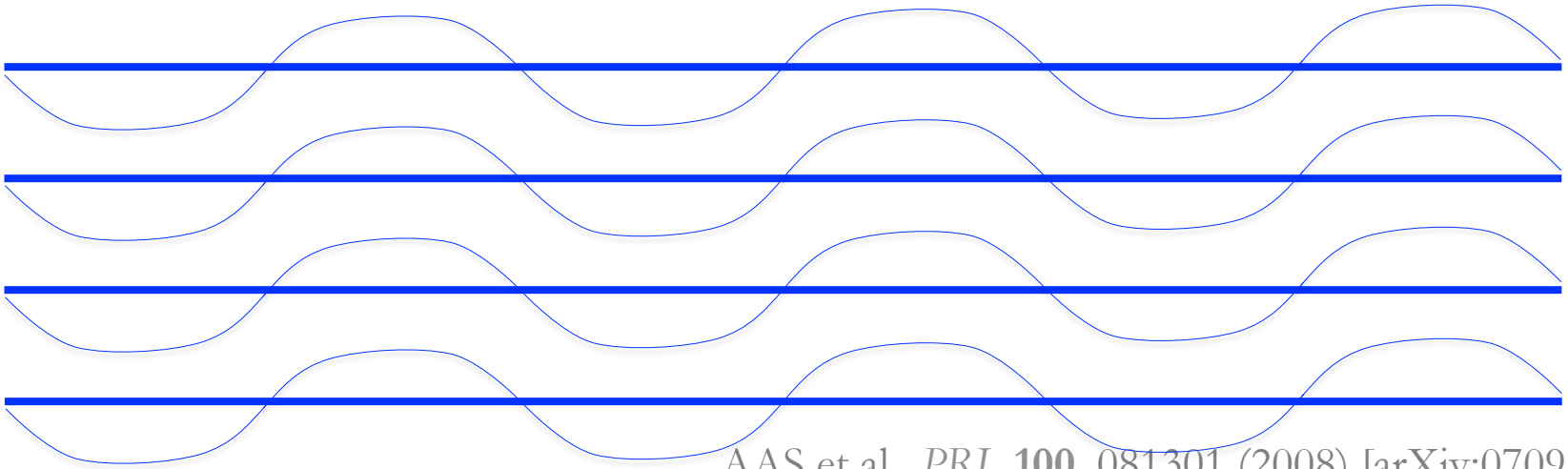

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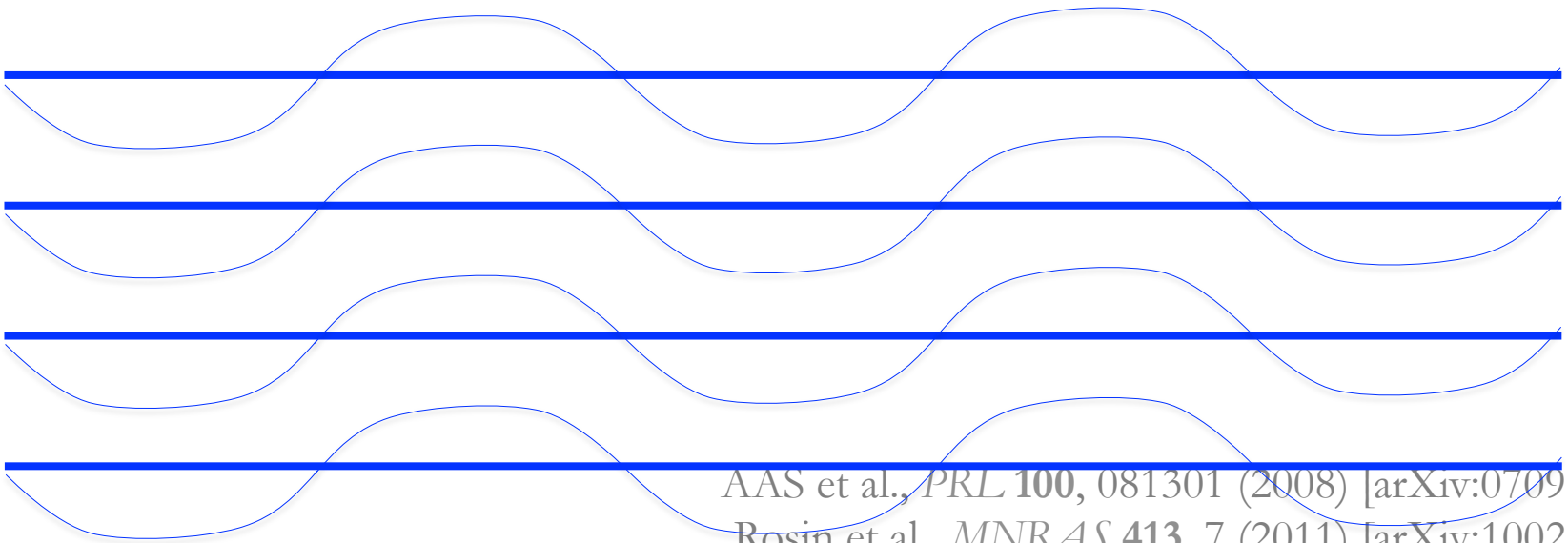

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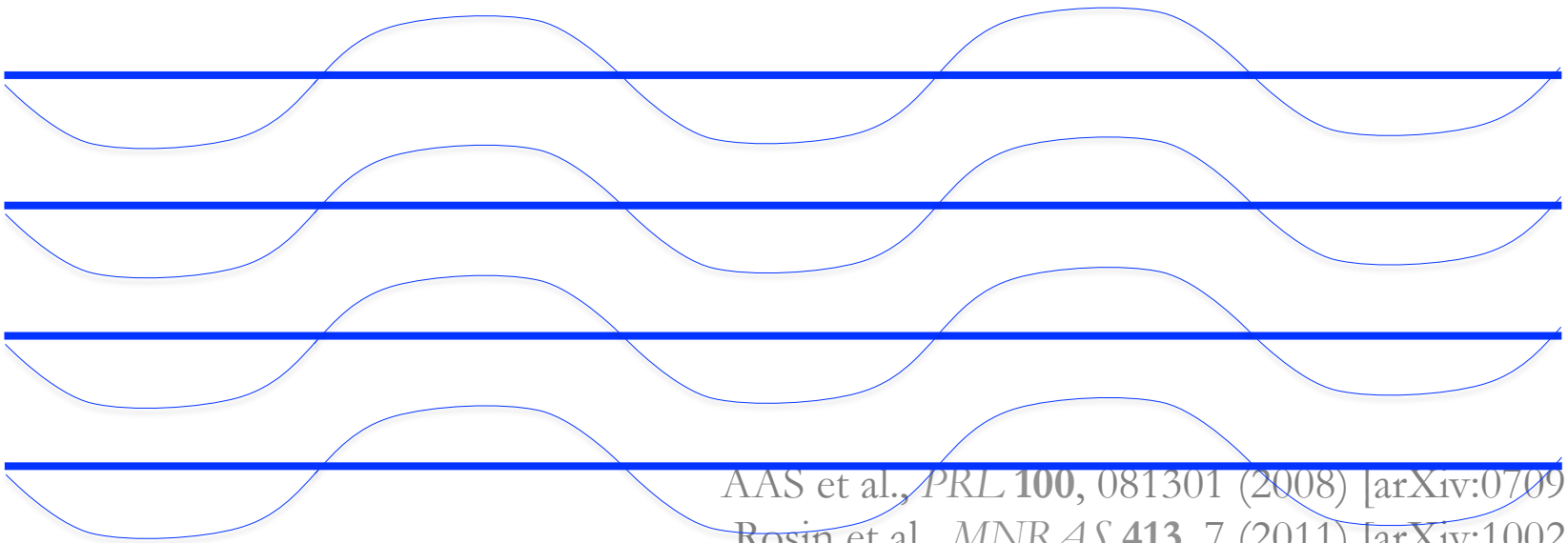
pressure anisotropy driven by shear

pressure anisotropy from firehose

marginal stability

$$\frac{3}{2}\frac{\overline{|\delta\mathbf{B}_\perp|^2}}{B_0^2} = 3S\int^t \mathrm{d}t' \, \hat{b}_x(t')\hat{b}_y(t') - \frac{2}{\beta} \sim St$$

secular growth



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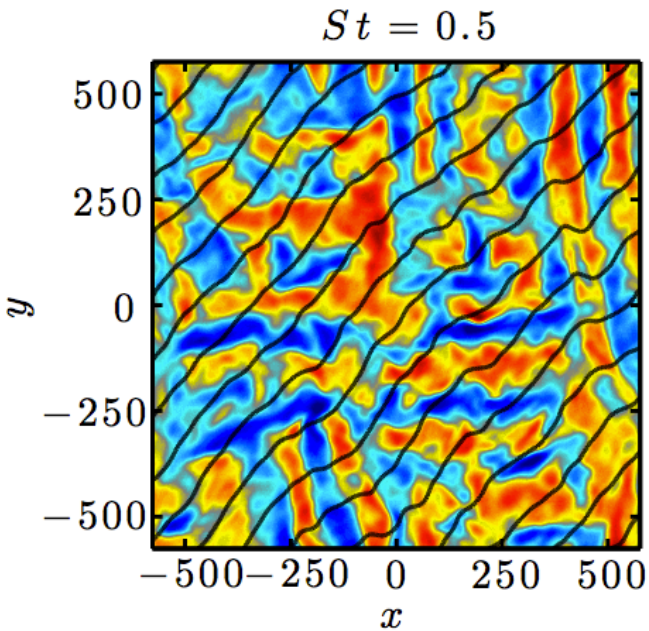
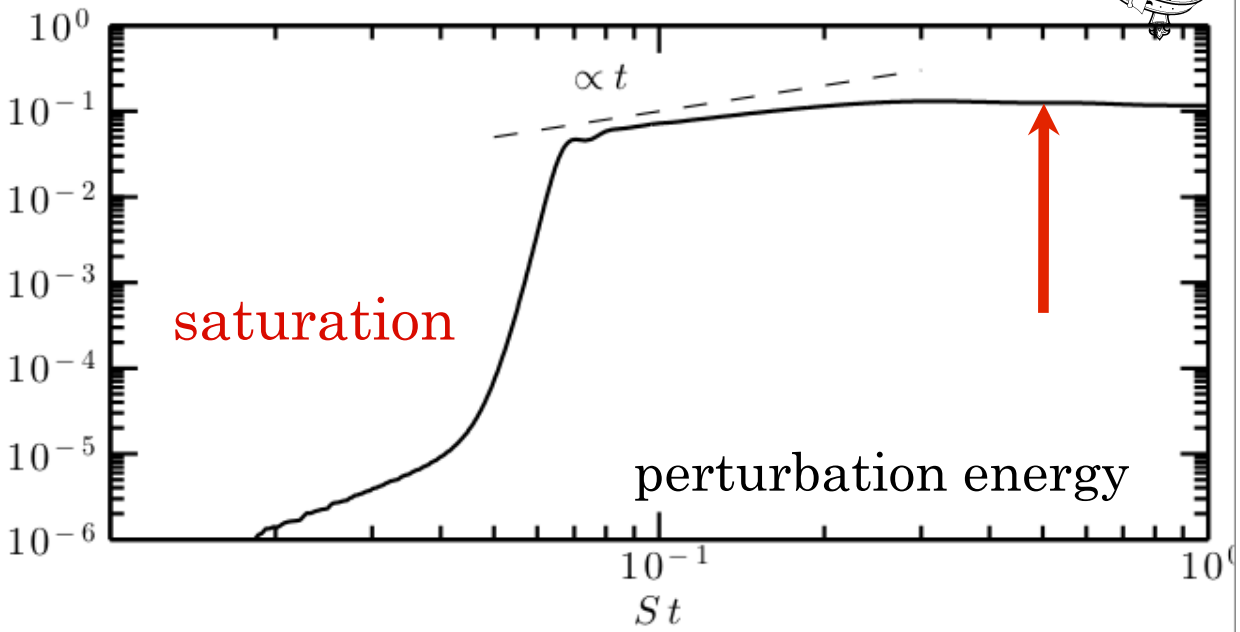
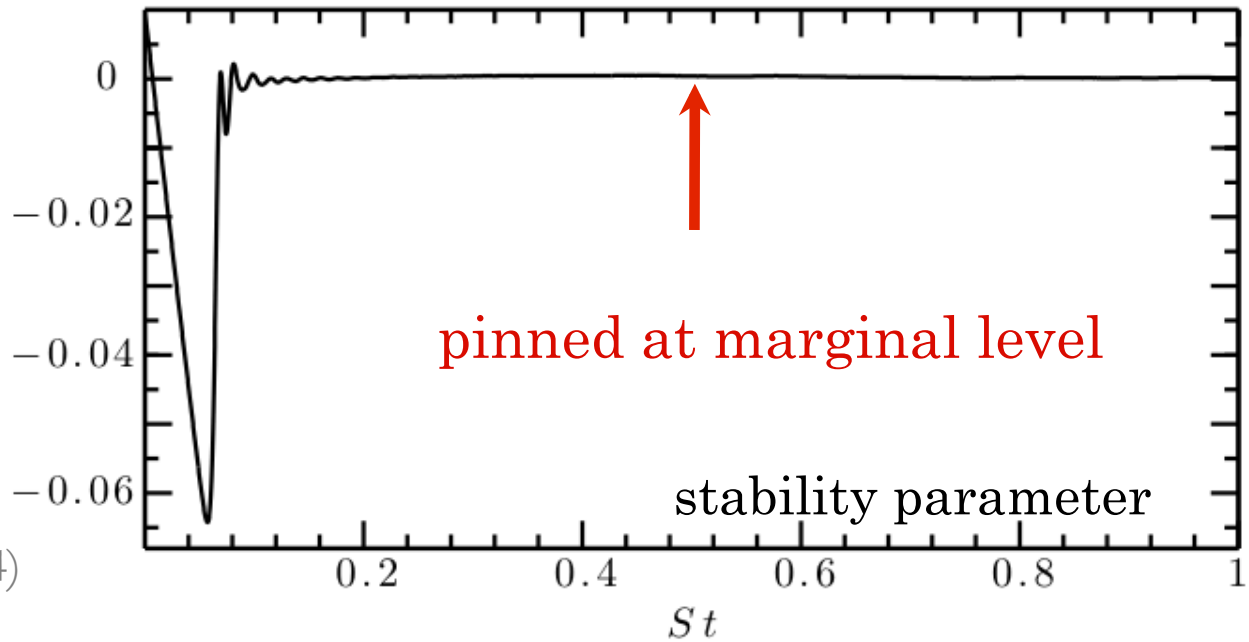
Firehose Instability: Saturated







