

§4 Firehose Instability

I will discuss the ways in which a pressure anisotropy might arise later on, after we learn how to calculate p_{\perp} and p_{\parallel}' , but first I want to take a pause in deriving equations and extract some fun physics from what we've got. Let us take our new momentum eqn (39) ~~*~~, together with the induction eqn for \vec{B} , linearise them and see what can be learned.

The linearisation will be around a putative equilibrium with

$$\rho = \rho_0 = \text{const}$$

$$p_{\perp} = p_{\perp 0} = \text{const} \quad (41)$$

$$p_{\parallel}' = p_{\parallel 0}' = \text{const}$$

$$\vec{B} = B_0 \hat{z} = \text{const}$$

and no eq. flow.

Induction eqn, as in MHD [see eq. (13.5) of Notes]

$$-\omega \delta \vec{B} = B_0 k_{\parallel} \vec{u} - \frac{1}{2} B_0 \vec{k} \cdot \vec{u} \quad (42)$$

turning, for $\frac{\delta \vec{B}_{\perp}}{B_0} = \delta \vec{b}$ and $\frac{\delta B_{\parallel}}{B_0} = \frac{\delta B}{B}$, into

$$\left\{ \begin{array}{l} -\omega \delta \vec{b} = k_{\parallel} \vec{u}_{\perp} \\ -\omega \frac{\delta B}{B_0} = -\vec{k}_{\perp} \vec{u}_{\perp} \end{array} \right. \quad (43)$$

The momentum equation becomes ⁽³⁹⁾

$$\begin{aligned} -\omega \rho_0 \vec{u} &= -\vec{k} \left(\delta p_{\perp} + \frac{B_0 \delta B}{4\pi} \right) \quad \vec{k} \cdot \delta \vec{b} = -k_{\parallel} \frac{\delta B}{B_0} \\ &\quad + \vec{k} \cdot \left[\left(\frac{1}{2} \delta \vec{b} + \delta \vec{b} \frac{1}{2} \right) \left(p_{\perp 0} - p_{\parallel 0}' + \frac{B_0^2}{4\pi} \right) \right. \\ &\quad \left. + \cancel{\vec{z} \vec{z}} \left(\delta p_{\perp} - \delta p_{\parallel}' + \frac{B_0 \delta B}{2\pi} \right) \right] \end{aligned}$$

$$\begin{aligned}
 &= -\vec{k}_L \left(\delta p_L + \frac{B_0 \delta B}{4\pi} \right) - \hat{z} k_{||} \left[\delta p_L + \frac{B_0 \delta B}{4\pi} + \frac{\delta B}{B_0} \left(p_{L0} - p'_{L0} + \frac{B_0^2}{4\pi} \right) \right. \\
 &\quad \left. - \delta p_{||} + \delta p'_{||} - \frac{B_0 \delta B}{2\pi} \right] + \delta \hat{B} k_{||} \left(p_{L0} - p'_{L0} + \frac{B_0^2}{4\pi} \right) \\
 &= -\vec{k}_L \left(\delta p_L + \frac{B_0 \delta B}{4\pi} \right) - \hat{z} k_{||} \left[\delta p'_{||} + \left(p_{L0} - p'_{L0} \right) \frac{\delta B}{B_0} \right] \\
 &\quad + \delta \hat{B} k_{||} \left(p_{L0} - p'_{L0} + \frac{B_0^2}{4\pi} \right) \quad (44)
 \end{aligned}$$

Obviously, we can't complete the job because we need the kinetic equation to calculate δp_L and $\delta p'_{||}$, but we can extract a piece that is entirely independent of kinetics perturbations:

$$\begin{aligned}
 \vec{k}_L \times | -\omega p_0 \vec{u}_L &= -\vec{k}_L \left(\delta p_L + \frac{B_0 \delta B}{4\pi} \right) + \delta \hat{B} k_{||} \left(p_{L0} - p'_{L0} + \frac{B_0^2}{4\pi} \right) \\
 -\omega p_0 \vec{k}_L \times \vec{u}_L &= + \cancel{k_{||}} \left(p_{L0} - p'_{L0} + \frac{B_0^2}{4\pi} \right) \underbrace{\vec{k}_L \times \delta \hat{B}}_{\frac{k_{||}}{\omega}} \quad (45)
 \end{aligned}$$

