BALLOONING THEORY: Part 2

- HIGHER ORDER THEORY AND RADIAL STRUCTURE

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1. RESUME OF PART 1: THE LOWEST ORDER THEORY

- Anomalous transport associated with micro-instabilities such as the electron drift wave
  - driven by electron Landau resonance, with long parallel wavelengths to minimise ion Landau damping

$$V_{Ti} < \frac{\omega}{k_{||}} < V_{Te}$$

- Pressure driven MHD instability depends on competition between destabilising effect of a pressure gradient in a region of unfavourable curvature and the stabilising effect of field line bending $\propto \frac{k_{\parallel}^2 B^2}{2\mu_0}$

  - so $k_{\parallel}$ plays an important role again and the most unstable modes have the smallest $k_{\parallel}$
  - find $k_{\parallel} \sim \frac{1}{Rq}$

- This talk will explore how the stability and mode structure responds to realistic magnetic geometry and radial profiles
  - leads to ballooning theory and more recent developments in this topic

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GEOMETRY

Cylinder

- $n(r)$, $T(r)$ and $q(r)$: $q = \frac{r B_z}{R B_\theta}$
- Fourier analyse: $\varphi = \varphi(r) e^{-i(m\theta - nz/R)}$
- $$k_{\|} = \frac{1}{Rq}\left(m - nq(r)\right)$$
- For electron Landau drive and to minimise ion Landau damping

$$V_{Ti} < \frac{\omega}{k_{\|}} < V_{Te}$$ ⇒ long parallel wavelength

- Shear $s = \frac{r}{q}\frac{dq}{dr} \neq 0 \Rightarrow$ mode localised around resonant surface

$$r_0 : m = nq(r_0)$$

$$\Rightarrow k_{\|} = -\frac{(nq)}{r}\frac{(r - r_0)}{L_s} \quad , \quad \frac{1}{L_s} = \frac{s}{Rq}$$

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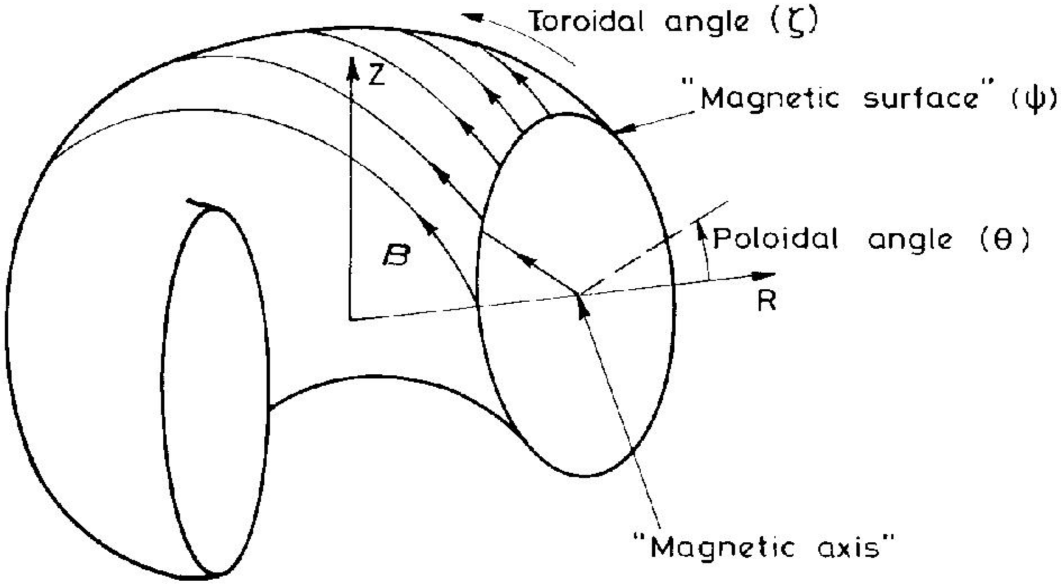
Axisymmetric Torus, B(r,θ)



- $\varphi = \varphi(r, \theta)e^{in\zeta}$
- 2D (r,θ) – periodic in θ ; different poloidal m coupled
- High-n – simplify with eikonal

$$\varphi(r,\theta) \sim \tilde{\varphi}(r,\theta)e^{inS(r,\theta)}$$

- Problem is to reconcile this with small $k_{||}$ and periodicity

$$B \cdot \nabla S = 0 \Rightarrow S = \zeta - q(r)\theta$$

but not periodic! ; shear ⇒ $q(r) \neq q(r_0)$

Preview of Model Drift Wave Eigenvalue Equation

$$\left\{\frac{\partial^2}{\partial X^2}-\sigma^2\left(\frac{\partial}{\partial\theta}+iX\right)^2-\kappa_1 X-\kappa_2 X^2-2\varepsilon\left(\cos\theta+i s\sin\theta\frac{\partial}{\partial X}\right)+i\gamma_e-\Lambda\right\}\varphi(X,\theta)=0$$

FLR	Ion Sound	Radial variation due to ω_* or Ω_E	Toroidicity	Electron Drive	Eigenvalue $\Lambda(\Omega)$

- $\mathbf{X = nq'\,(r - r_0)}$ $\quad \Rightarrow \Delta x = 1/nq'$

- Resonant surfaces

- **Different cases, depending on magnitudes of ε, κ_1, κ_2**

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SHEARED SLAB/CYLINDER

- Include magnetic shear, $s \neq 0$, and density profile $n(r)$

- Eigenvalue equation

$$\left\{ \frac{d^2}{dX^2} + \sigma^2 X^2 - \kappa_2 X^2 + i\gamma_e(X) - \Lambda(\Omega) \right\} \varphi(X) = 0$$

FLR shear density profile electron Landau drive eigenvalue

(shear, density profile and electron Landau drive terms:) Potential $Q(X)$

- 1D problem
- Seek localised (or ‘outgoing wave’) solutions
- $\kappa_2 << 1$, have outgoing wave solutions: $\varphi(x)=\exp(-i\sigma X^2/2)$
- Outgoing waves exhibit ‘shear damping’

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QUASI-MODES

- In periodic cylinder

$$k_{\|} \propto m - nq\,(r)$$

- Fix n, different m have resonant surfaces r_m: $n = q(r_m)$

- For large n, m they are only separated by

$$\frac{\Delta X}{r} \sim \frac{1}{nq'} << 1 \qquad \Rightarrow \quad \kappa_2 \sim 0(1/n^2) \ll 1$$

- Then each radially localised ‘m-mode’ ‘looks the same’ about its own resonant surface