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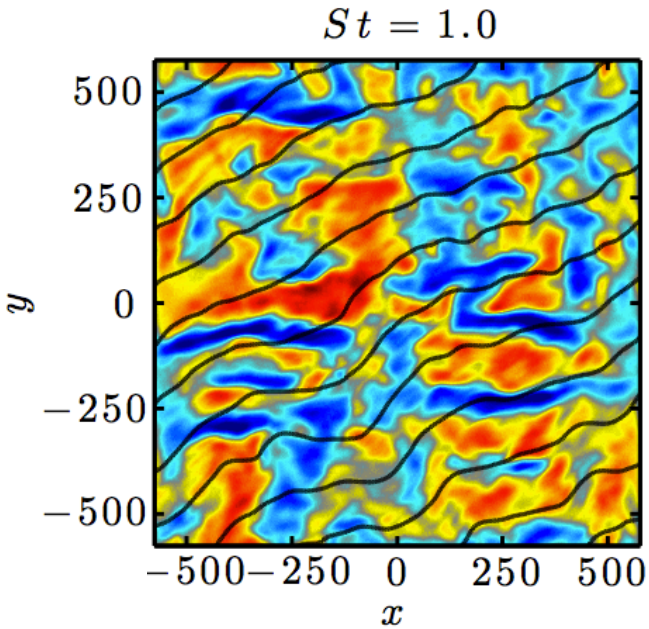
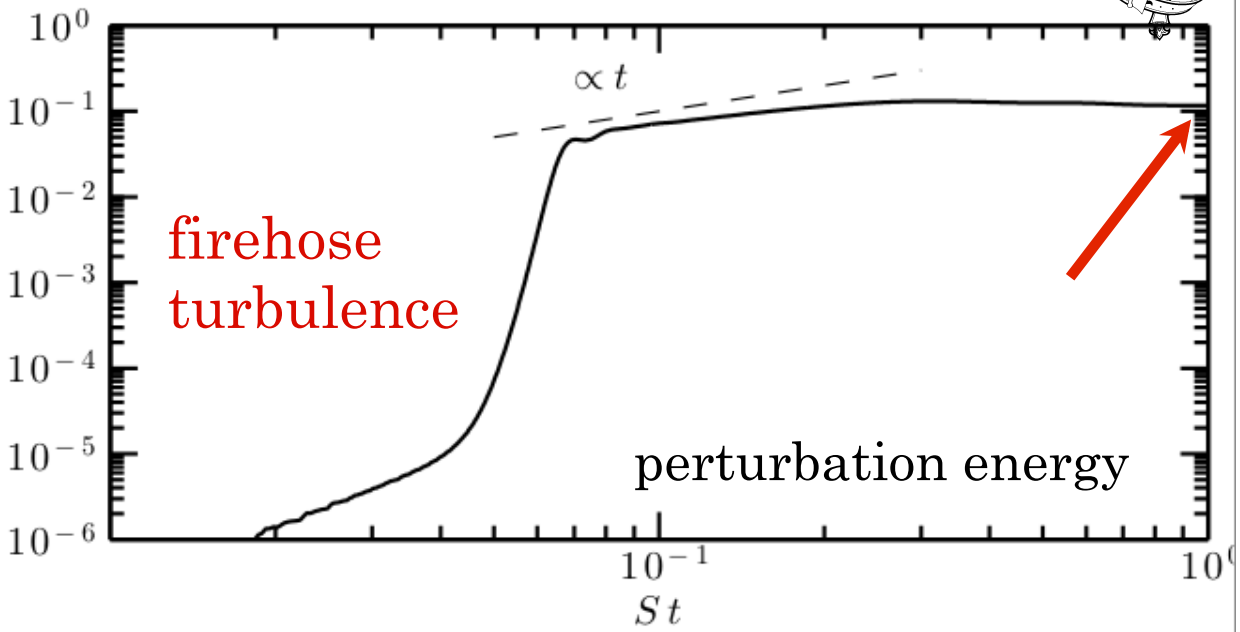
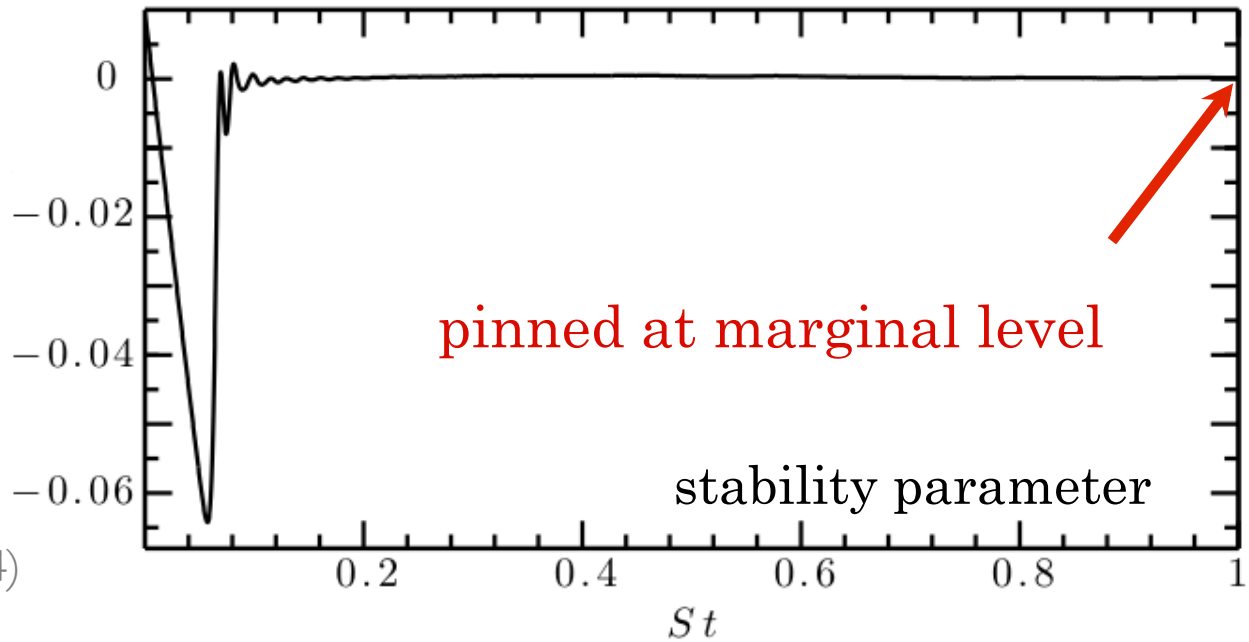




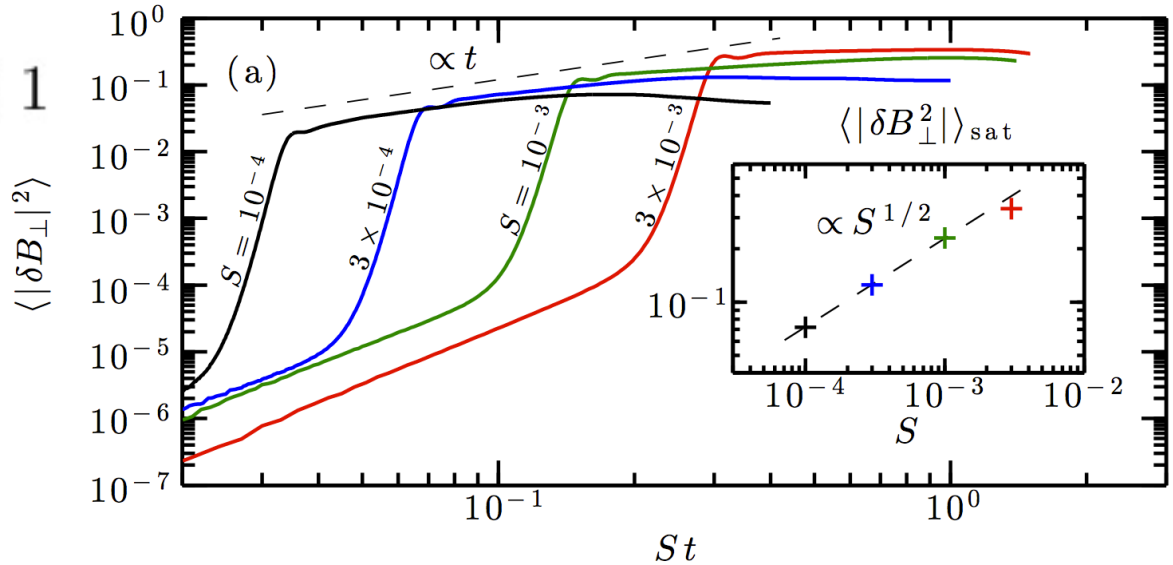


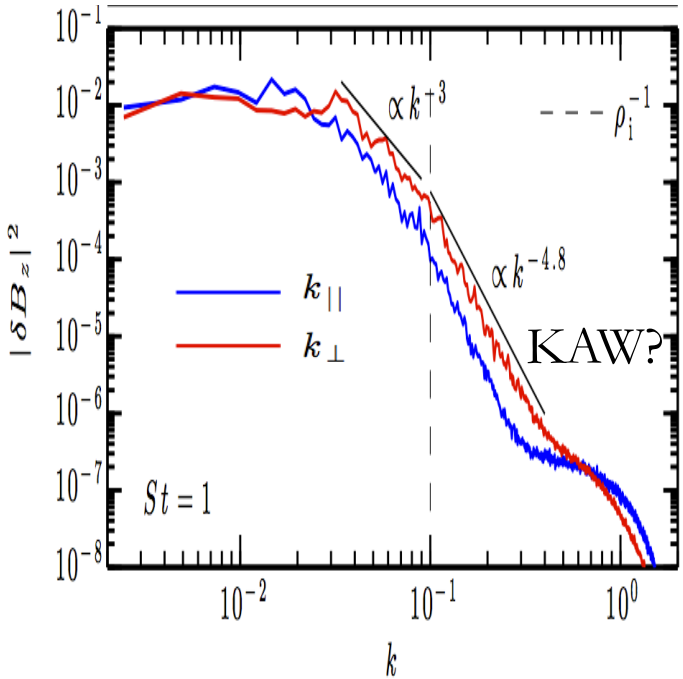
Kunz et al., *PRL* **112**, 205003 (2014)
[arXiv:1402.0010]