You guessed it right, these are

$$-\frac{k_{||}}{\omega} \vec{k}_L \times \vec{u}_L$$

Alfvén waves:

$$\boxed{\omega^2 = k_{||}^2 \left(\frac{p_{L0} - p'_{L0}}{\rho_0} + v_A^2 \right)} \quad (46)$$

$$\hookrightarrow v_A = \frac{B_0}{\sqrt{4\pi \rho_0}}$$

Like in MHD, Alfvén waves do not care about (do not involve) compressive perturbations — and, therefore, they do not care about kinetics, just about the equilibrium stress.

We shall need kinetics to get the rest of the waves.

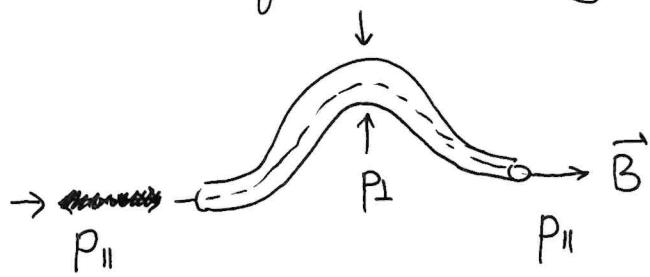
But look at horrible thing that is manifest in (46):

if $\frac{P_{\perp 0} - P'_{\perp 0}}{P_0} + V_A^2 < 0$

or
$$\left[\frac{P'_{\perp 0} - P_{\perp 0}}{P_0} > \frac{B_0^2}{4\pi P_0} = \frac{2}{\beta} \right] \quad (47)$$

the Alfvén wave ceases to be a wave and becomes an instability — this is called the firehose instability

Pressure anisotropy weakens the restoring tension force and, when large enough, turns the restoring force negative — field lines lose elasticity and develop a tendency to buckle:



A truly bad implication of (46) is that the instability growth rate

$$\gamma = k_{\parallel} \sqrt{\frac{P'_{\perp 0} - P_{\perp 0}}{P_0} - V_A^2} \propto k_{\parallel} \quad (48)$$

so the fastest-growing modes are at $k_{\parallel} \rightarrow \infty$, i.e., outside ~~our~~ our approximation $k P_0 \rightarrow 0$.

Thus, the equations we are deriving are ill-posed in physical regimes where they can develop large enough pressure anisotropy to satisfy (47).

We shall see later that $P_{\perp 0} > P'_{\perp 0}$ is also bad.

You might wonder what happens in nature: these instabilities grow, then what? — how do they saturate? There is some active research literature on this that I invite you to read.

Some useful papers (super-biased selection):

AAS + MNRAS 405, 291 (2010) — easiest way to expand to next order in k_{\parallel} and find k_{\parallel} at which the firehose growth rate peaks

Melville + MNRAS 459, 2701 (2016) — how firehose saturates (see also refs therein)

It can also
saturate in
a different
way, see
refs in the
paper.

{ Basically, firehose fluctuations scatter particles
and thus make plasma more collisional,
to pin its pressure anisotropy to the
instability threshold (47).

Squire + PRL 119, 155101 (2017) — here you try to propagate a finite-amplitude Alfvén wave but it gets ~~absorbed~~ interrupted by loss of tension due to developing pressure anisotropy

The interaction of large scale MHD dynamos at high β with seas of firehose fluctuations usually triggered by it at small scales is an active research topic — jumping in!

If you want to know more about magnetized kinetics, you need the kinetic equation after all. So back to derivation of equations...

§5 Gyroaveraged kinetic equation

OK, let us gyroaverage (36), using (37).