- Each mode has almost same frequency – i.e. almost degenerate and satisfies

$$\frac{k_{\|}}{k_{\perp}} \sim \frac{\rho_i}{L_n} << 1$$

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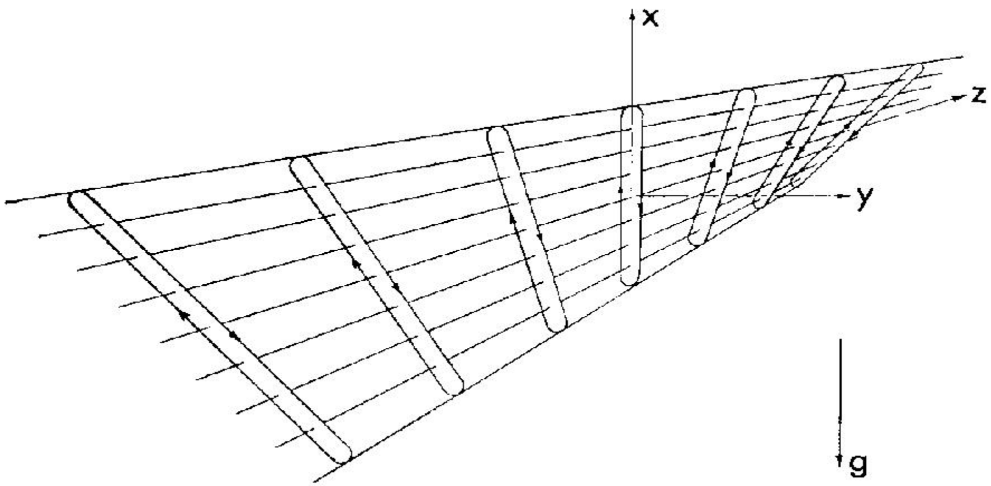
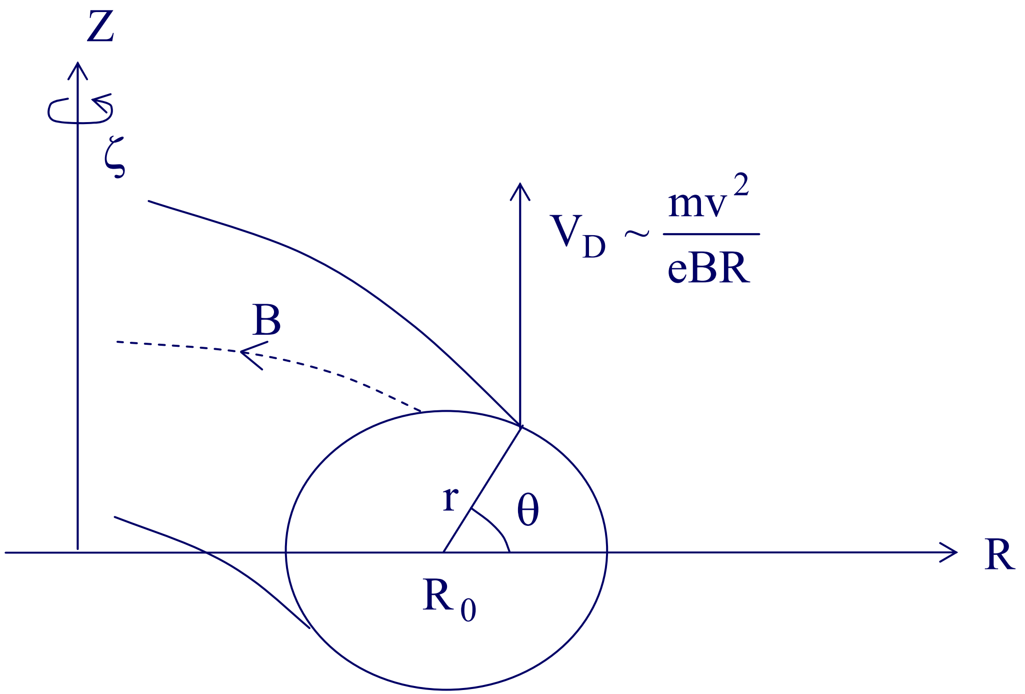
- Roberts and Taylor realised it was possible to superimpose them to form a radially extended mode
  - the twisted slicing or quasi-mode which maintains $k_{\parallel} << k_{\perp}$



TOROIDAL GEOMETRY

- In a torus new effects arise from inhomogeneous magnetic fields
  - changes in mode structure from magnetic drifts: affects shear damping

- $B_\zeta \sim \frac{B_0}{R} \Rightarrow$ a vertical drift



- Doppler shift: $\omega \to \omega$ - **k** . **v**$_D$

$$\varepsilon_T\left(\cos\theta + i s \sin\theta \frac{\partial}{\partial x}\right) \quad ; \quad \varepsilon_T = \frac{r}{R} << 1$$

normal geodesic curvatures

- φ (r) $\to$ φ(r,θ): two-dimensional

$$\Rightarrow k_{\|} \to \frac{1}{Rq}\left(-\frac{i\partial}{\partial\theta} + m - nq(r)\right) \propto \left(-\frac{i\partial}{\partial\theta} + X\right)$$

- Eigenvalue equation (absorb γ_e into Λ)

$$\left[\frac{\partial^2}{\partial X^2} - \sigma^2\left(\frac{\partial}{\partial\theta} + iX\right)^2 - 2\varepsilon\left(\cos\theta + i s \sin\theta \frac{\partial}{\partial X}\right) - \kappa_2 X^2 - \Lambda(\Omega)\right]\varphi = 0$$

$\varepsilon \sim 0(1)$

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EIKONAL SOLUTION

- To minimise $k_{\square}$ try $\varphi \sim \varphi(\theta) e^{-iX\theta + i\int^{X} k dX}$ to obtain

$$\left\{ \sigma^2 \frac{\partial^2}{\partial\theta^2} + (\theta - k)^2 + 2\varepsilon[\cos\theta + s(\theta - k)\sin\theta] + \Lambda(\Omega) + \kappa_2 X^2 \right\} \varphi = 0$$

$\kappa_2 << \varepsilon$ - one-dimensional equation in θ

But solution must be periodic in θ over 2π

- must reconcile with secular terms!

- Solve problem with Ballooning Transformation (Connor, Hastie, Taylor)

$$\varphi(X,\theta) = \sum_m e^{-im\theta} \underbrace{\int_{-\infty}^{\infty} d\eta e^{-i(X-m-k)\eta}}_{\varphi_m(X)} \tilde{\phi}(\eta, X; k)$$

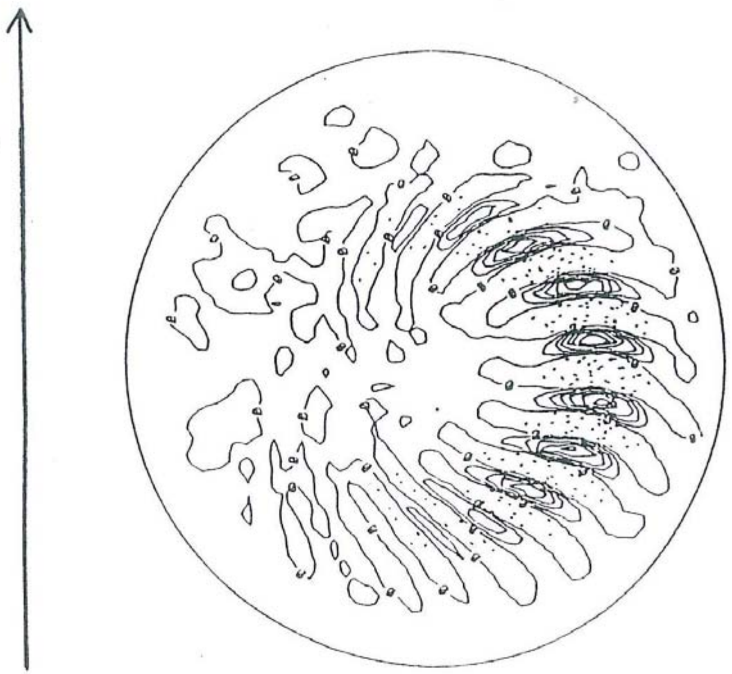
$$\left\{\sigma^2 \frac{\partial^2}{\partial \eta^2} + \underbrace{(\eta - k)^2 + 2\varepsilon(\cos\eta + s(\eta - k)\sin\eta)}_{\text{Potential } Q(\eta)} + \Lambda + \kappa_2 X^2\right\}\hat{\varphi}(\eta, X, k) = 0$$

Potential $Q(\eta)$

$-\infty < \eta < \infty \Rightarrow$ no longer need periodic solution!