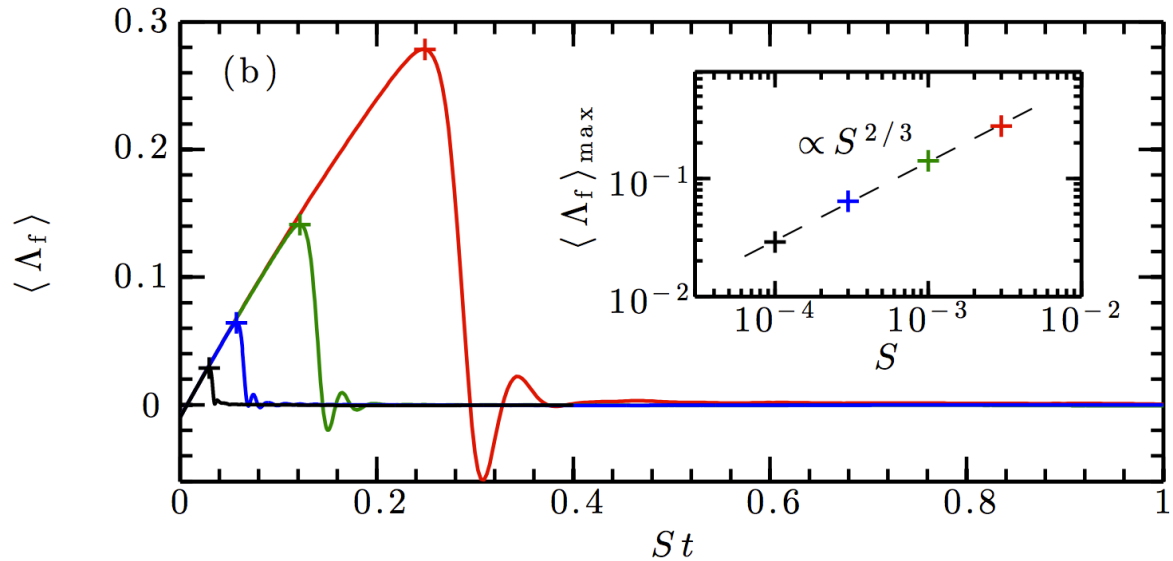
Firehose Saturates at Small Amplitudes

$$\frac{\langle|\delta\mathbf{B}_\perp|^2\rangle}{B_0^2} \propto \left(\frac{S}{\Omega_i}\right)^{1/2} \ll 1$$

small-amplitude
Larmor-scale
firehose turbulence







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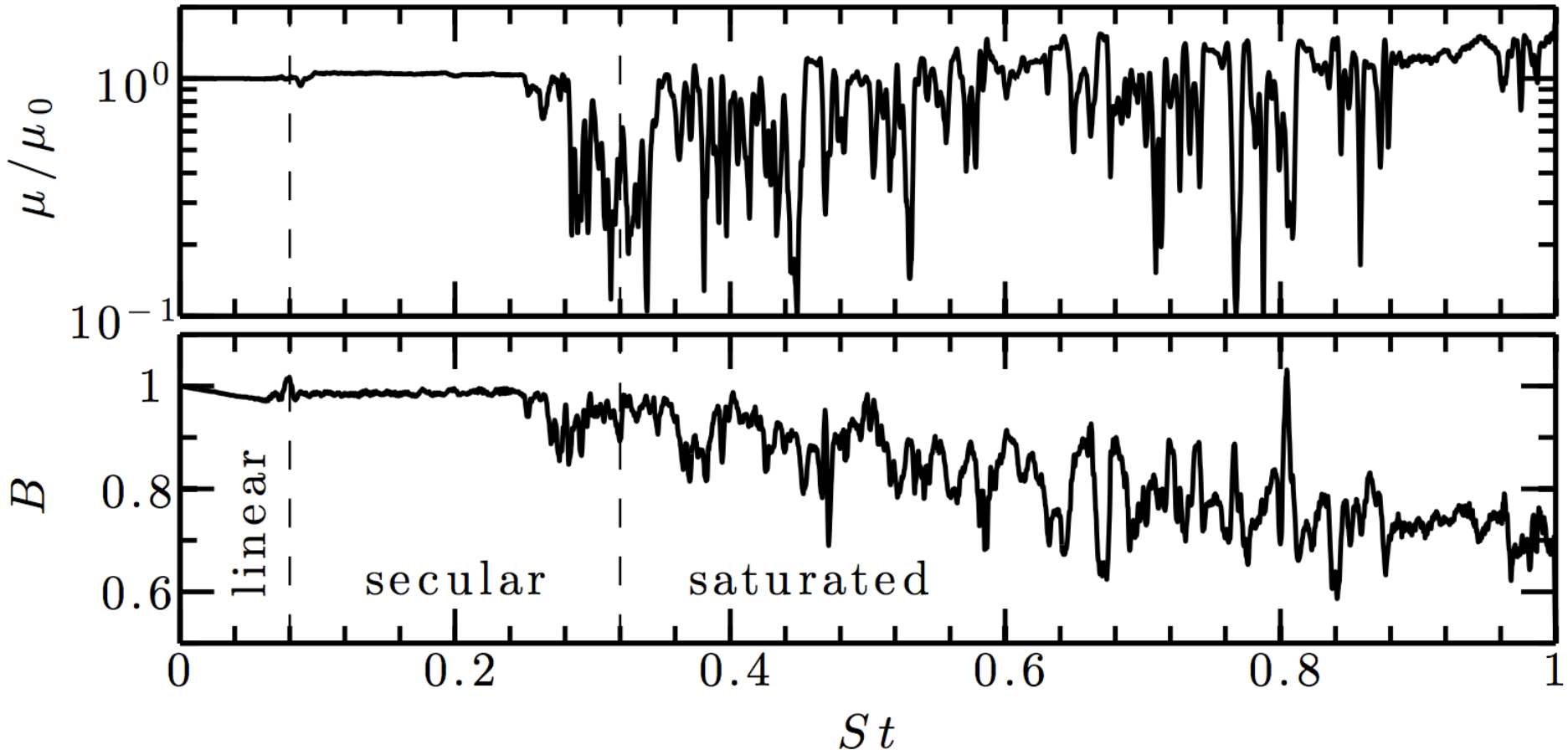
Saturated Firehose Scatters Particles



μ conservation is broken at long times, firehose fluctuations scatter particles to maintain pressure anisotropy at marginal level

Kunz, AAS & Stone, *PRL* **112**, 205003 (2014) [arXiv:1402.0010]

Saturated Firehose Scatters Particles

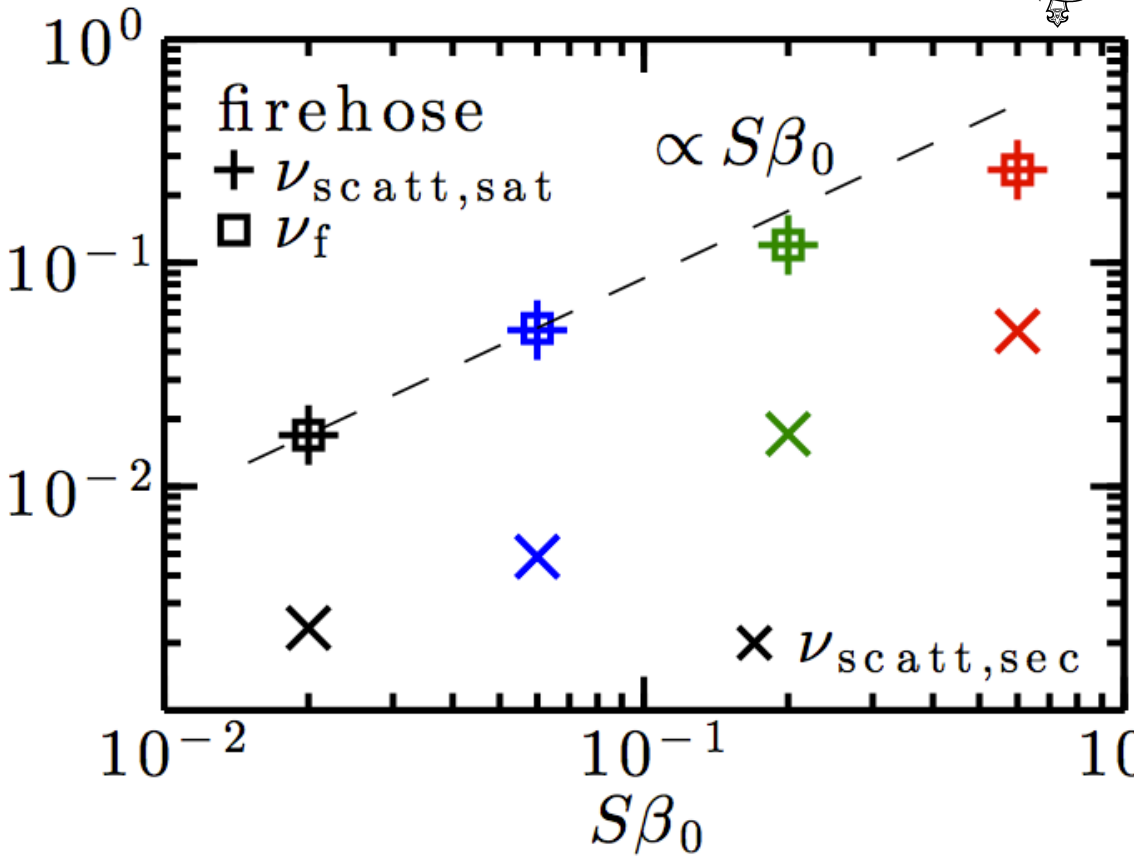
$$\frac{\mathrm{d}\Delta}{\mathrm{d}t} = 3\frac{\mathrm{d}\ln|\langle\mathbf{B}\rangle|}{\mathrm{d}t} - \nu_\mathrm{f}\Delta$$

$$= 0$$

$$\nu_\mathrm{f} = \frac{3}{\Delta}\frac{\mathrm{d}\ln|\langle\mathbf{B}\rangle|}{\mathrm{d}t}$$

$$= -\frac{3\beta}{2}\frac{\mathrm{d}\ln|\langle\mathbf{B}\rangle|}{\mathrm{d}t}$$

$$\sim S\beta$$



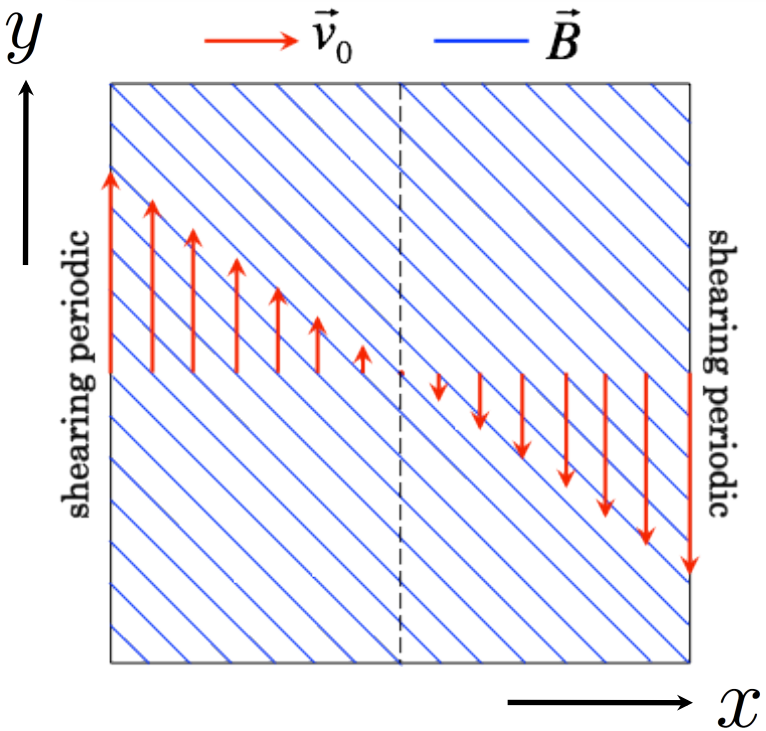
□ effective collisionality required to maintain marginal stability