In fact we see

$$\begin{aligned} \left\langle \left(\frac{df_d}{dt_d} \right) \right\rangle_{\vec{w}} &= \left(\frac{df_d}{dt_d} \right)_{w_\perp, w_\parallel} + \underbrace{\left\langle \left(\frac{dw_\perp}{dt_d} \right) \right\rangle_{\vec{w}}}_{\vec{w} \cdot \frac{d\vec{b}}{dt_d}} \frac{\partial f_d}{\partial w_\perp} + \underbrace{\left\langle \left(\frac{dw_\parallel}{dt_d} \right) \right\rangle_{\vec{w}}}_{\vec{w} \cdot \frac{d\vec{b}}{dt_d}} \frac{\partial f_d}{\partial w_\parallel} \\ &\quad - \left\langle \frac{w_\parallel}{w_\perp} \vec{w} \cdot \frac{d\vec{b}}{dt_d} \right\rangle = 0 \quad w_\perp = \sqrt{w^2 - w_\parallel^2} \\ &= \left(\frac{df_d}{dt_d} \right)_{w_\perp, w_\parallel} \end{aligned} \quad (49)$$

$$w_\parallel = \vec{w} \cdot \hat{b}$$

$$w_\perp = \sqrt{w^2 - w_\parallel^2}$$

$$\left\langle \vec{w} \cdot \frac{d\vec{b}}{dt_d} \right\rangle$$

$$= w_\parallel \hat{b} \cdot \frac{d\hat{b}}{dt_d} = 0$$

$$\begin{aligned} \left\langle \vec{w} \cdot (\nabla f_d)_{\vec{w}} \right\rangle_{\vec{w}} &= w_\parallel (\nabla_\parallel f_d)_{w_\perp, w_\parallel} + \underbrace{\left\langle \vec{w} \cdot (\nabla w_\perp)_{\vec{w}} \right\rangle}_{\vec{w} \cdot \frac{\partial \vec{b}}{\partial w_\perp}} \frac{\partial f_d}{\partial w_\perp} + \\ &+ \underbrace{\left\langle \vec{w} \cdot (\nabla w_\parallel)_{\vec{w}} \right\rangle}_{\vec{w} \cdot \frac{\partial \vec{b}}{\partial w_\parallel}} \frac{\partial f_d}{\partial w_\parallel} = \\ &\quad \left\langle \vec{w} \vec{w} \right\rangle : \nabla \hat{b} = \frac{w_\perp^2}{2} \nabla \cdot \hat{b} + (w_\parallel^2 - \frac{w_\perp^2}{2}) \hat{b} : \nabla \hat{b} \quad \left\langle \vec{w} \cdot (\nabla \hat{b}) \cdot \vec{w} \right\rangle_{w_\perp} \\ &\quad \frac{w_\perp^2}{2} (\hat{1} - \hat{b} \hat{b}) + w_\parallel^2 \hat{b} \hat{b} \quad \left\langle \vec{w} \cdot \frac{\partial \vec{b}}{\partial w_\perp} \right\rangle_{w_\parallel} = - \frac{w_\perp}{2} (\nabla \cdot \hat{b}) w_\parallel \quad \text{by the same calculation} \end{aligned}$$

$$= w_\parallel (\nabla_\parallel f_d)_{w_\perp, w_\parallel} + \underbrace{\nabla \cdot \hat{b}}_{-\frac{\nabla_\parallel B}{B}} \left(- \frac{w_\perp w_\parallel}{2} \frac{\partial}{\partial w_\perp} + \frac{w_\perp^2}{2} \frac{\partial}{\partial w_\parallel} \right) f_d =$$

$$= w_\parallel \nabla_\parallel f_d + \frac{\nabla_\parallel B}{B} \frac{w_\perp}{2} \left(w_\parallel \frac{\partial}{\partial w_\perp} - w_\perp \frac{\partial}{\partial w_\parallel} \right) f_d$$

