which is comparable to the value 10^{-11} to 10^{-10} $cm^2 \cdot s^{-1}$ determined by Pillath et al. [11] for H_2 molecules in a-C:H layers.

It seems that the background levels in our samples were somewhat lower than those found by Wampler et al. [12] in an investigation of graphite on TFTR. In this case, the deuterium level in the bulk of exposed tiles was found to be higher than that of tiles not exposed to plasma. Further investigations of the possibility of deep penetration of deuterium should be performed. Of special importance is the study of carbon based substrates exposed to a very high hydrogen load (10^{20}–10^{27} m^{-2}) because of the consequences for the use of graphite materials in fusion devices.

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TEARING MODE STABILIZATION BY LOCAL CURRENT DENSITY PERTURBATIONS

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ABSTRACT. The stability of tearing modes is analysed for current density profiles with a small, well localized, additional current around the resonant mode rational surface. The stability parameter Δ' is calculated with a perturbative treatment of the additional current. For the case of a Gaussian profile of the additional current, this is shown to lead to a simple and illustrative condition for the amount of additional current required for stabilization. The consequences of this condition are analysed. In particular, the dependence of the effect of the additional current on its exact location is clearly shown. In the case of a co-driven additional current, far less current is required than in the case of a counter-driven current.

1. INTRODUCTION

Tearing modes are known to play a crucial role in the stability and performance of tokamaks [1–5]. Hence, their stability will be of great importance for future tokamak reactors. It is well known that the stability of tearing modes is very sensitive to the details of the current density profile and, in particular, to the current density gradient around the mode rational surface. In view of this, it has been proposed to generate a small current perturbation around the mode rational surface to stabilize the mode [6]. The most direct way to generate such a perturbation is by means of non-inductive current drive.

An analysis of the requirements for the localization and the amount of non-inductively driven current to stabilize the $m = 2$, $n = 1$ tearing mode was presented previously [7]. Simple criteria for stability, based on intuitive arguments, were given and confirmed by parameter scans. A number of points, however, were not addressed in that paper. In particular, the influence of the strength of the instability in the absence of additional current, the role of shear at the mode resonant surface, and the differences in effectiveness for mode stabilization of co- and counter-driven currents remained unclear. In a recent report, Zakharov and Subbotin [8] analysed the influence of an additional current layer on the stability of tearing modes. On the basis of a perturbative treatment of the additional current layer, they calculated its effect on the stability parameter Δ'. From their results it can be concluded that the additional current required for stabilization is proportional to the square width of the current layer (confirming the conclusion of Ref. [7]), the strength of the instability and the shear at the mode resonant surface. In this Letter, the analysis of Ref. [8] is extended. A particularly illustrative expression for Δ' and a new stability criterion are obtained for the case of a Gaussian profile of the additional current. This criterion shows explicitly the role of the exact position of the additional current and the differences between co- and counter-driven currents. To simplify the complexity of the stability calculations, however, the cylindrical approximation is used here. A comparison with the results of Ref. [8], which include some effects due to toroidicity and non-circularity of the flux surfaces, shows that these effects only lead to an additional geometrical factor of order unity. The consequences of the new stability criterion will be analysed in the context of two different assumptions for the equilibrium current density profiles. Under the assumption of a Gaussian equilibrium current density profile, the stability criterion leads to the same scaling of the current required for stabilization as the one that has been derived in a more intuitive manner in Ref. [7]. On the other hand, the strength of the instability in the case of a simple block profile is used as an indication of the limitations for stabilization of very flat equilibrium current density profiles.

2. STABILITY CRITERION WITH A LOCAL CURRENT DENSITY PERTURBATION

In cylindrical geometry, the stability of the tearing mode with poloidal mode number m and toroidal mode number n is determined by the parameter Δ' [9]

$$\Delta' \equiv \lim_{\epsilon \downarrow 0} \frac{\dfrac{\partial\psi}{\partial r}(r_s + \epsilon) - \dfrac{\partial\psi}{\partial r}(r_s - \epsilon)}{\psi(r_s)} \tag{1}$$

with $\Delta' > 0$ meaning instability. Here, r_s denotes the mode rational surface where $q(r_s) = m/n$, and ψ is the perturbed helical flux, which satsifies

$$\frac{d^2\psi}{dr^2} + \frac{d\psi}{r\,dr} - \left(\frac{m^2}{r^2} + \frac{\dfrac{2\pi}{r}\dfrac{\partial j}{\partial r}}{\dfrac{1}{q} - \dfrac{n}{m}} \right) \psi = 0 \tag{2}$$

The following normalizations are used: The minor radius r is normalized to 1 at the plasma edge and the current density is normalized by the factor $2\pi B_T/\mu_0 R_0$.