+ measured scattering rate during the saturated phase

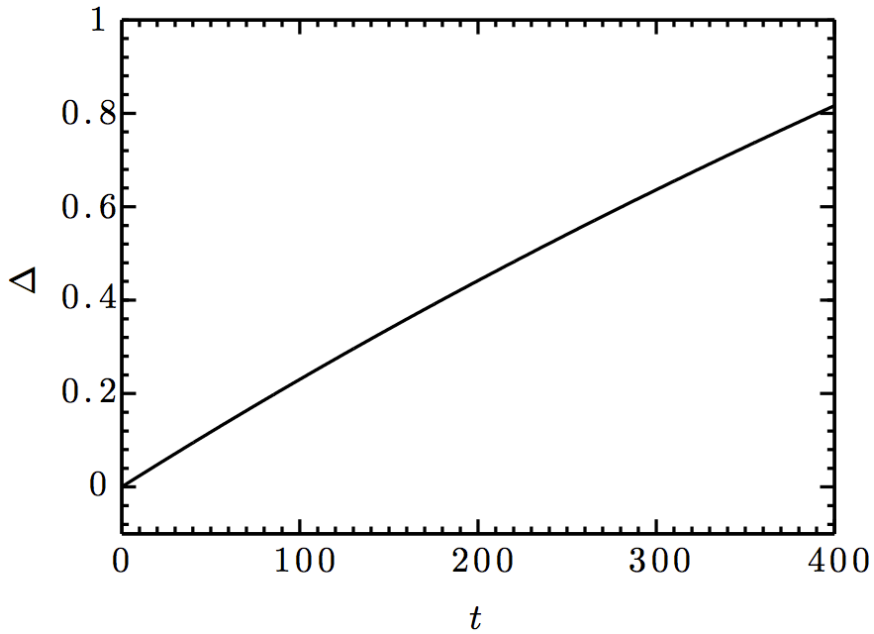
× measured scattering rate during the secular phase

Kunz, AAS & Stone, *PRL* **112**, 205003 (2014) [arXiv:1402.0010]

Mirror Instability



$$\frac{\mathrm{d}B}{\mathrm{d}t} > 0 \quad \Rightarrow \quad \Delta = \frac{p_\perp - p_\parallel}{p} > 0$$



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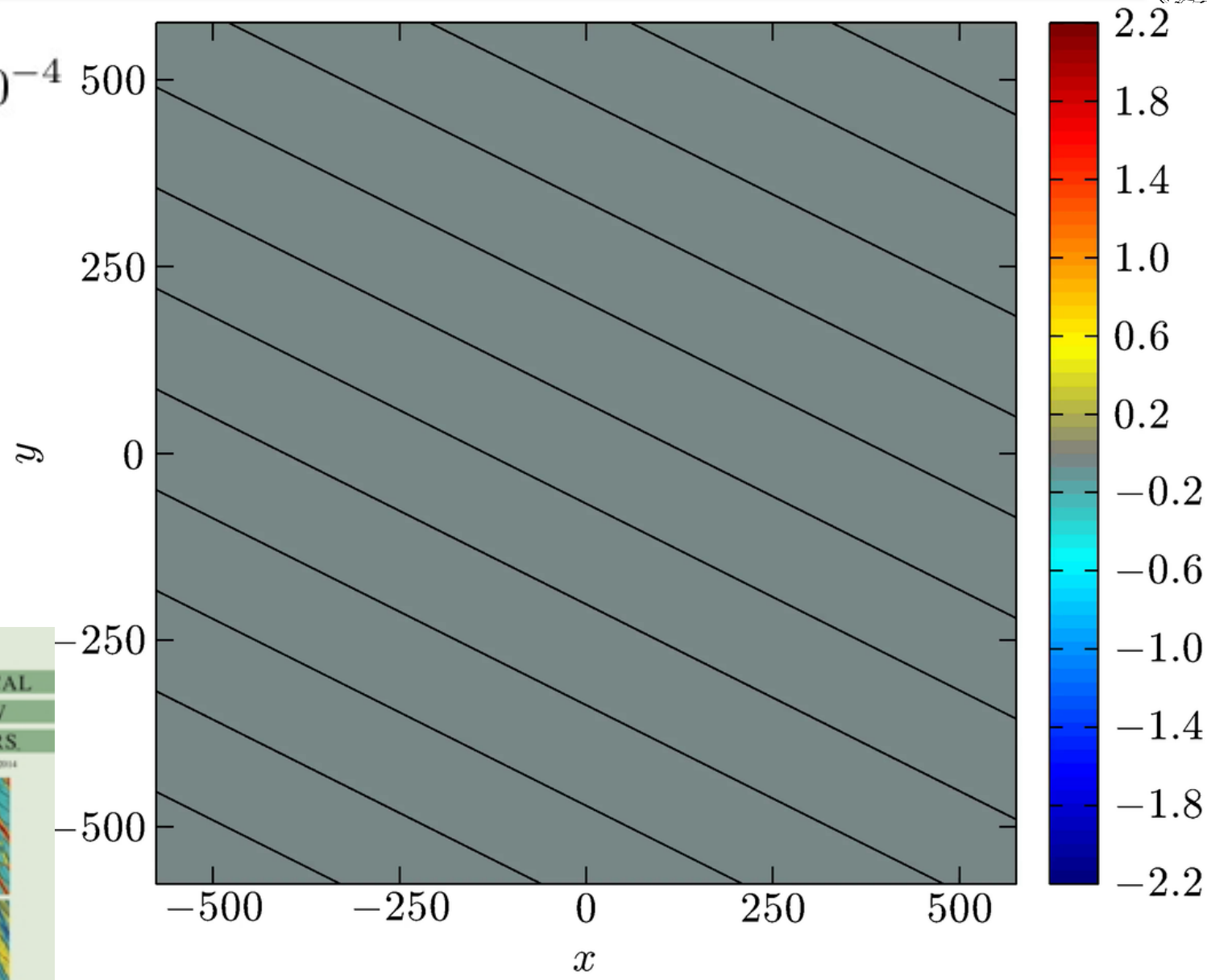
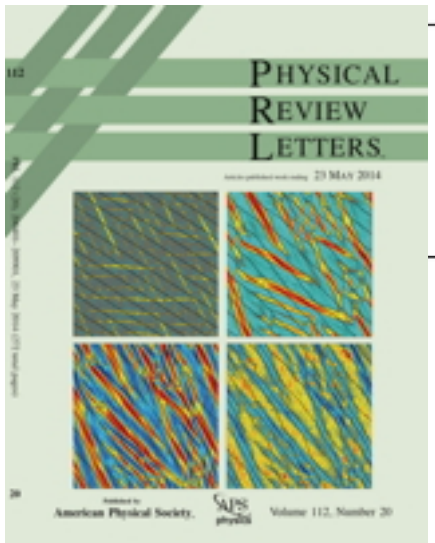
Riquelme, Quataert & Verscharen, arXiv:1402.0014 (2014)

Mirror Instability

$$\frac{S}{\Omega_i} = 3 \times 10^{-4}$$

$$\beta_i = 200$$





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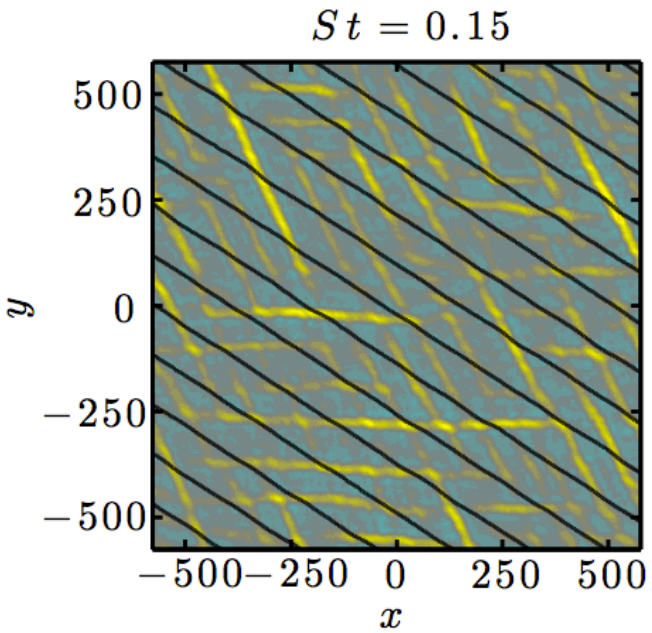
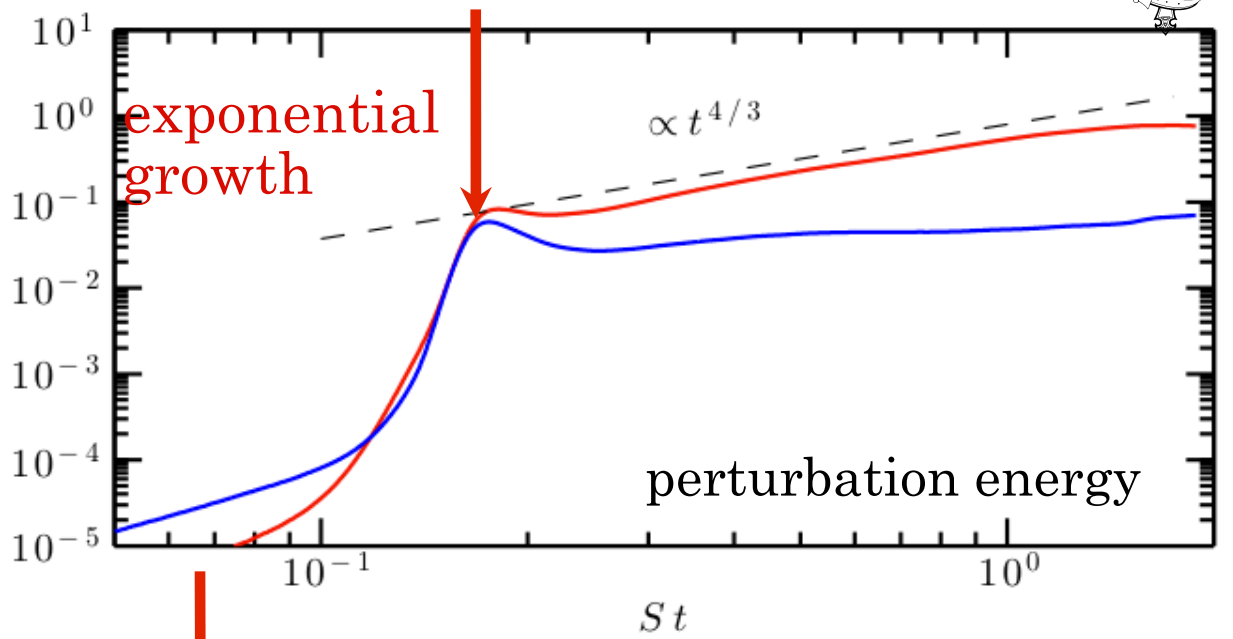
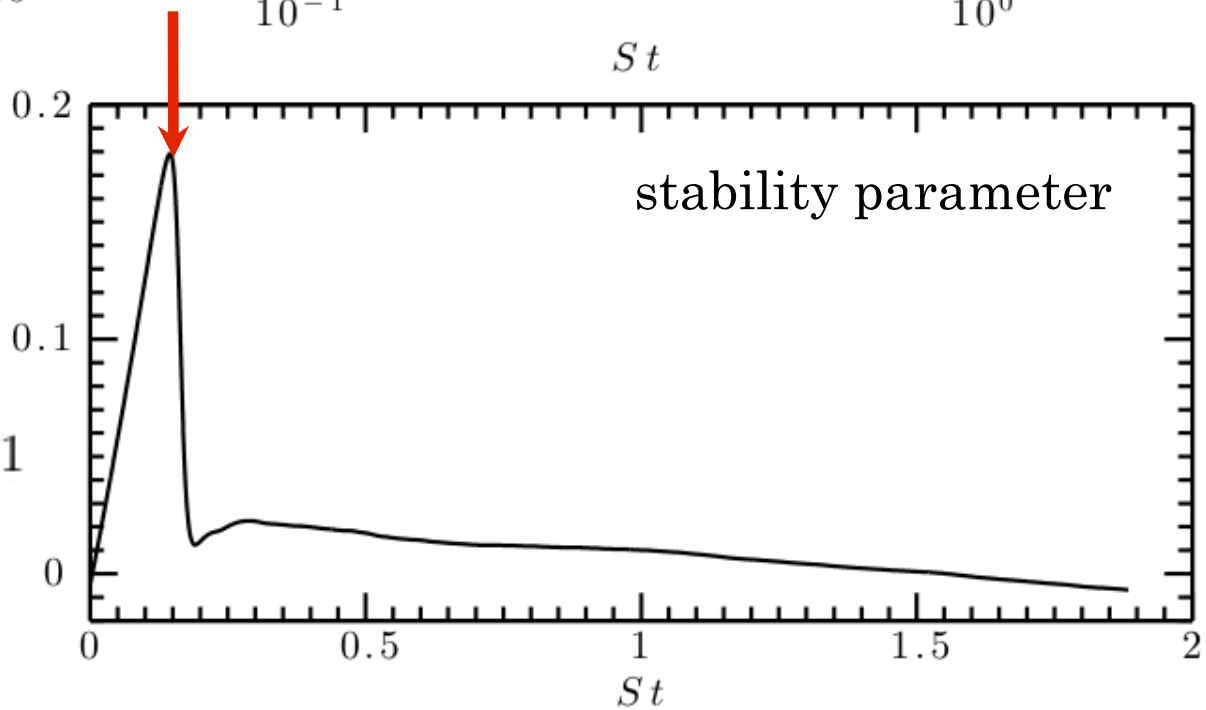
Mirror Instability: Linear







long, oblique modes

$$k_{\parallel}\rho_i \sim \left(\Delta - \frac{1}{\beta}\right)^{1/2} \quad k_{\perp}\rho_i \ll 1$$