- Often consider 'Lowest Order' equation, $\kappa_2 = 0$
  - 1D Schrődinger equation with potential $Q(\eta)$ with k a parameter, $\Lambda(\Omega)$ a 'local' eigenvalue
  - k usually chosen to be 0 or π (to give most unstable mode)
  - localisation of solutions in η reduces shear damping (waves reflected by toroidal coupling)

- Reconstructed φ(X,θ) is a quasi-mode
  - ‘balloons’ in θ
  - ‘m’ varies with X
  - radially extended
    - ⇒ determine slowly varying radial envelope A(m) by reintroducing κ_2 << 1 in Part 2
  - seen in gyro-kinetic simulations of ITG modes (e.g. W Lee)



SUMMARY OF PART 1

- Shown how the ballooning transform allows one to use the simplification of a WKB ansatz for short wave-length (high-n) modes in a torus while allowing the most unstable long parallel wavelengths but still respecting both the toroidal and poloidal periodicities

- This provides an efficient calculational method for micro-instabilities and high-n MHD stability in a torus – the 'lowest order theory'

- The 'higher order theory' determines radial mode structures and normally provides only a small correction to mode frequencies

  - its importance then is to justify choosing the most unstable value of k and to show there is a fully consistent theory for the radial variation

  - Situations with low magnetic shear and finite rotation shear pose new problems for ballooning theory

  - A subject for Part 2!

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PART 2: RADIAL MODE STRUCTURE

- ‘Higher order’ theory: determines radial envelope A(X) (Taylor, Connor, Wilson)
- Reintroduce $\kappa_{1,2} << 1 \Rightarrow k = k(X)$
  - yields radial envelope in WKB APPROXIMATION

$$A(X) = e^{i\int^X k(X)dX}$$

  - eigenvalue condition

$$\oint k\,dX = (\ell + 1/2)\pi$$

Conventional Version: ω_* has a maximum

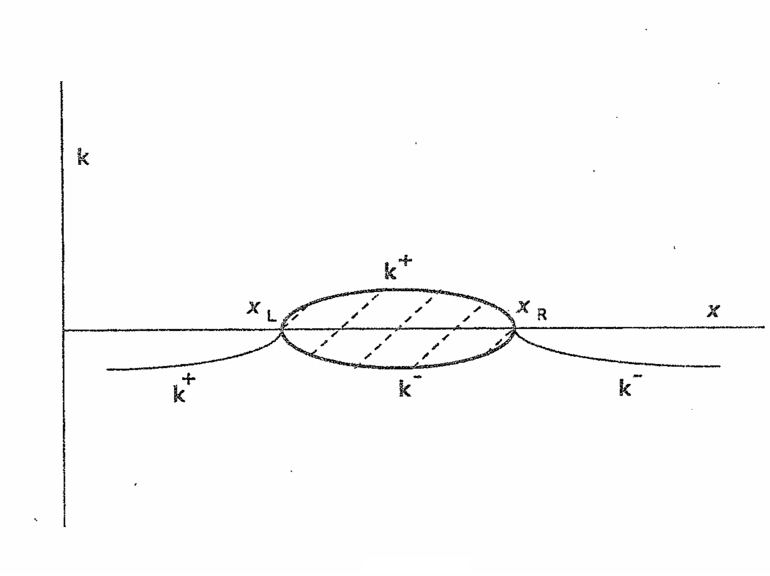
- Lowest order theory gives ‘local’ eigenvalue

$$\Omega = \underbrace{\Omega_*(0) - \kappa_2 X^2}_{\text{'local' } \omega_*(x)} + \underbrace{i\gamma_s(k)}_{\text{shear damping}}$$

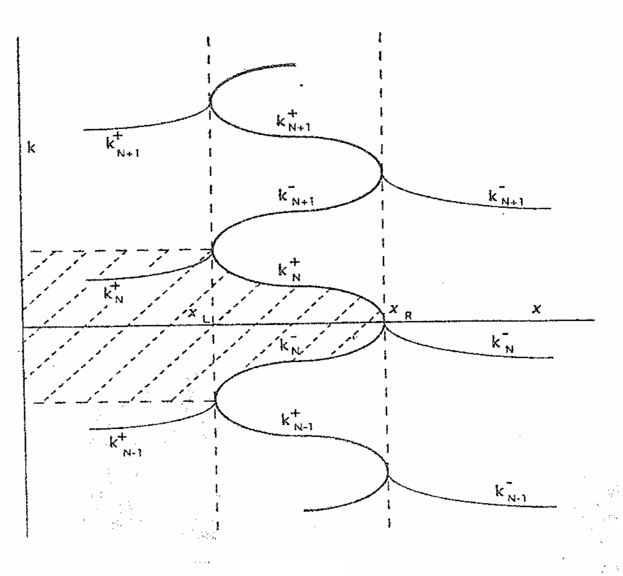
- Expand Ω about k_0 where shear damping is minimum

$$\Omega = \Omega(0) - \kappa_2 X^2 + i\gamma_{kk}(k - k_0)^2 \quad (\gamma_{kk} \sim \varepsilon)$$

$$\Rightarrow k(X, \Omega)$$



Quadratic profile



Linear profile

- Implications
  - mode width $X_M \sim \left(\frac{\varepsilon}{\kappa_2}\right)^{1/4} \sim n^{1/2}, \quad \Rightarrow \quad \Delta r \sim \frac{a}{n^{1/2}} << a$
  - mode localised about $X = 0$, i.e. $\Omega_{*max,}$, but covers many resonant surfaces
  - spread in k: $\Delta k \sim n^{-1/2} << 1$

    $\Rightarrow k \cong k_0$ (minimum in shear damping)

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HIGHER ORDER THEORY: EXPANSION IN 1/n

Ballooning equation : $L(\eta,\ k,\ x;\ \Omega)\varphi(\eta,\ k,x) = 0$

$$\varphi = A(x)\exp(inq'\int^{x} k(x)dx)\ \hat{\varphi}(\eta,k) \qquad \Rightarrow \frac{d}{dx} \to \frac{d}{dx} + inq'k$$

$$\Rightarrow L \to L\left(\eta, k - \frac{i}{nq'}\frac{d}{dx}, x;\ \Omega\right)$$

Expand in $n^{-1/2}$: $\varphi = \varphi_0 + \frac{1}{n^{1/2}}\varphi_1 + \frac{1}{n}\varphi_2 + ...$

$$L = L_0(\eta,k,x,\omega(x)) - \frac{i}{n^{1/2}q'}\frac{\partial L_0}{\partial k}\frac{\partial}{\partial x} - \frac{1}{2nq'^2}\frac{\partial^2 L_0}{\partial^2 k}\frac{\partial^2}{\partial x^2} + \frac{\partial L_0}{\partial \Omega}(\Omega - \omega(x))$$

$$\equiv \qquad L_0 \qquad + \frac{1}{n^{1/2}}L_1 \qquad + \frac{1}{n}L_2$$

Lowest order :
$$L_0(\eta, k, x, \omega(x))\varphi_0 = 0$$

$$\Rightarrow \omega = \omega(k, x), \varphi_0 = A(x)\hat{\varphi}_0(\eta, k)$$

First order :

$$L_0\varphi_1 + L_1\varphi_0 = 0, \qquad \text{where } L_1 = -\frac{i}{n^{1/2}q'}\frac{\partial L_0}{\partial k}\frac{\partial}{\partial x}$$

$$\therefore \qquad L_0\varphi_1 - \frac{i}{n^{1/2}q'}\frac{\partial L_0}{\partial k}\frac{\partial A}{\partial x}\hat{\varphi}_0 = 0$$

But $L_0\hat{\varphi}_0 = 0 \Rightarrow \dfrac{\partial(L_0\hat{\varphi}_0)}{\partial k} = \dfrac{\partial L_0}{\partial k}\hat{\varphi}_0 + L_0\dfrac{\partial \hat{\varphi}_0}{\partial k} = 0$

$$\Rightarrow \varphi_1 = -\frac{i}{n^{1/2}q'}\frac{\partial A}{\partial x}\hat{\varphi}_1 \text{ with } \hat{\varphi}_1 = \frac{\partial \hat{\varphi}_0}{\partial k}$$

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Determining k_0

Let $\int_{-\infty}^{\infty} d\eta ...... \equiv \langle ...... \rangle$, where $\langle \varphi L_0 \psi \rangle = \langle \psi L_0 \varphi \rangle$

Then $\langle \varphi_0 L_0 \varphi_1 \rangle + \langle \varphi_0 L_1 \varphi_0 \rangle = 0 \Rightarrow \left\langle \hat{\varphi}_0 \frac{\partial L_0}{\partial k} \hat{\varphi}_0 \right\rangle = 0$