A small, well localized current density perturbation is now assumed to affect only $\psi'(r_s)$ and to leave $\psi(r_s)$ unchanged (see Ref. [8]). With this approximation, the following expression for the stability parameter Δ' can be derived:

$$\Delta' = \Delta_0' + P \int_{-\infty}^{+\infty} dx \, \frac{2\pi}{x} \frac{\partial \delta j}{\partial x} \left[\frac{r\,dq}{q^2 dr} \right]^{-1}_{r = r_s} \tag{3}$$

where Δ_0' is the stability parameter in the absence of a current perturbation, δj is the current density perturbation and $x \equiv r - r_s$ is the normalized distance to the rational surface. The extension of the integration domain to infinity is allowed, since the additional current is supposed to be well localized. As in Ref. [7], the current perturbation is assumed to have a Gaussian profile:

$$\delta j = \frac{I_{CD}}{2\pi r_s \sqrt{\pi/\beta}} \, e^{-\beta(x - x_0)^2} \tag{4}$$

where $\beta^{-1/2}$ is a measure of its width and $x_0 = r_{CD} - r_s$ is the position of the centre of the current perturbation with respect to the rational surface. Then, the integration in Eq. (3) can be carried out to obtain

$$\Delta' = \Delta_0' - I_{CD} \left[\frac{r^2\,dq}{q^2\,dr} \right]^{-1}_{r = r_s} 2\beta \times [1 + \sqrt{\beta}\, x_0 \, \mathrm{Re}\, Z(\sqrt{\beta}\, x_0)] \tag{5}$$

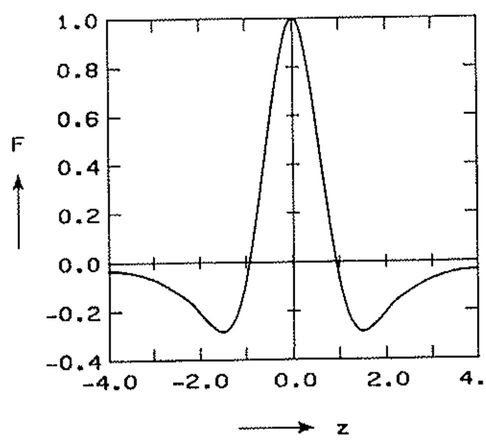

FIG. 1. Profile function $F(z) \equiv 1 + z\,\mathrm{Re}\,Z(z)$.

where $\mathcal{Z}(z)$ is the plasma dispersion function. The profile function, $F(z) \equiv 1 + z \,\mathrm{Re}\, \mathcal{Z}(z)$, is given in Fig. 1. A criterion for the integrated current perturbation I_{CD} required for stabilization can be derived from Eq. (5). Assuming that the mode is located close to the plasma edge, the total plasma current I_p can be approximated by the current inside the mode rational surface, i.e. $I_p \approx r_s^2/q_s$. Then, we obtain the following requirement for stability ($\Delta' \leq 0$):

$$\frac{I_{CD}F(\sqrt{\beta}\, x_0)}{I_p} \geq \frac{\left[\frac{1}{q}\frac{dq}{dr}\right]_{r=r_s} \Delta_0'}{2\beta} \tag{6}$$

This criterion confirms that high peaking factors β are favourable for stabilization [7] and illustrates the role of the strength of the instability and of the shear at the rational surface [8]. In addition, the present result clarifies the differences in the effectiveness for mode stabilization of co- and counter-driven currents.

A number of conclusions can be drawn from Eq. (6) and Fig. 1. The current required for stabilization is proportional to Δ_0' [8], to the shear at r_s [8] and to the square width β^{-1} of the current perturbation [7, 8]. A co-driven current should be centred in the interval $r_s \pm \sqrt{1/\beta}$, while a counter-driven current should be localized around $r_s \pm \sqrt{2/\beta}$ [7]. A current driven in the co-direction is about three times as effective in stabilizing tearing modes as a counter-driven current. When the current is driven far from the rational surface for a given mode, its influence is negligible. Thus, the effect of an appropriately localized current on modes other than the mode that is to be stabilized is negligible.