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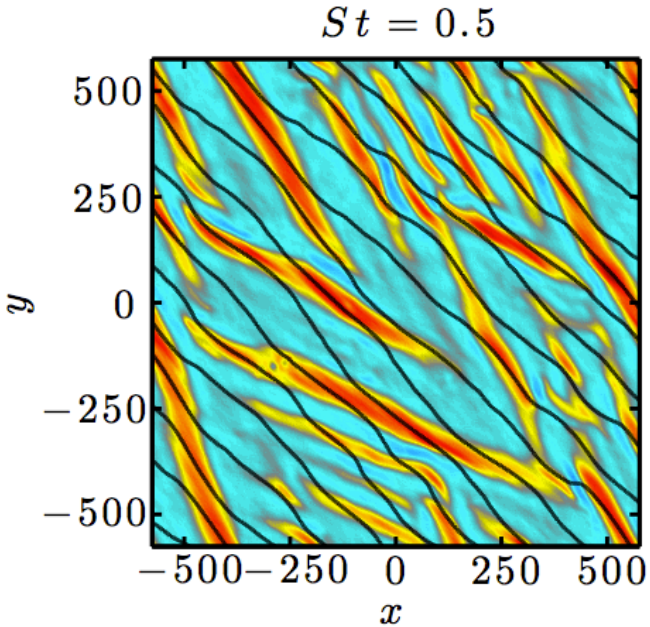
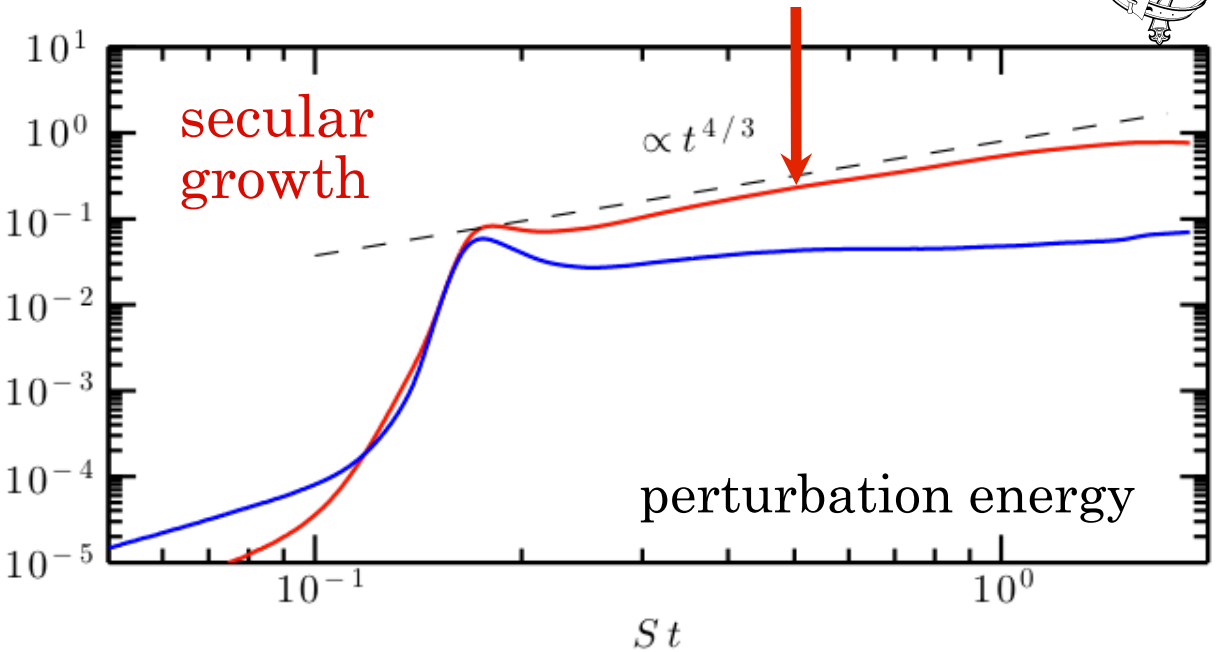
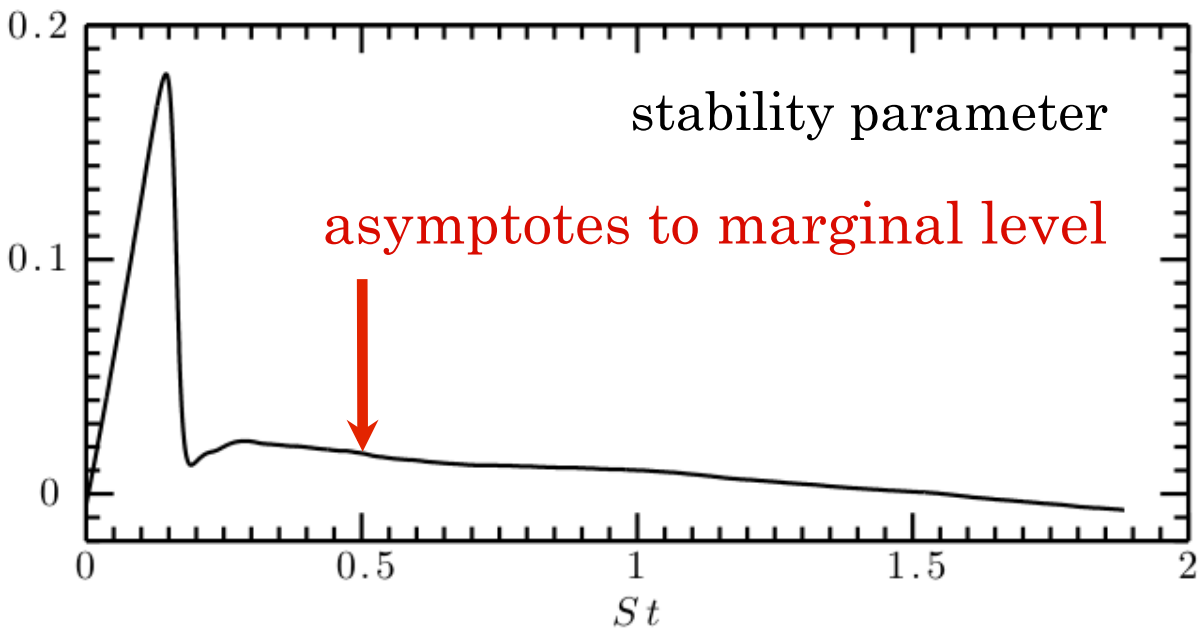
Mirror Instability: Secular







Kunz et al., *PRL* **112**, 205003 (2014)
[arXiv:1402.0010]

Mirror Instability: Secular

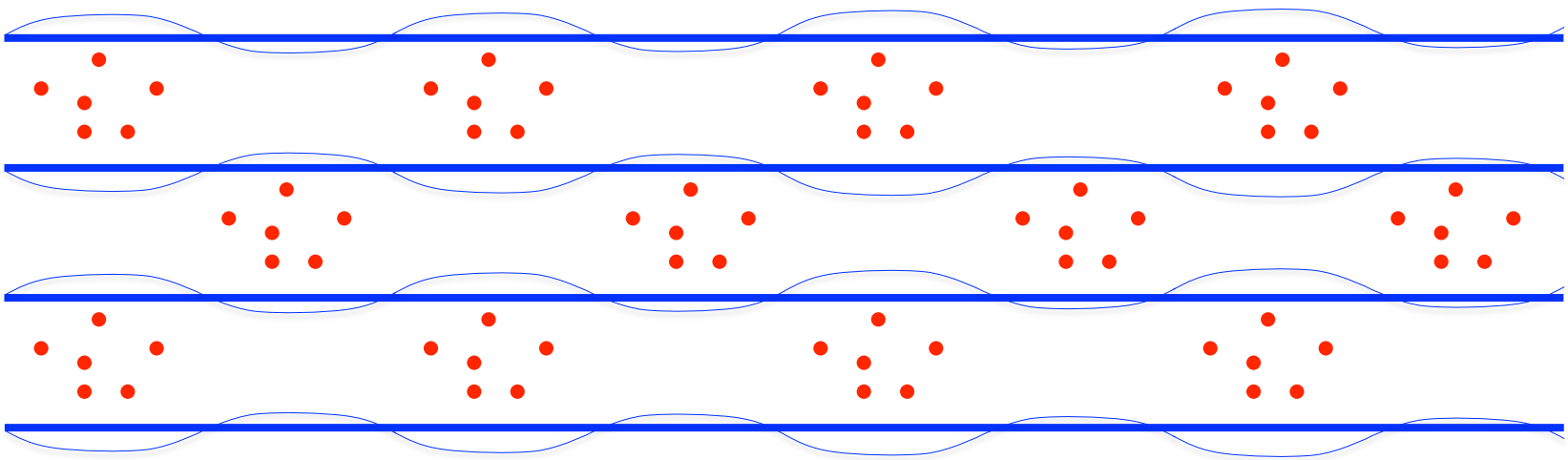
$$\overline{B} = B_0 + \overline{\delta B_{\|}}$$

$$\Delta = 3\int^t \mathrm{d}t' \, \overline{\frac{\mathrm{d}\ln B}{\mathrm{d}t}} = 3\int^t \mathrm{d}t' \left(\frac{\mathrm{d}\ln B_0}{\mathrm{d}t} + \frac{\mathrm{d}}{\mathrm{d}t}\overline{\frac{\delta B_{\|}}{B_0}} \right) \rightarrow \frac{1}{\beta}$$

pressure anisotropy driven by shear

pressure anisotropy from mirror-trapped particles in holes (fraction $\sim |\delta B_{\|}/B_0|^{1/2}$)

marginal stability



Rincon, AAS & Cowley, *MNRAS* **447**, L45 (2015) [arXiv:1407.4707]
Kunz, AAS & Stone, *PRL* **112**, 205003 (2014) [arXiv:1402.0010]

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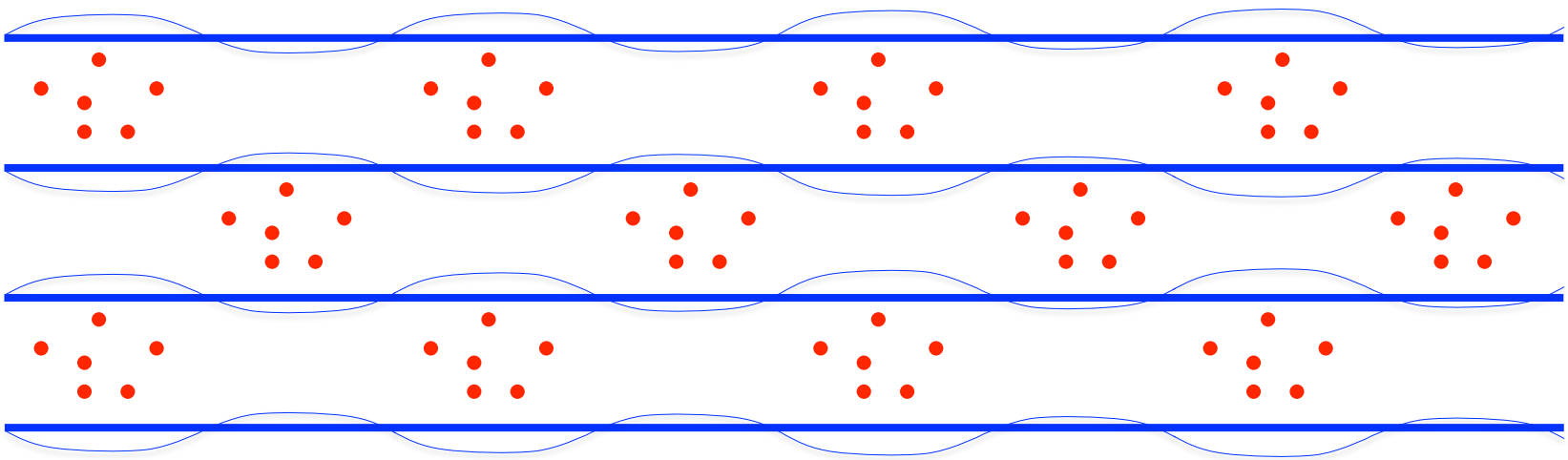