$$E_\parallel \frac{q_d}{m_d} - \frac{d\vec{u}_d}{dt_d} \cdot \hat{b}$$

$$\begin{aligned} \left\langle \vec{a}_d \cdot \frac{\partial f_d}{\partial \vec{w}} \right\rangle_{\vec{w}} &= \vec{a}_d \cdot \left\langle \frac{\partial w_\perp}{\partial \vec{w}} \frac{\partial f_d}{\partial w_\perp} + \frac{\partial w_\parallel}{\partial \vec{w}} \frac{\partial f_d}{\partial w_\parallel} \right\rangle_{\vec{w}} = \\ &= \vec{a}_d \cdot \left[- \frac{\vec{w} - w_\parallel \hat{b}}{w_\perp} \frac{\partial f_d}{\partial w_\perp} + \hat{b} \frac{\partial f_d}{\partial w_\parallel} \right] = (\vec{a}_d \cdot \hat{b}) \frac{\partial f_d}{\partial w_\parallel} \end{aligned} \quad (51)$$

$$\begin{aligned}
 -\langle \vec{w} \cdot (\nabla \vec{u}_2) \cdot \frac{\partial f_2}{\partial \vec{w}} \rangle_{\vec{v}} &= -\langle \vec{w} \cdot (\nabla \vec{u}_2) \cdot \left(+ \frac{\vec{w} - w_{\perp} \hat{b}}{w_{\perp}} \frac{\partial f_2}{\partial w_{\perp}} + \hat{b} \frac{\partial f_2}{\partial w_{\parallel}} \right) \rangle_{\vec{v}} \\
 &= -\underbrace{\langle \vec{w} \cdot \vec{w} \rangle}_{\frac{w_{\perp}^2}{2} (1 - \hat{b} \hat{b}) + w_{\parallel}^2 \hat{b} \hat{b}} \cdot (\hat{b} \vec{u}_2) \frac{1}{w_{\perp}} \frac{\partial f_2}{\partial w_{\perp}} + \frac{w_{\parallel}^2}{w_{\perp}} (\hat{b} \hat{b} : \nabla \vec{u}_2) \frac{\partial f_2}{\partial w_{\perp}} - (\hat{b} \hat{b} : \nabla \vec{u}_2) w_{\parallel} \frac{\partial f_2}{\partial w_{\parallel}} \\
 &= -(\nabla \cdot \vec{u}_2) \frac{w_{\perp}}{2} \frac{\partial f_2}{\partial w_{\perp}} + (\hat{b} \hat{b} : \nabla \vec{u}_2) \left(+ \frac{w_{\perp}}{2} - \frac{w_{\parallel}^2}{w_{\perp}} + \frac{w_{\parallel}^2}{w_{\perp}} \right) \frac{\partial f_2}{\partial w_{\perp}} - (\hat{b} \hat{b} : \nabla \vec{u}_2) w_{\parallel} \frac{\partial f_2}{\partial w_{\parallel}} \\
 &= + \frac{w_{\perp}}{2} \frac{\partial f_2}{\partial w_{\perp}} \underbrace{(\hat{b} \hat{b} : \nabla \vec{u}_2 - \nabla \cdot \vec{u}_2)}_{\frac{1}{B} \frac{d\vec{B}}{dt_2}} - (\hat{b} \hat{b} : \nabla \vec{u}_2) w_{\parallel} \frac{\partial f_2}{\partial w_{\parallel}} \quad (52)
 \end{aligned}$$