In the present approximation the results are symmetric around $r = r_s$. This is a consequence of the Gaussian profile for δj (i.e. δj is symmetric around x_0) and of the approximation of the local shear by the shear at the rational surface. In practice, the shear outside r_s will be larger than the shear inside r_s and, consequently, a counter-driven current will be more effective when it is located inside r_s. For example, in the case of current drive by electron cyclotron waves the current drive efficiency is proportional to the electron temperature. In this case, the efficiency with which the required current can be driven is considerably higher inside r_s than outside r_s because of the higher temperature. The effects of counter-driven currents outside r_s will not be considered further.

3. APPLICATION TO A GAUSSIAN EQUILIBRIUM CURRENT PROFILE

For a more quantitative comparison with the results presented in Ref. [7], we concentrate here on the $m = 2$, $n = 1$ tearing mode and consider an equilibrium with a current profile

$$j(r) = j(0)\, e^{-1.5\lambda r^2} \tag{7}$$

The stability of the $m = 2$, $n = 1$ tearing mode is analysed in Ref. [7] over a large range of values for the parameter λ and the position $r_{q=2}$ of the $q = 2$ surface. Typical current profiles as considered for future tokamaks (NET, ITER) would be relatively broad (i.e. $\lambda = 1$–2), with the $q = 2$ surface close to the plasma edge. Such a profile would be well within the unstable parameter regime [7]. For example, assuming $\lambda = 1$ and $r_{q=2} = 0.85$, we obtain a strongly unstable profile with $\Delta_0' = 6.2$. For this profile, the value of q on axis is $q_0 = 1.22$ and that of q at the edge is $q_a = 2.36$. The shear at the $q = 2$ surface is $[r dq/q dr]_{r=r_{q=2}} = 0.9$. After substitution of these values into Eq. (6), we obtain the minimum requirement

$$\frac{I_{CD}}{I_p} \gtrsim \frac{2.5}{\beta} \tag{8}$$

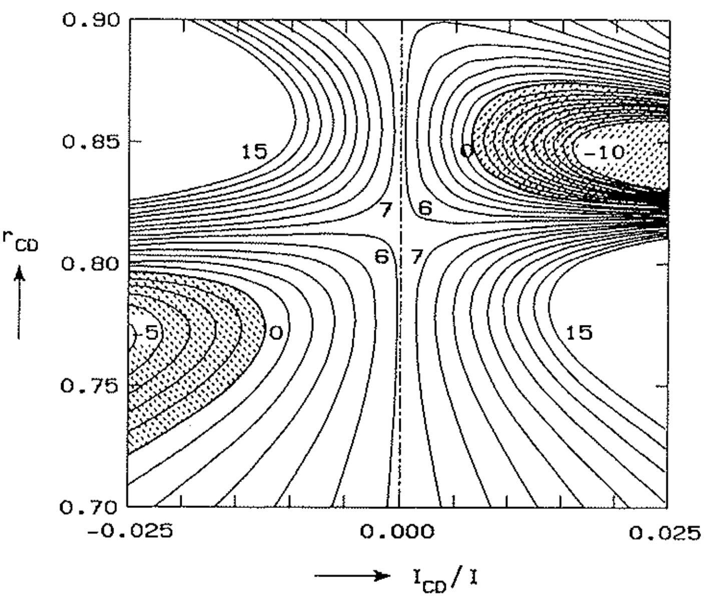

FIG. 2. Results of stability calculations for $\lambda = 1$ and $\beta = 400$. Contours of constant Δ' are given. The shaded area indicates the stable parameter regime.

This result is also in excellent agreement with the results presented in Ref. [7]. As a further check on these results, the effect on the instability of a current perturbation with peaking factor $\beta = 400$ is calculated for a scan of the driven current I_{CD} and its position r_{CD}. The full equations (1) and (2) are used to calculate Δ'. The results are given in Fig. 2 as level curves of Δ' in the I_{CD}–r_{CD} plane. The shaded areas indicate the stable parameter regime. These results clearly confirm, both qualitatively and quantitatively, the criteria for stabilization given above. The positions and widths of the stable regions are as expected from the profile function $F(\sqrt{\beta}\, x_0)$, and the current needed to obtain stability is a factor of two lower in the co-direction than in the counter-direction.

4. APPLICATION TO A BLOCK SHAPED EQUILIBRIUM CURRENT PROFILE

The Gaussian equilibrium current profile used above is, of course, only one special case out of all possible current density profiles. As an alternative model for a broad current density profile, one could use block profiles with a constant and finite current inside the radius r_c and zero current outside this radius (see Ref. [6]). This has the advantage that the stability parameter Δ_0' is known analytically, so that the dependence of the stability requirements on the equilibrium profile parameters can be more easily assessed.