But $\frac{\partial}{\partial k} \langle \hat{\varphi}_0 L_0 \hat{\varphi}_0 \rangle = \left\langle \hat{\varphi}_0 \frac{\partial L_0}{\partial k} \hat{\varphi}_0 \right\rangle + \left\langle \hat{\varphi}_0 \frac{\partial L_0}{\partial \omega} \frac{\partial \omega}{\partial k} \hat{\varphi}_0 \right\rangle = 0$

$$\Rightarrow \frac{\partial \omega}{\partial k} = 0 \Rightarrow k = k_0$$

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Second Order

$$L_0\varphi_2 + L_1\varphi_1 + L_2\varphi_0 = 0 \qquad \Rightarrow \quad \langle \varphi_0 L_1 \varphi_1 \rangle + \langle \varphi_0 L_2 \varphi_0 \rangle = 0 \qquad \text{(I)}$$

Consider $\left\langle \hat{\varphi}_0 \frac{\partial L_0}{\partial k} \hat{\varphi}_0 \right\rangle = 0$

$$\frac{d}{dk}\left\langle \hat{\varphi}_0 \frac{\partial L_0}{\partial k} \hat{\varphi}_0 \right\rangle$$

$$= 2\left\langle \frac{\partial \hat{\varphi}_0}{\partial k} \frac{\partial L_0}{\partial k} \hat{\varphi}_0 \right\rangle + \left\langle \hat{\varphi}_0 \frac{\partial^2 L_0}{\partial k^2} \hat{\varphi}_0 \right\rangle + \left\langle \hat{\varphi}_0 \frac{\partial L_0}{\partial \omega} \frac{\partial^2 \omega}{\partial k^2} \hat{\varphi}_0 \right\rangle = 0 \qquad \text{(II)}$$

as $\frac{\partial \omega}{\partial k} = 0$

Now $\langle \varphi_0 L_1 \varphi_1 \rangle = \left\langle \hat{\varphi}_0 \left(-\frac{i}{nq'} \frac{\partial L_0}{\partial k} \frac{\partial}{\partial x} \right) \left(-\frac{i}{nq'} \frac{\partial \hat{\varphi}_0}{\partial k} \frac{dA}{dx} \right) \right\rangle$

From (II) $= -\frac{1}{2n^2 q'^2} \left[\left\langle \hat{\varphi}_0 \frac{\partial^2 L_0}{\partial k^2} \hat{\varphi}_0 \right\rangle + \left\langle \hat{\varphi}_0 \frac{\partial L_0}{\partial \omega} \frac{\partial^2 \omega}{\partial k^2} \hat{\varphi}_0 \right\rangle \right] \frac{d^2 A}{dx^2}$ (III)

Radial Equation

Combining (I) and (III)

$$\frac{1}{2n^2q'^2}\left\langle \hat{\varphi}_0 \frac{\partial L_0}{\partial \omega}\frac{\partial^2 \omega}{\partial k^2}\hat{\varphi}_0 \right\rangle \frac{d^2A}{dx^2} + \left\langle \hat{\varphi}_0 \frac{\partial L_0}{\partial \omega}\hat{\varphi}_0 \right\rangle \left(\Omega - \omega(x)\right) A = 0$$

or, writing $\omega = \omega_0 - \frac{x^2}{2}\frac{\partial^2 \omega}{\partial x^2} = \omega_0 - \omega_{xx}\frac{x^2}{2}$

$$\frac{1}{2n^2q'^2}\frac{\partial^2 \omega}{\partial k^2}\frac{d^2A}{dx^2} + \left(\Omega - \omega_0 - \frac{x^2}{2}\frac{\partial^2 \omega}{\partial x^2}\right) A = 0$$

Thus $A = A_0 \exp\left(-nq'x^2\left(\omega_{xx}/\omega_{kk}\right)^{1/2}/2\right)$, $\Omega = \omega_0 + \frac{\left(\omega_{xx}/\omega_{kk}\right)^{1/2}}{2nq'}$

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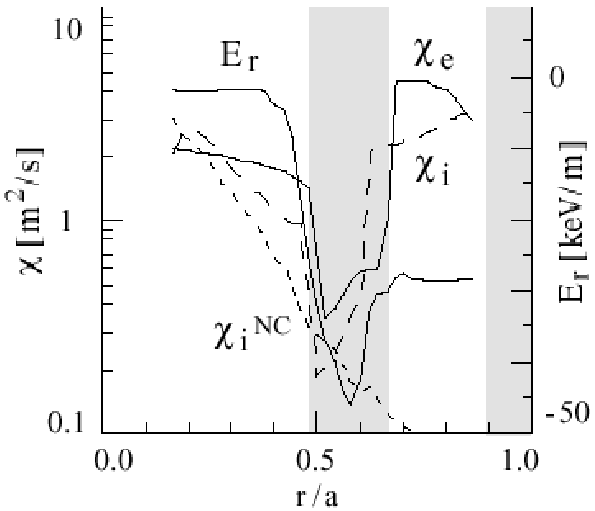
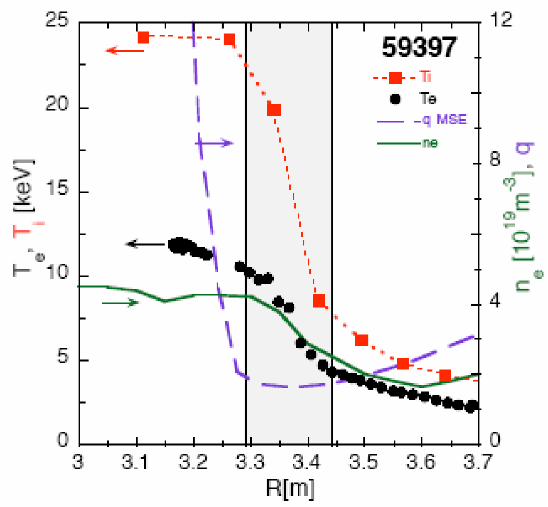
LIMITATIONS

1. Low magnetic shear

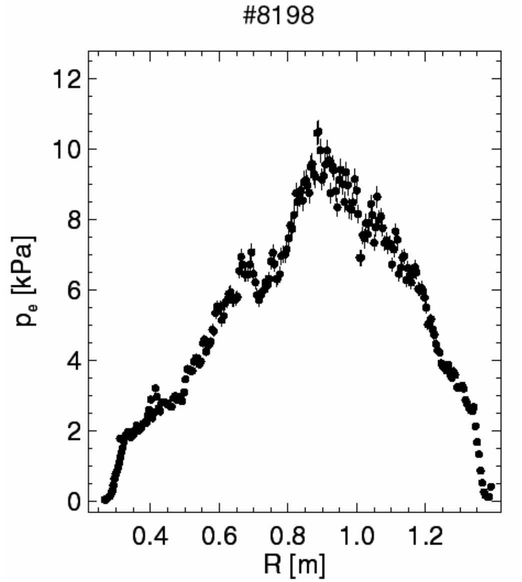
2. High velocity shear

3. The edge
   - unfortunately characteristics of transport barriers!





ITB



H-mode

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FOURIER OR BALLOONING MODES? - (1)

Introduce Fourier expansion and treat ε εand κ as perturbations

Seek solution

$$\varphi^{(0)} = \sum_n A_m e^{im\theta} \varphi_m (X - m); \quad \varphi_m (X) = \exp(-i\sigma(X - m)^2 / 2);$$

$$\Lambda^{(0)} = -i\sigma$$

i.e. φ_m satisfy lowest order 'cylindrical' equation with $\kappa = 0$

Coefficients A_m are determined in first order through degenerate perturbation theory, by annihilating $\varphi^{(1)}$

FOURIER OR BALLOONING MODES? - (2)

Applying annihilator

$$\int_{-\infty}^{\infty} dX \oint d\theta \, \varphi_m(X) \exp(im\theta)$$

We obtain a recurrence relation

$$\rho(A_{m+1} + A_{m-1}) = (a - m^2)A_m; \;\; \rho = \frac{\varepsilon}{2\kappa}\left(1 + \frac{i\sigma s}{2}\right)\exp\left(-\frac{i\sigma}{4}\right), \;\; a = \left(\frac{i\sigma}{2} - \frac{\Lambda^{(1)}}{\kappa}\right)$$

Recurrence relation equivalent to Mathieu equation

FOURIER OR BALLOONING MODES? - (3)

Small ρ : $A_0 = 1$, $A_1 = -2\rho$, $A_2 = \rho^2/2$………; $a = -2\rho^2 + 7\rho^4/2$…..