Assume: $\frac{1}{B} \frac{d\vec{B}}{dt_2}$

$$\begin{aligned}
 \frac{df_2}{dt_2} + w_{\parallel} \nabla_{\parallel} f_2 + \underbrace{\frac{\nabla_{\parallel} B}{B} \frac{w_{\perp}}{2} \left(w_{\parallel} \frac{\partial}{\partial w_{\perp}} - w_{\perp} \frac{\partial}{\partial w_{\parallel}} \right) f_2}_{+} + \\
 + \left(\frac{q_d}{m_2} E_{\parallel} - \underbrace{\frac{D\vec{u}_2}{Dt_2} \cdot \hat{b}}_{\frac{1}{B} \frac{d\vec{B}}{dt_2}} \right) \frac{\partial f_2}{\partial w_{\parallel}} + \underbrace{\frac{1}{B} \frac{dB}{dt_2} \frac{w_{\perp}}{2} \frac{\partial f_2}{\partial w_{\perp}}}_{-} - \underbrace{(\nabla_{\parallel} \vec{u}_2) \cdot \hat{b} w_{\parallel} \frac{\partial f_2}{\partial w_{\parallel}}}_{= 0}
 \end{aligned}$$

$$\text{Let } \frac{D}{Dt_2} = \frac{d}{dt_2} + w_{\parallel} \nabla_{\parallel} = \frac{\partial}{\partial t} + (\vec{u}_2 + w_{\parallel} \hat{b}) \cdot \nabla. \text{ Then} \quad (53)$$

$$\boxed{\frac{Df_2}{Dt_2} + \frac{1}{B} \frac{DB}{Dt_2} \frac{w_{\perp}}{2} \frac{\partial f_2}{\partial w_{\perp}} + \left(\frac{q_d}{m_2} E_{\parallel} - \frac{D\vec{u}_2}{Dt_2} \cdot \hat{b} - \frac{w_{\perp}^2}{2B} \nabla_{\parallel} B \right) \frac{\partial f_2}{\partial w_{\parallel}} = \left(\frac{\partial f_2}{\partial t} \right)_c}$$

Note that the only place where drifts appear is

S.2 Hiding
The drifts

$$\frac{D}{Dt_2} = \frac{\partial}{\partial t} + \vec{u} \cdot \nabla + \underbrace{(u'_{\parallel 2} + w_{\parallel}) \hat{b} \cdot \nabla}_{w'_{\parallel}} \equiv \frac{D}{Dt}$$

" w'_{\parallel} Let's shore them back in!

So, we have

$$\left(\frac{Df_2}{Dt_2} \right)_{w'_{\parallel}} = \left(\frac{Df_2}{Dt} \right)_{w'_{\parallel}} + \frac{Du'_{\parallel 2}}{Dt} \frac{\partial f_2}{\partial w'_{\parallel}}, \quad \frac{\partial f_2}{\partial w'_{\parallel}} = \frac{\partial f_2}{\partial w_{\parallel}}$$

$$\text{NB: } \frac{D\vec{u}_2}{Dt} \cdot \hat{b} = \frac{D\vec{u}}{Dt} \cdot \hat{b} + \frac{Du'_{\parallel 2} \hat{b}}{Dt} \cdot \hat{b} = \frac{D\vec{u}}{Dt} \cdot \hat{b} + \frac{Du'_{\parallel 2}}{Dt}$$

This gives us

(54)

$$\frac{Df_2}{Dt} + \frac{1}{B} \frac{DB}{Dt} \frac{w_{\perp}}{2} \frac{\partial f_2}{\partial w_{\perp}} + \left(\frac{q_2}{m_2} E_{\parallel} - \frac{D\vec{U}_{\parallel} \cdot \hat{b}}{Dt} - \frac{w_{\perp}^2}{2} \frac{D_{\parallel\parallel} B}{B} \right) \frac{\partial f_2}{\partial w_{\parallel}} = \left(\frac{\partial f_2}{\partial t} \right)$$

and the drifts, if we want them, are moments of f_2 :

$$U'_{\parallel 2} = \frac{1}{n_2} \int d^3 \vec{w} w_{\parallel}^2 f_2. \quad (55)$$

But in fact we do not want them all that much because the only place they appear is in this combination:

$$P'_{\parallel 2} = \int d^3 \vec{w} m_2 w_{\parallel}^2 f_2 = \underbrace{\int d^3 \vec{w} m_2 w_{\parallel}^2 f_2}_{\parallel} + m_2 n_2 U'_{\parallel 2}^{1/2} \quad (56)$$

(If we don't do that, we need $\vec{b} \cdot (\vec{v})$ to calculate the drifts.)