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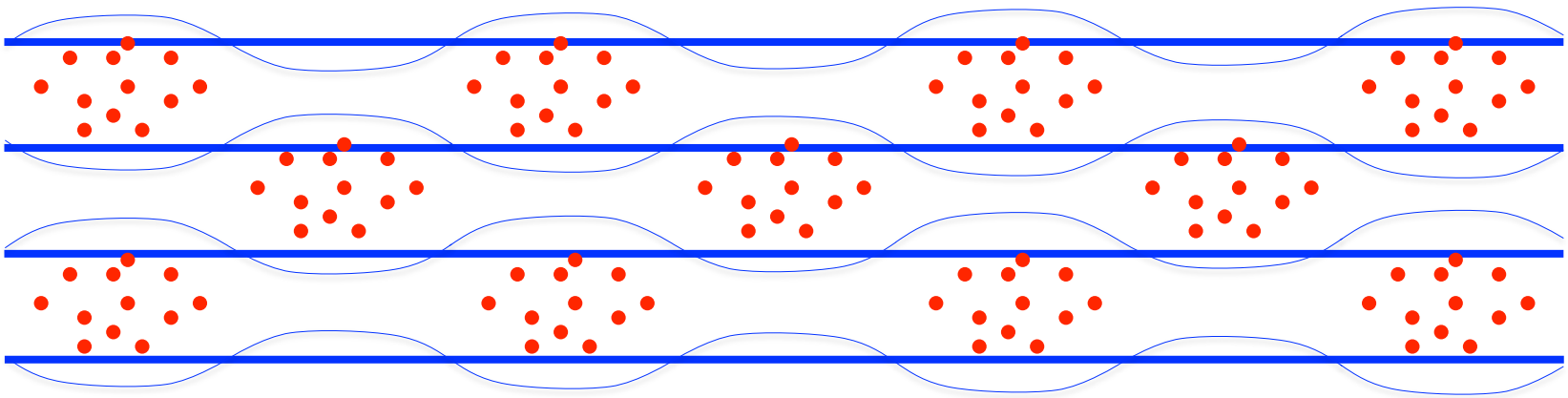

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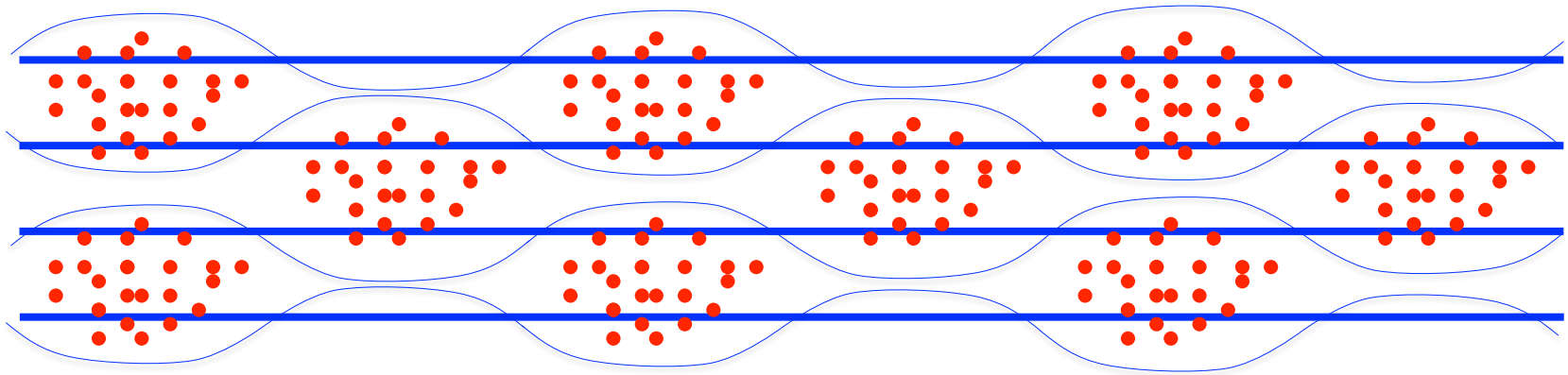

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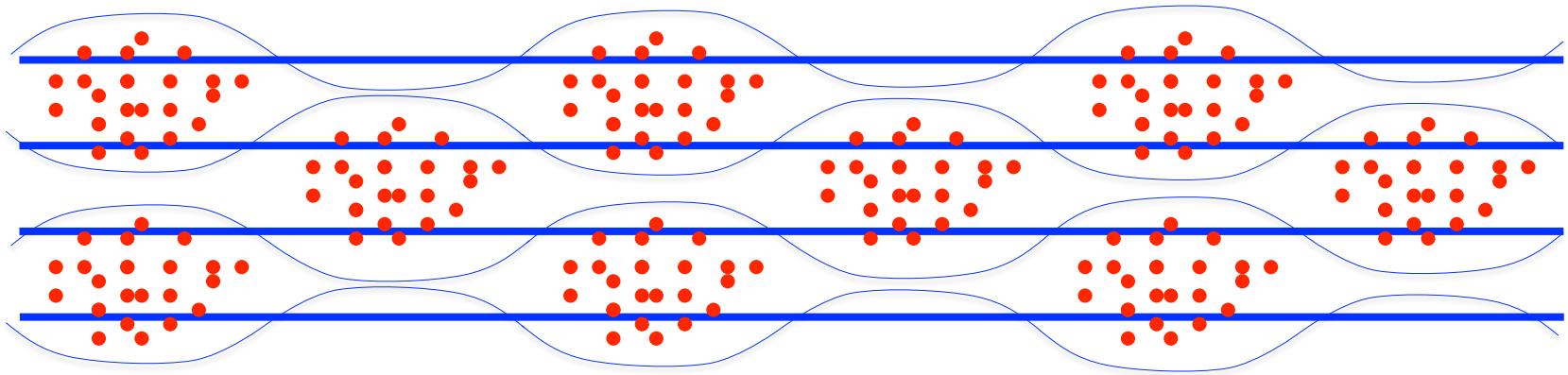
$$\overline{\left|\frac{\delta B_\parallel}{B_0}\right|^{3/2}} = S\int^t \mathrm{d}t' \, \hat{b}_x(t')\hat{b}_y(t') - \frac{1}{\beta} \quad \Rightarrow \quad \frac{\delta B_\parallel^2}{B_0^2} \sim (St)^{4/3}$$

secular growth



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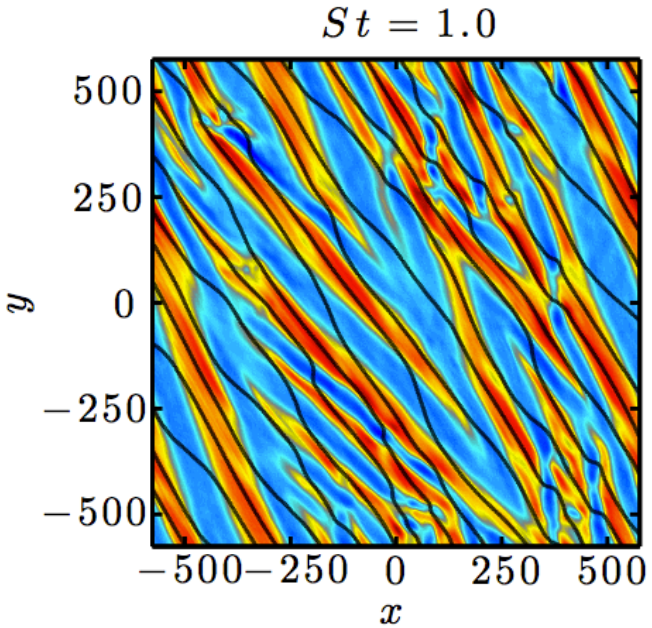