A single Fourier mode with weak sidebands

Large ρ : A_m **slowly varying in m**

$$\rho \frac{d^2 A_m}{dm^2} + \left(m^2 - a + 2\rho\right) A_m = 0$$

$$\Rightarrow A_m = \exp(-im^2/2\rho^{1/2}); \qquad a = 2\rho - i\rho^{1/2};$$

$\therefore A_m$ essentially constant out to $m \sim \pm|\rho|^{1/4}$

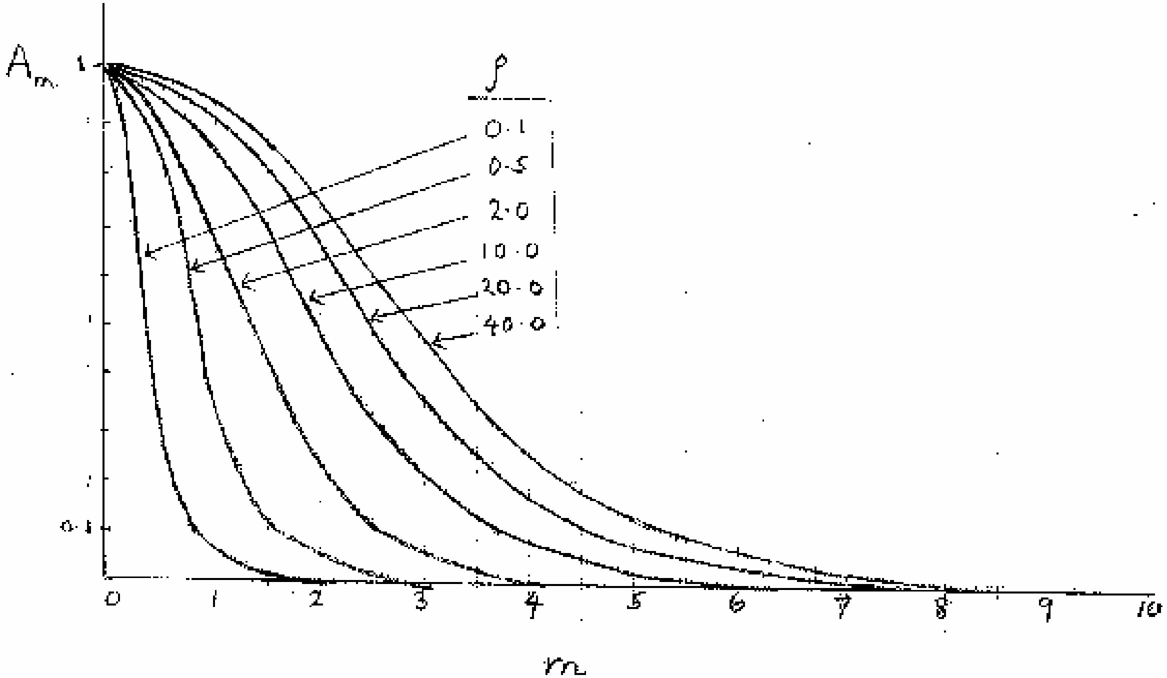
and then fall off exponentially

If ω **unstable then** σ **has a negative imaginary part**

Since $\sigma \sim 1/s$, $\rho \ll 1$ **for** $s \ll 1$

Example with ρ **real**

Fourier Amplitudes A_m as function of real $\rho = \varepsilon/2\kappa$



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LOW MAGNETIC SHEAR ($s \ll 1$, ε finite)

- Two-scale analysis of ballooning equations $\eta \rightarrow (\eta, u)$: η periodic equilibrium scale, $u = s\eta$
  - $\Rightarrow$'averaged' eqn., independent of $k \Rightarrow \Omega$ independent of k!
  - corresponds to uncoupled Fourier harmonics at each m^{th} surface, localised within $X \sim |s|$: i.e. non-overlapping radially

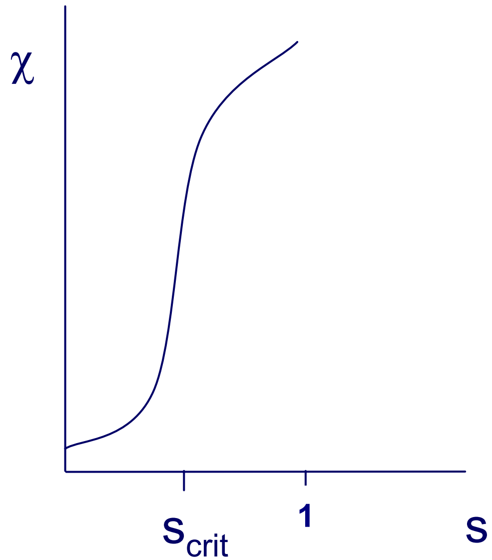
- Recover k-dependence by using these as trial functions in variational approach (Romanelli and Zonca)
  - exponentially weak contribution

$$\Omega = \Omega_*(0) - \kappa_2 X^2 + i \exp(-c/|s|)\hat{\gamma}_{kk}(k - k_0)^2$$

$$\Rightarrow X_M \sim \left(\frac{\varepsilon}{\kappa_2}\right)^{\frac{1}{4}} \exp\left(-\frac{s_{crit}}{|s|}\right) \qquad s_{crit} \propto (k\rho_i)^{-2}$$

  - becomes very narrow as $|s| \rightarrow 0$, or $k_\perp \rho_i \rightarrow 0$

- Estimate anomalous transport: $\chi \propto \gamma_{Lin} X_M^2$

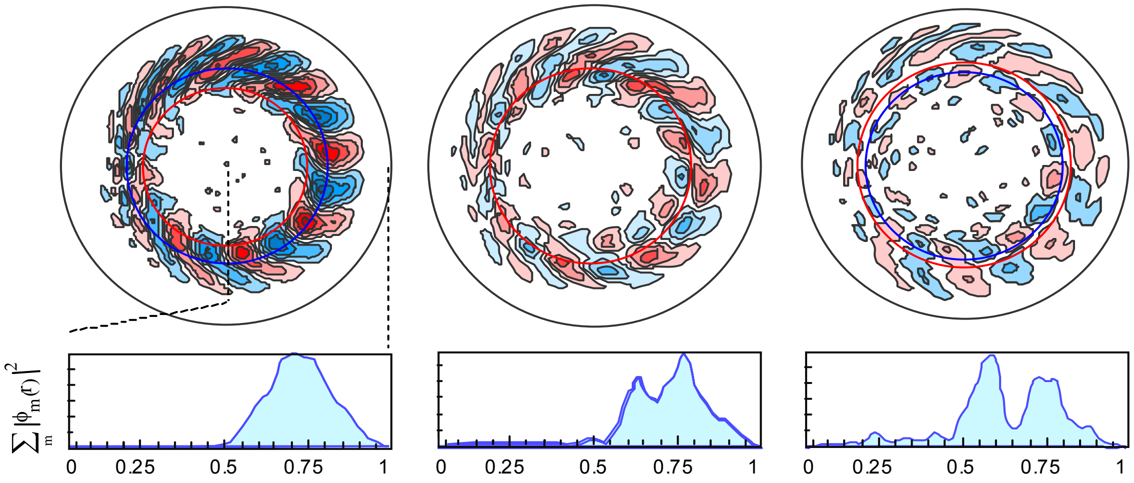


suggests link between low shear and ITBs

(Romanelli, Zonca)