In the case of a block profile, the stability of a given mode with mode numbers m and n at the rational surface r_s is given by

$$\Delta_0' = \frac{-2m\left[1 - \frac{1}{m} - \frac{nq_0}{m}\right]}{r_s\left[1 - \frac{1}{m} - \frac{nq_0}{m} + \frac{1}{m}\left(\frac{nq_0}{m}\right)^{m}\right]} \qquad (9)$$

where q_0 is the safety factor on axis, corresponding to the assumed current density inside r_c. Instability is found for $0 < m - nq_0 < 1$ [6], where the lower limit is due to the absence of a rational surface with $q = m/n$ for $q_0 > m/n$. Note, however, that when q_0 approaches m/n from below, Δ' can become arbitrarily large. In this case the mode will become increasingly more difficult to stabilize. This situation is further aggravated by the fact that the current on the rational surface is zero and, consequently, the shear is maximum, $dq/dr = 2q/r$. For example, for the $m = 2$, $n = 1$ mode, Eq. (9) becomes

$$\Delta_0' = \frac{-4\,(1 - q_0)}{r_s\,(1 - \frac{1}{2}q_0)^2} \qquad (10)$$

which means that already for $q_0 \gtrsim 1.2$ the current required to stabilize the $m = 2$, $n = 1$ tearing mode becomes larger than $8I_p/\beta$, which is more than three times the minimum required current as estimated above on the basis of a Gaussian current density profile.
As pointed out in Ref. [6], however, the stability of tearing modes is significantly improved when there is a modest pedestal in the current density profile. In the present case, such a pedestal not only helps to reduce Δ_0' but also reduces the shear at the rational surface and, thereby, the current required for stabilization.
In a tokamak reactor, such a pedestal may well be provided by the bootstrap current, which will be highest near the outer plasma region where the density gradient is expected to be relatively steep. Nevertheless, the requirements for stabilization of the $m = 2$, $n = 1$ tearing mode may well become prohibitively large in the case of broad current density profiles with high q_0 (≈ 1.5 or more) but with very little or no current outside the $q = 2$ surface. It seems advisable to avoid such current profiles in a tokamak reactor.

5. CONCLUSIONS AND DISCUSSION

We have derived an expression (Eq. (5)) for the tearing mode stability parameter Δ' in a plasma with an additional current that is localized near the mode rational surface. The stability requirement $\Delta' \leq 0$ then leads to a criterion for the additional current (Eq. (6)). This criterion shows that the current required for stabilization is proportional to the total plasma current and to the square width of the profile of the additional current. This confirms the results of earlier work [7] on the stabilization of the $m = 2$, $n = 1$ tearing mode. In agreement with the results of Ref. [8], the required current is found to be proportional to the strength of the instability Δ_0' in the absence of additional current and proportional to the shear at the mode rational surface. In addition, the difference in the mode stabilization effectiveness of co- and counter-driven currents is shown explicitly, i.e. co-driven currents are far more effective than counter-driven currents. Application of these results to the $m = 2$, $n = 1$ tearing mode in the case of a relatively broad Gaussian equilibrium current density profile also provides a quantitative confirmation of the results presented in Ref. [7]. However, in the case of block shaped equilibrium current profiles, very large values of Δ_0' are possible. In this case, the current required for stabilization of the m = 2, n = 1 tearing mode could well become prohibitively large. In practice, such profiles are not likely to occur. In any case, a significant current density outside the q = 2 surface, which has a stabilizing effect [6], is expected to be present.

For a practical application of this method of mode stabilization it is necessary to have an efficient method to generate the required current density perturbation. As shown in Ref. [10], non-inductive current drive by electron cyclotron waves is a prime candidate for generating such a current. Because of the good localization of wave-particle interaction in the electron cyclotron range of frequencies, narrow current profiles with peaking factors $\beta \approx 400$ can easily be generated around the q = 2 surface without excessive demands on the wave launching system. A detailed analysis of this application to NET is presented in Ref. [10].

Finally, it must be mentioned that all considerations so far are based on linear theory. In particular, the presence of finite size magnetic islands has important consequences for the effects of the additional current. When the island size becomes comparable to the width of the additional current, it no longer affects the stability parameter Δ', but rather changes the internal non-linear dynamics of the magnetic island. For example, if the additional current were driven inside a magnetic island itself [11] it would lead to a reduction of the growth rate and, possibly, a total suppression of the mode, provided the current is driven in the co-direction. On the other hand, if this current were located close to an X-point between two magnetic islands, it would have exactly the opposite effect, or else it would have to be driven in the opposite direction to suppress the mode.

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* Note that Figs 3 and 4, but not their captions, have been interchanged in this paper.

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