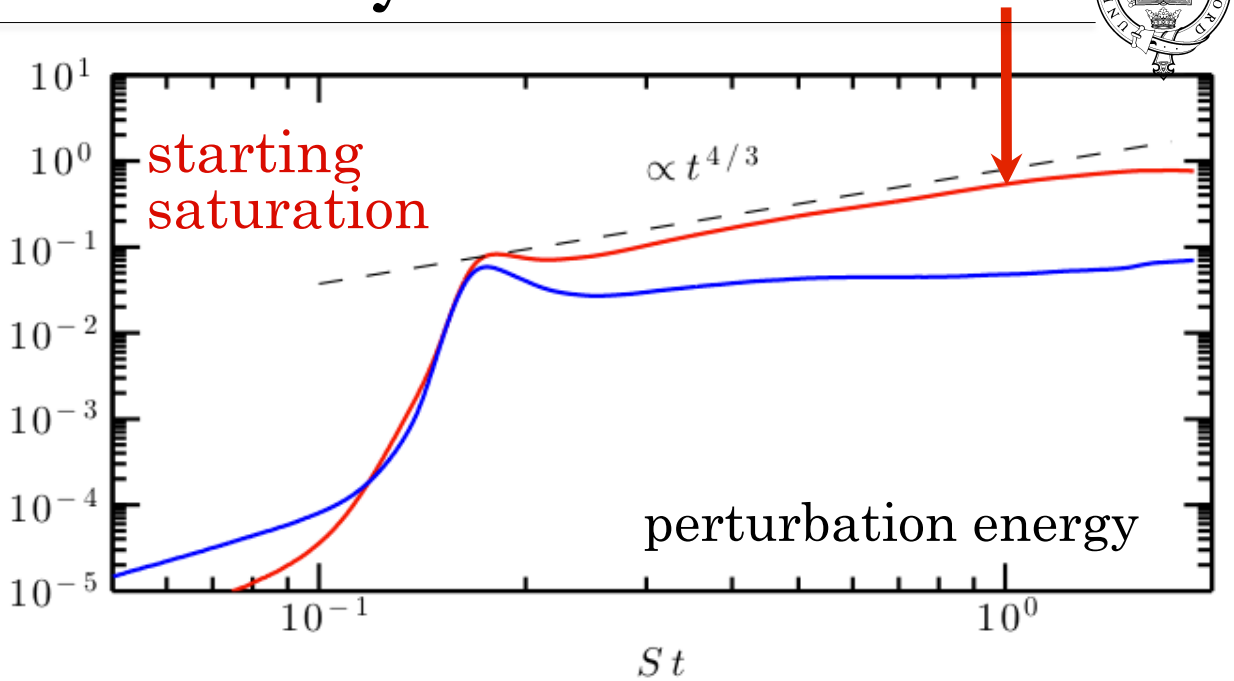

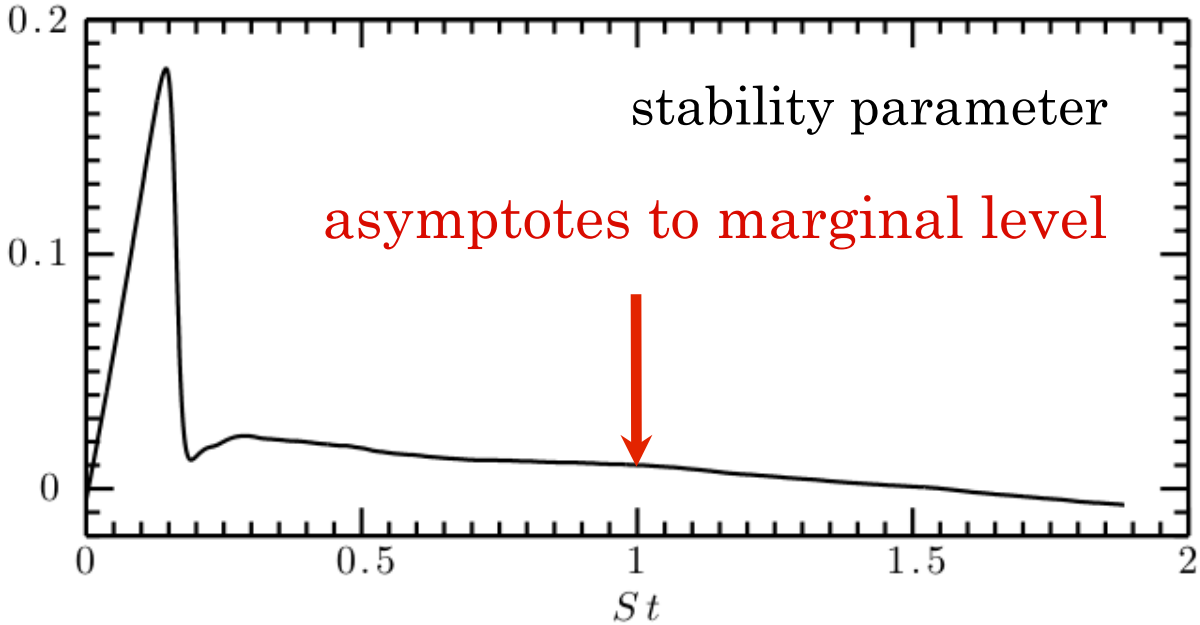

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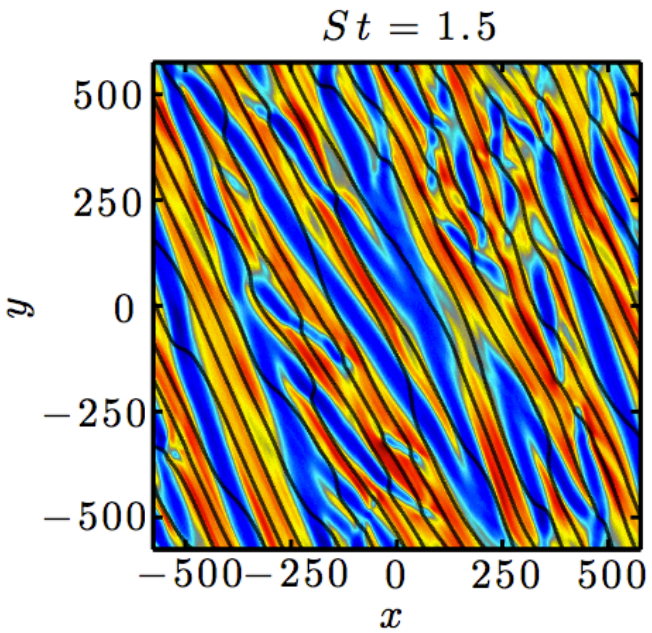
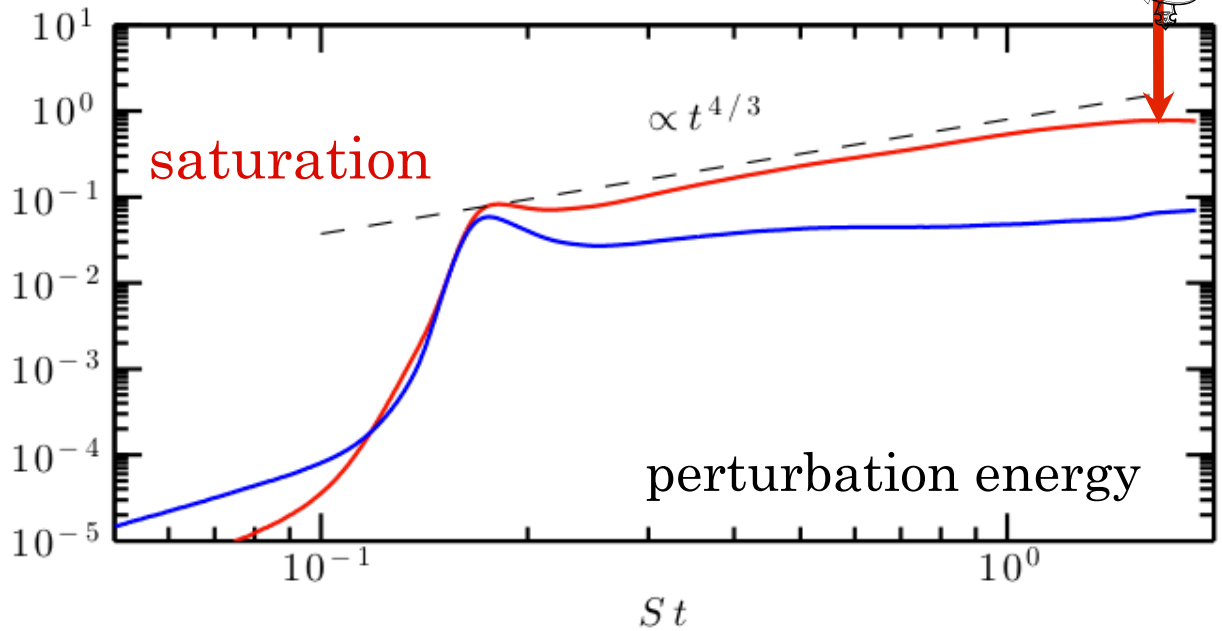
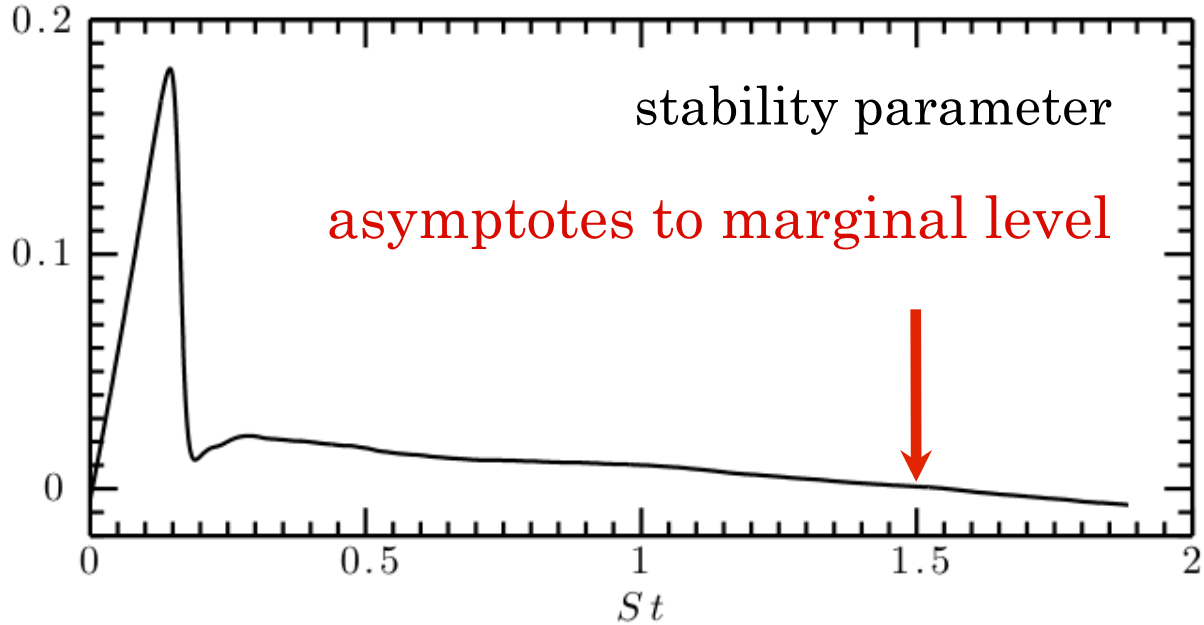
[arXiv:1402.0010]

Mirror Instability: Saturated





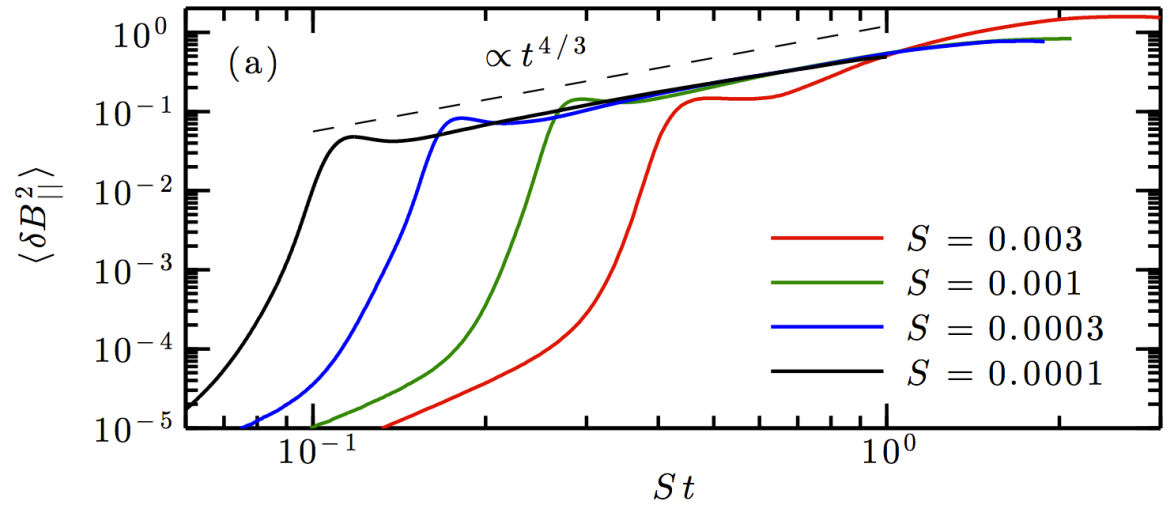


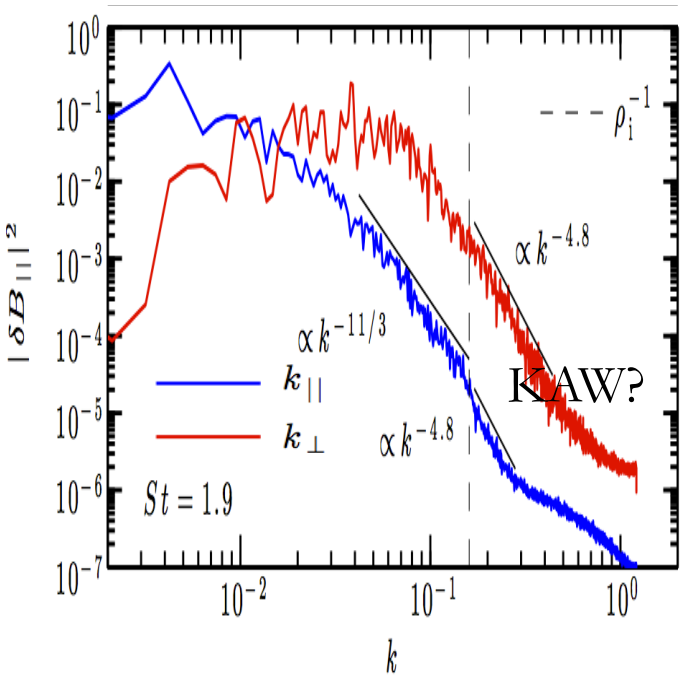
Kunz et al., *PRL* **112**, 205003 (2014)
[arXiv:1402.0010]

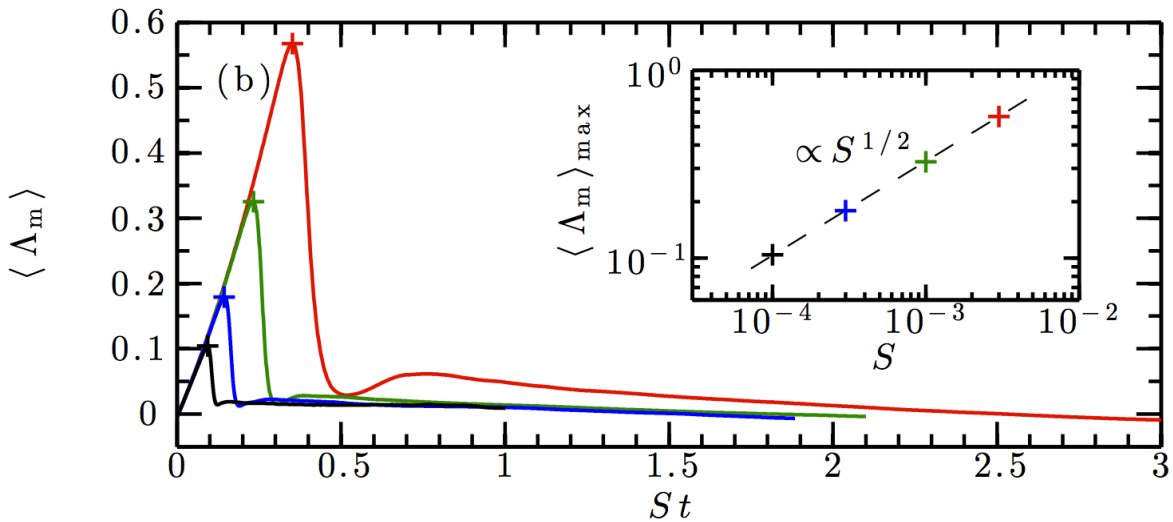
Mirror Saturates at Order-Unity Amplitudes