- Presence of q_{min} acts as barrier to mode structures

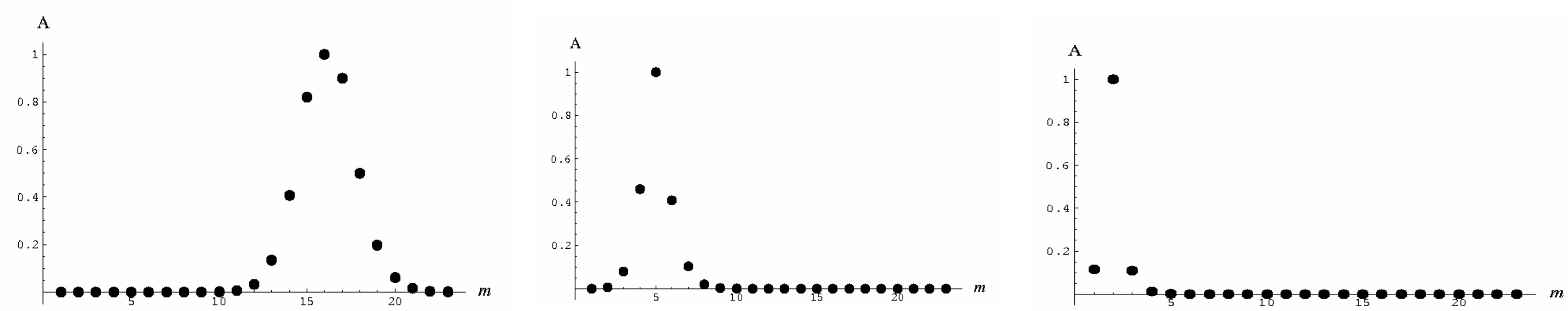
Gyro-kinetic simulation by Kishimoto



(a) $r_{min}/a = 0.5$ (b) $r_{min}/a = 0.6$ (c) $r_{min}/a = 0.65$

- Ballooning mode theory fails for sufficiently low s (or long wavelength)
  - reverts to weakly coupled Fourier harmonics, amplitude A_m, when

$$\frac{n\rho_i}{r} < \sqrt{\frac{L_n}{4sq^2R}}$$



  - spectrum of A_m narrows as mode centre moves towards q_{min}

- In practice, ballooning theory holds to quite low n
  e.g. ITG modes with $k_\perp\rho_I$ ~ 0(1) largely unaffected by q_{min}

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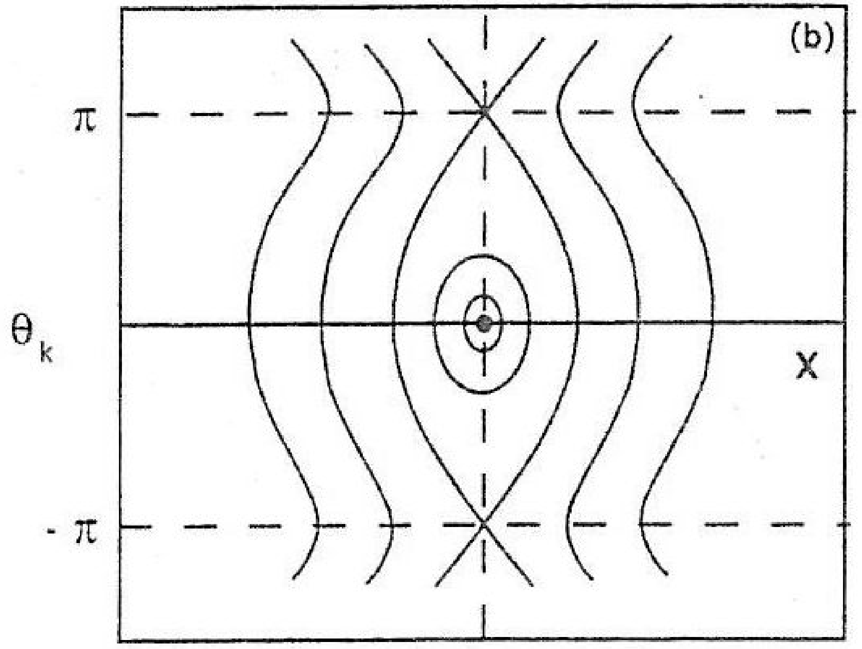
THE WAVE-NUMBER REPRESENTATION

- More general contours of $\mathbf{k}\,(\mathbf{X}, \Omega)$ $\qquad (\, k \rightarrow \theta_k \,)$



(Romanelli, Zonca)

- 'Closed' contours already discussed; 'passing' contours sample all $\mathbf{k}$
  - WKB treatment in $\mathbf{X}$-space still possible
  - easier to use alternative, but entirely equivalent, Wave-number Representation (Dewar, Mahajan)

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$$\varphi(X,\theta) = \int_{-\infty}^{\infty} dk\hat{\varphi}(\theta,k)\exp\left[-iX(\theta-k)-S(k)\right]$$

- $\hat{\varphi}(\theta,k)$ **satisfies ballooning eqn. on -∞ < θ < ∞, i.e. not periodic in θ**
- $\hat{\varphi}(\theta + 2\pi, k + 2\pi) = \hat{\varphi}(\theta, k)$

  – **φ is periodic in θ if S(k) is periodic in k: eigenvalue condition**

Example

- **Suppose linear profile:** $\Omega = \Omega_*(0) + \kappa_1 X + i\gamma_s(k)$, $\kappa_1 \sim \dfrac{1}{nq'L} \sim 0\left(\dfrac{1}{n}\right)$

  $$\Rightarrow \qquad \kappa_1 \frac{dS}{dk} + i(\Omega - \Omega_*(0) - i\gamma_s(k)) = 0$$

- **Periodicity of S yields eigenvalue condition**

$$\oint dk[\Omega - \Omega_* - i\gamma_s(k)] = 2\pi\ell\kappa_1$$

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Implications

- Re Ω related to local $\omega_*(x)$: $\Omega_* + 2\pi\ell\kappa_1$
- $\mathrm{Im}\ \Omega = \frac{1}{2\pi}\oint dk\gamma_s(k)$
  - k not restricted to near k_0 (where γ is maximum): for the electron drift wave, all k contribute to give an average of the shear damping!
  - some shear damping restored: more STABLE

    e.g. ε = 4: $\gamma_s(0)$ = - 0.02, $\frac{1}{2\pi}\oint\gamma_s(k)dk = -0.35$

- $\Delta k = 2\pi \Rightarrow \Delta X \sim \frac{1}{\kappa_1}\Delta\gamma(k) \sim \frac{\varepsilon}{\kappa_1} >> 1$ if $\varepsilon >> \kappa_1$
- Mode width:

  (i) Ω real $\Rightarrow \Delta X \sim \varepsilon n$, or $\Delta r \sim \varepsilon a$

  (ii) Ω complex $\Rightarrow \Delta X \sim n^{1/2}\varepsilon^{1/2}$, or $\Delta r \sim (\varepsilon/n)^{1/2}a$