$$\frac{\langle \delta \mathbf{B}_{\parallel}^2 \rangle}{B_0^2} \sim 1$$

order-unity-amplitude (independent of S) long-parallel-scale mirror turbulence







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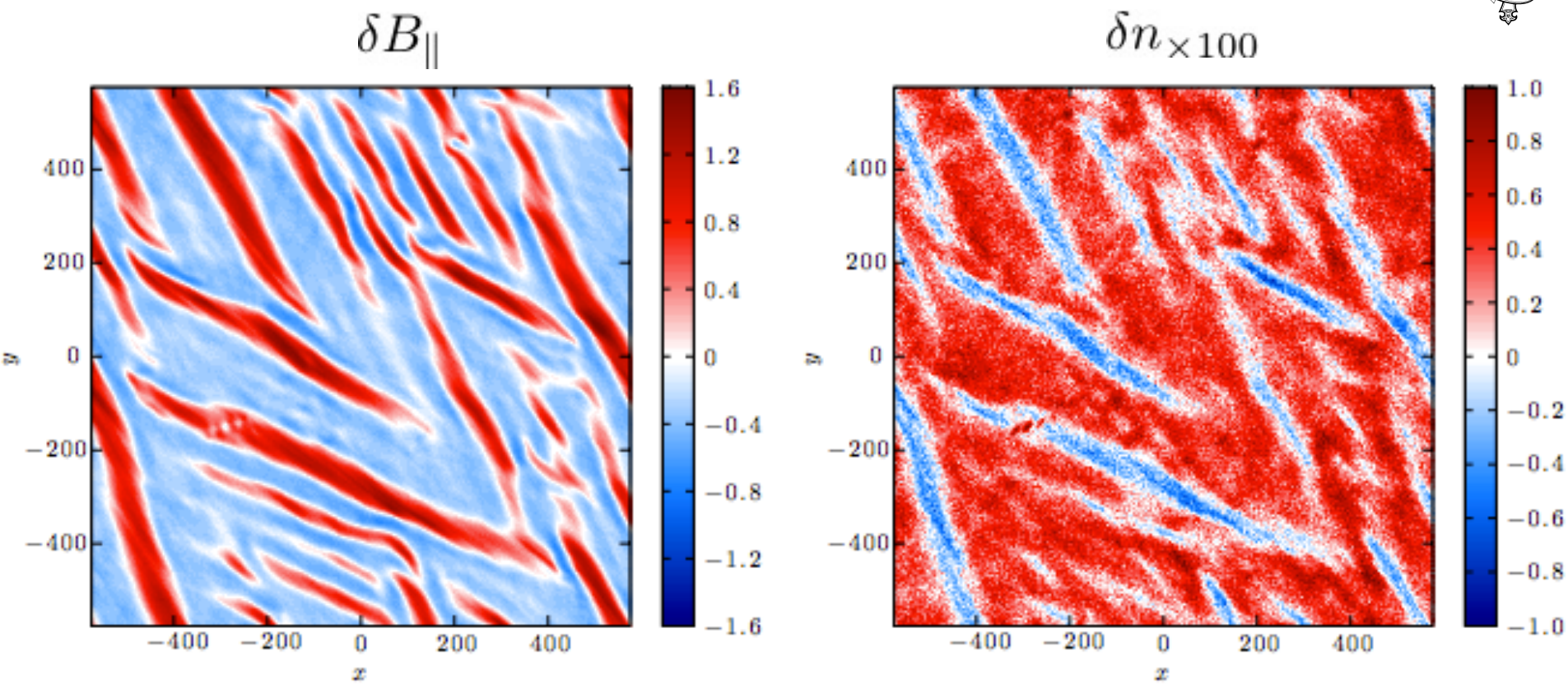
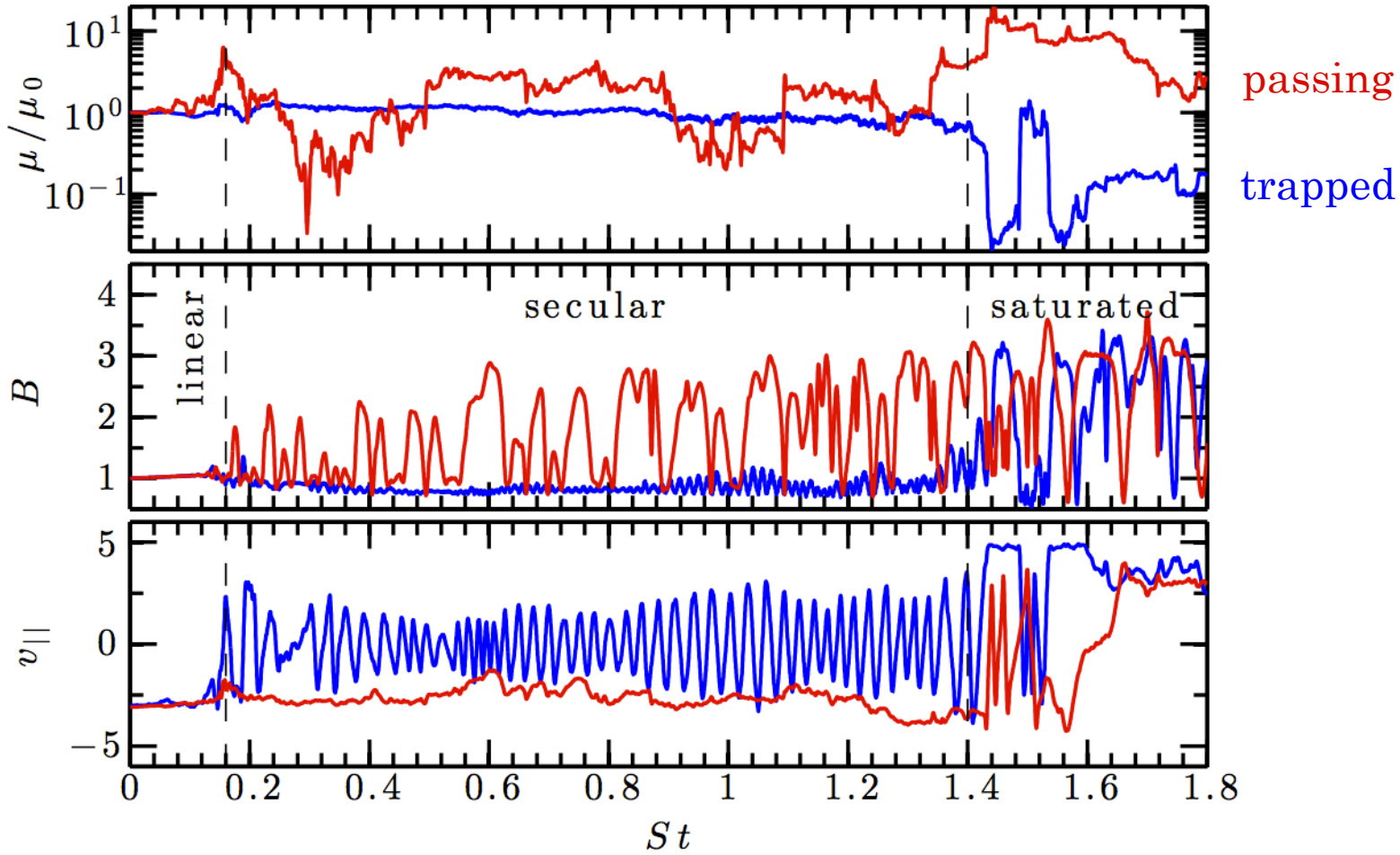
Mirror Instability: Trapped Particles



pressure anisotropy is regulated by trapped particles in magnetic mirrors, where field strength stays constant on average…

Kunz, AAS & Stone, *PRL* **112**, 205003 (2014) [arXiv:1402.0010]

Secular Mirror Doesn't Scatter Particles



pressure anisotropy is regulated by trapped particles in magnetic mirrors, where field strength stays constant on average…

no particle scattering until (late) saturation (off mirror edges)

Kunz, AAS & Stone, *PRL* **112**, 205003 (2014) [arXiv:1402.0010]

Secular Mirror Doesn't Scatter Particles

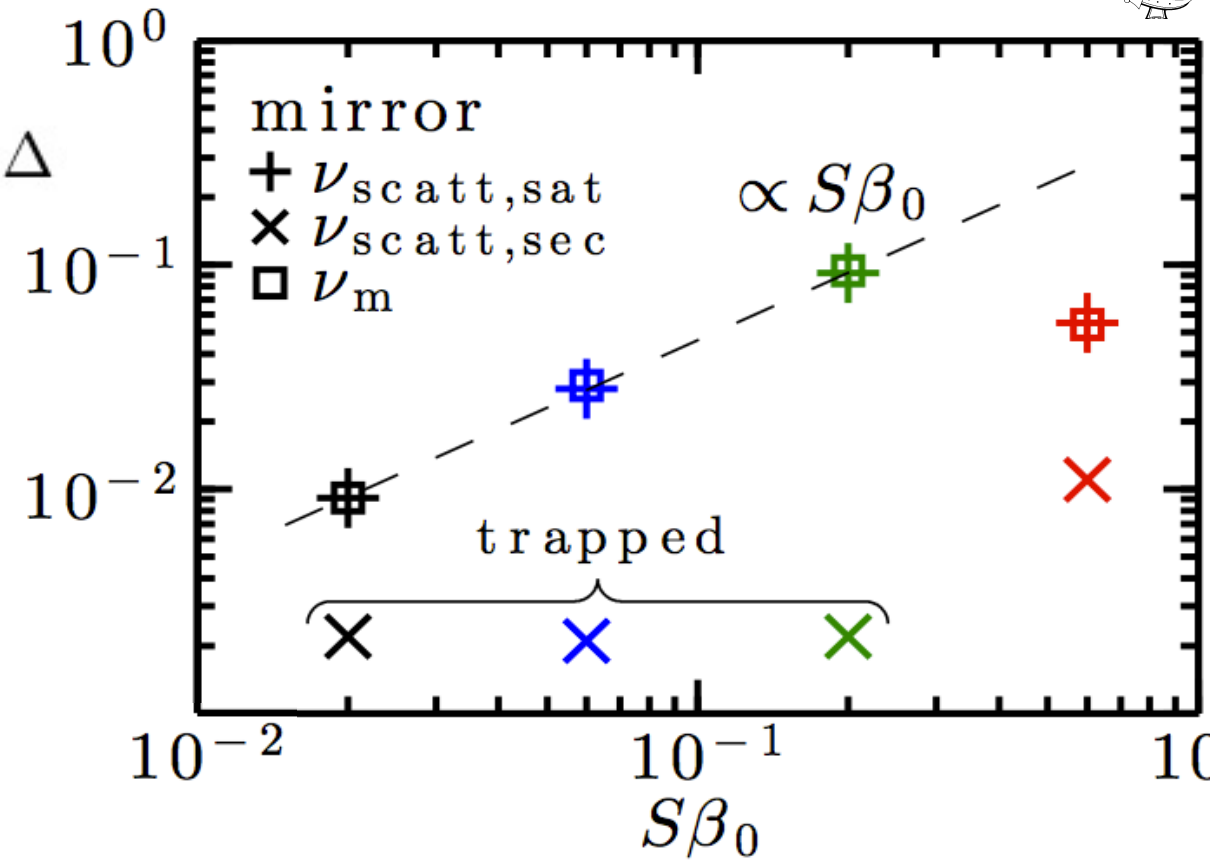
$$\frac{\mathrm{d}\Delta}{\mathrm{d}t} = 3\frac{\mathrm{d}\ln|\langle\mathbf{B}\rangle|}{\mathrm{d}t} - \nu_\mathrm{f}\Delta$$

$$= 0$$

$$\nu_\mathrm{f} = \frac{3}{\Delta}\frac{\mathrm{d}\ln|\langle\mathbf{B}\rangle|}{\mathrm{d}t}$$

$$= 3\beta\frac{\mathrm{d}\ln|\langle\mathbf{B}\rangle|}{\mathrm{d}t}$$

$$\sim S\beta$$



□ effective collisionality required to maintain marginal stability

+ measured scattering rate during the saturated phase

× measured scattering rate during the secular phase

Kunz, AAS & Stone, *PRL* **112**, 205003 (2014) [arXiv:1402.0010]

Conclusions So Far

WE DON'T REALLY KNOW (YET) HOW MAGNETISED, HIGH β PLASMA MOVES

- In a high-beta plasma, any mascroscopic MHD-scale solution involving changing magnetic field or density or significant parallel heat fluxes will be instantly unstable to firehose and/or mirror. We do not know what happens next.
- Mascroscopic dynamics depends sensitively on whether nonlinear firehose and mirror fluctuations regulate pressure anisotropy by scattering particles or by adjusting rate of change of the magnetic field

- Driven **firehose** saturates at low amplitudes, scatters particles
- Driven **mirror** grows to $\delta B/B \sim 1$ without doing much scattering (marginal state achieved via trapped population in mirrors)

- Plasma Dynamo: the race is on!

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