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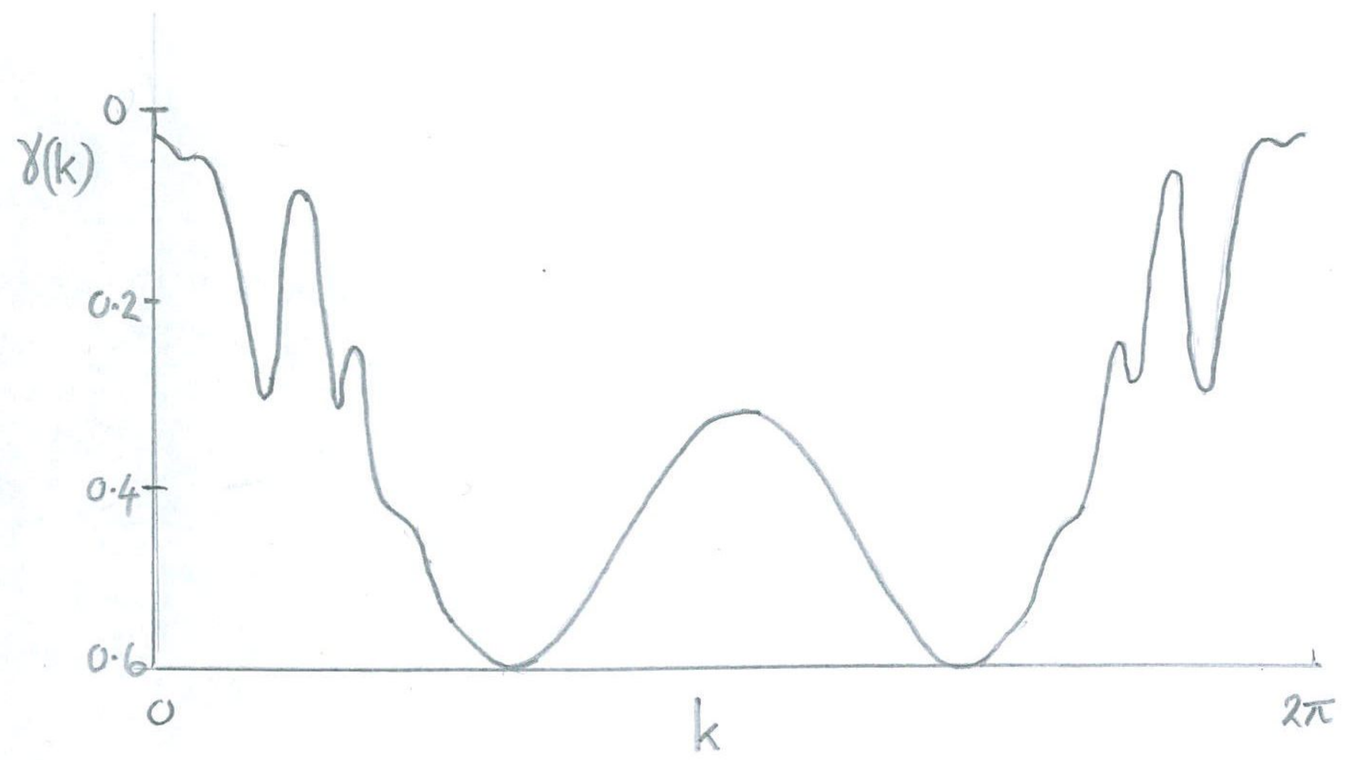

0
γ(k)
0·2
0·4
0·6
0
k
2π

SHEARED RADIAL ELECTRIC FIELDS

- Believed to reduce instability and turbulence – prominent near ITBs
- $\omega \to \omega$ - n Ω_E (x) (Doppler Shift); suppose $\Omega_E = \Omega' x$

  $$\Rightarrow \kappa_1 \to \kappa_E = \frac{d\Omega_E}{dq} \equiv \Omega_q \sim 0(1)\ !$$

- $\Delta X \sim \varepsilon / \Omega_q$

  $\Rightarrow$ mode narrows as $d\Omega_E/dq$ increases, reducing estimates of ΔX and transport

- Are these modes related to conventional ballooning modes?
  - introduce density profile variation
- Model: $\Omega = \Omega_*(0) - i\gamma_0 + i\gamma_{xx} X^2 - i\gamma_k \cos k - \Omega_q X$

  i.e. γ has maximum at $X = 0$

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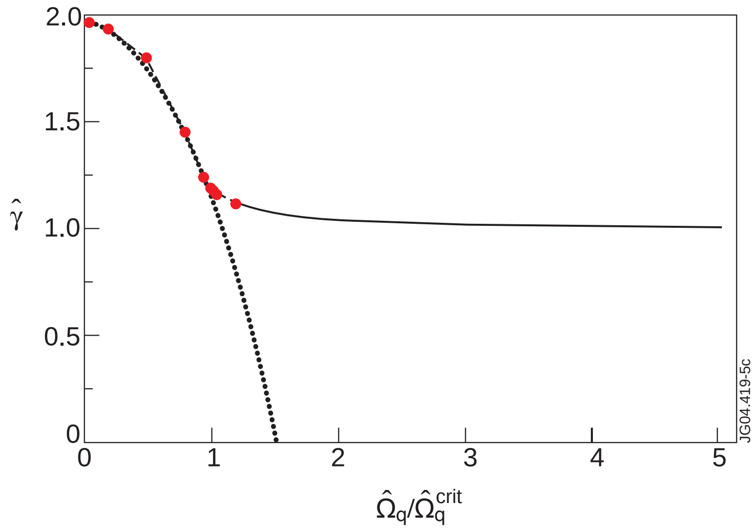
- Wave-number representation produces quadratic eqn for dS/dk
  - $\exp(inS) \rightarrow \varphi(k)$
    - periodic $S(k) \Rightarrow \varphi(k)$ is Floquet solution of Mathieu eqn: yields eigenvalue γ



Analytic solution for transition region possible (Connor)

$$\Omega_q^{crit} \sim 0\left(\varepsilon_T^{1/2} / n\right)$$

$\Rightarrow$ continuous evolution from conventional mode to more STABLE 'passing' mode

- $\Delta X \sim \dfrac{\varepsilon}{(d\Omega_E / dq)} \leq 1$ for large $d\Omega_E/dq$

$\Rightarrow$ reverts to Fourier modes!

FULL CIRCLE?

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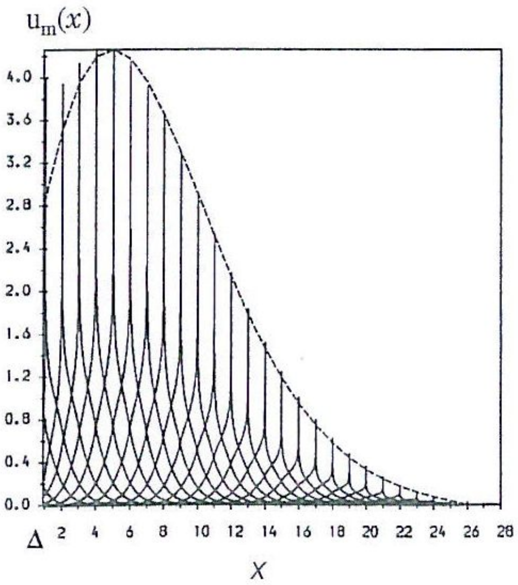
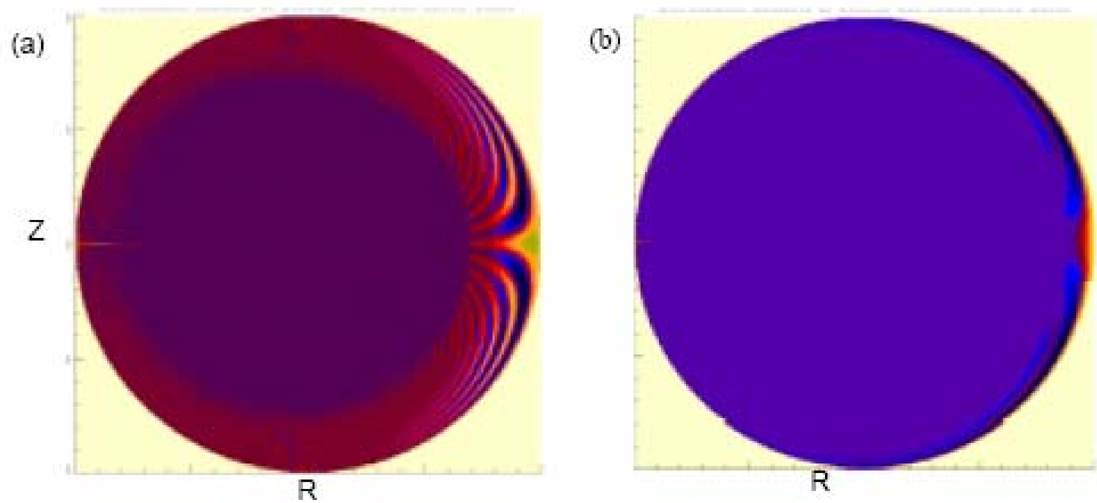
EXTENSIONS TO BALLOONING THEORY

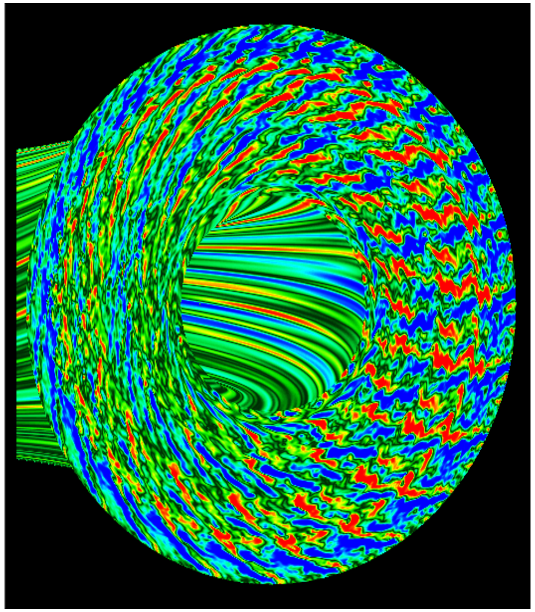
- Have seen limitations imposed by low magnetic shear and high flow shear

- The presence of a plasma edge clearly breaks translational invariance
  - have used 2D MHD code to study high-n edge ballooning modes; mode structure resembles ballooning theory 'prediction'

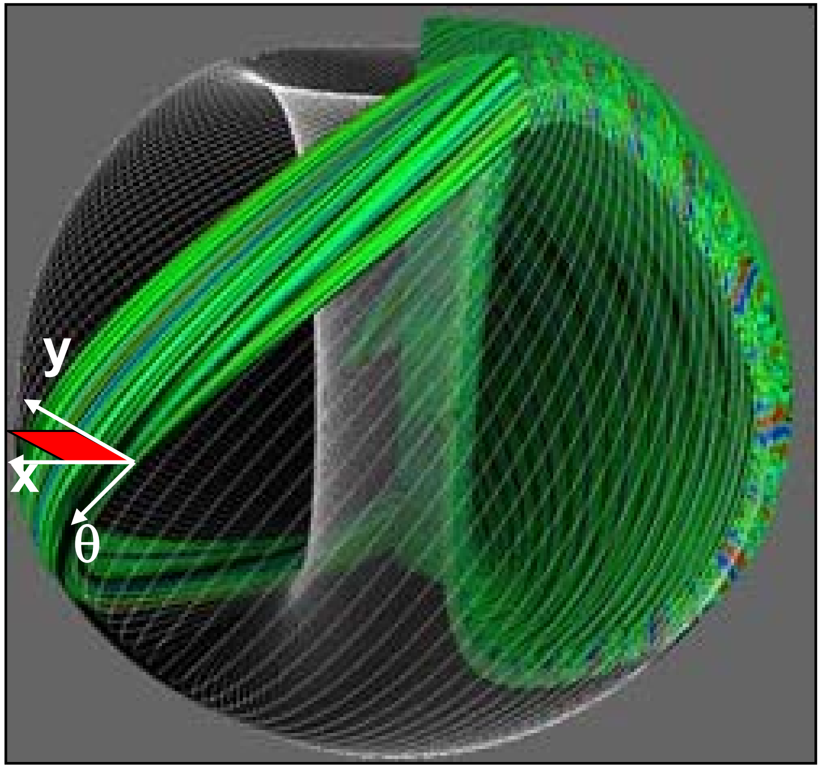




- Non-linear theory
  - the ‘twisted slices’ of Roberts and Taylor form a basis for flux-tube gyrokinetic simulations



Conventional Tokamak



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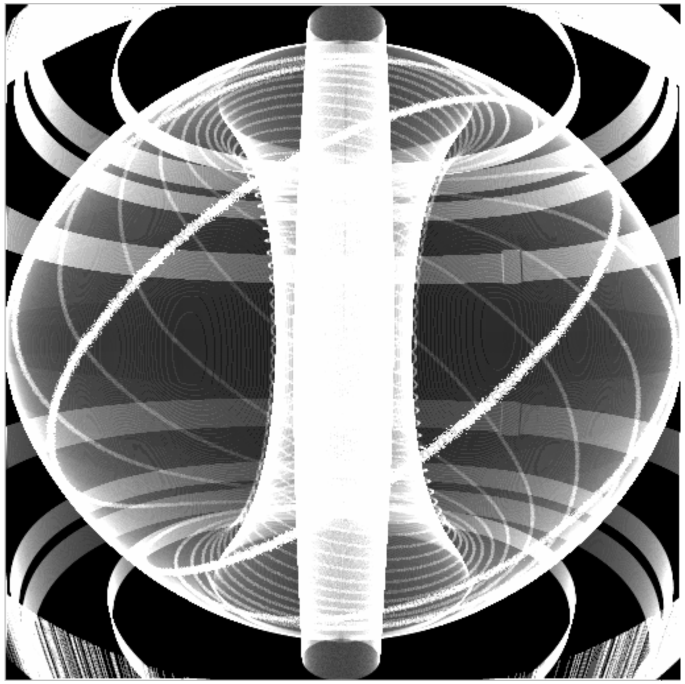
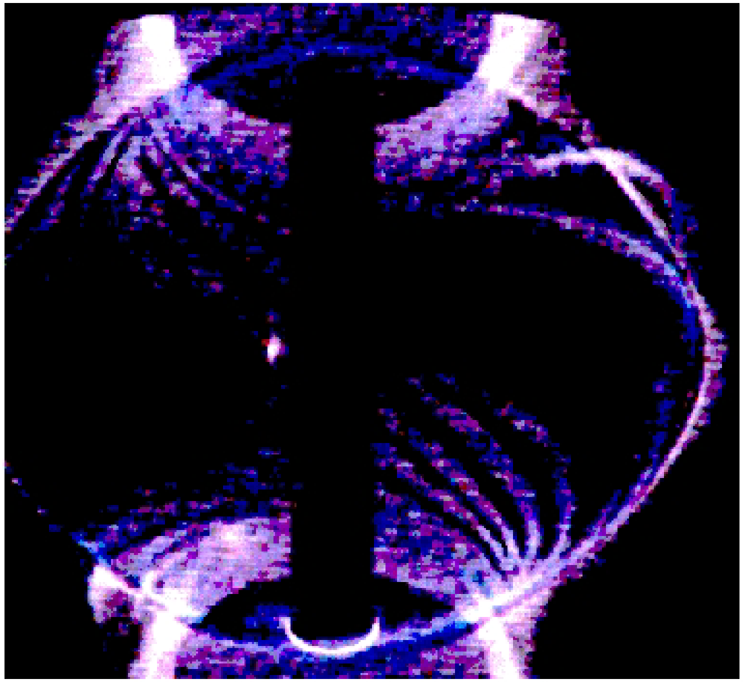
– introducing non-linearities into the theory of high-n MHD ballooning modes predicts explosively growing filamentary structures, seen on MAST



Simulation



Experiment

SUMMARY AND CONCLUSIONS

- Problems of toroidal periodicity in the presence of magnetic shear resolved by Ballooning theory

- Ballooning theory provides a robust and widely used tool, but its validity can break down for:
  - Low magnetic shear
  - Rotation shear
  - Plasma edge

  when the higher order theory is considered

- Re-emergence of Fourier modes in the torus for low s and high $d\Omega_E/dq$

- Ballooning theory also provides a basis for some non-linear theories and simulations

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