This is really just about which part of particle motion we interpret as "internal" — peculiar velocity wrt mean flow of a given species or peculiar velocity wrt mean flow.

Note that sometimes this kinetic equation is written in variables where $v_{\parallel} = u_{\parallel} + w'_{\parallel}$ is used, i.e., the parallel kinetic variable is not peculiar. Straightforwardly,

$$\frac{Df_2}{Dt} + \frac{1}{B} \frac{DB}{Dt} \frac{w_{\perp}}{2} \frac{\partial f_2}{\partial w_{\perp}} + \left(\frac{q_2}{m_2} E_{\parallel} - \frac{D\vec{U}_{\parallel} \cdot \hat{b}}{Dt} - \frac{w_{\perp}^2}{2} \frac{D_{\parallel\parallel} B}{B} \right) \frac{\partial f_2}{\partial v_{\parallel}} = \left(\frac{\partial f_2}{\partial t} \right)$$

$$\text{where } \frac{D}{Dt} = \frac{\partial}{\partial t} + \vec{U}_{\parallel} \cdot \nabla + V_{\parallel} \hat{b} \cdot \nabla \quad (\text{Kulsrud 1967}) \quad (57)$$

Parra Lectures.

This is awkward though because it makes U_{\parallel} determined from the mean flow momentum equation redundant.

Finally, let me make another variable transformation that will make eq. (54) even simpler:

$$(t, \vec{r}, w_{\perp}, w_{\parallel}') \rightarrow (t, \vec{r}, \mu, w_{\parallel}')$$

where $\mu = \frac{w_{\perp}^2}{2B}$

a.l.c.a. "magnetic moment"

this is the angular momentum of a gyrating particle:

$$m_2 w_{\perp} r = \frac{m_2 w_{\perp}^2}{2} \times \frac{w_{\perp}^2}{B}$$

(more of that later)

$$\left(\frac{Df_2}{Dt} \right)_{w_{\perp}} = \left(\frac{\partial f_2}{\partial t} \right)_{\mu} + \underbrace{\left(\frac{\partial \mu}{\partial t} \right)_{w_{\perp}} \frac{\partial \vec{F}_2}{\partial \mu}}$$

$$- \frac{w_{\perp}^2}{2B^2} \frac{DB}{Dt} = - \mu \frac{1}{B} \frac{DB}{Dt}$$

$$\frac{\partial f_2}{\partial w_{\perp}} = \frac{\partial \mu}{\partial w_{\perp}} \frac{\partial \vec{F}_2}{\partial \mu} = \frac{w_{\perp}}{B} \frac{\partial \vec{F}_2}{\partial \mu}$$

Therefore, eq. (54) becomes

$$\frac{D\vec{F}_2}{Dt} - \cancel{\mu \frac{1}{B} \frac{DB}{Dt} \frac{\partial \vec{F}_2}{\partial \mu}} + \cancel{\frac{1}{B} \frac{DB}{Dt} \frac{w_{\perp}^2}{2B} \frac{\partial \vec{F}_2}{\partial \mu}} + \text{the rest}$$

μ derivatives disappear, which is a manifestation of the conservation of μ — it enters the distribution "as a parameter"

So:

$$\boxed{\frac{D\vec{F}_2}{Dt} + \left(\frac{q_2}{m_2} E_{\parallel} - \frac{D\vec{u}}{Dt} \cdot \vec{b} - \mu \nabla_{\parallel} B \right) \frac{\partial \vec{F}_2}{\partial w_{\parallel}'}} = \left(\frac{\partial \vec{F}_2}{\partial t} \right)_c \quad (58)$$

parallel electric field.

"mirror force"

a non-inertial force to do with \parallel motion and with the plane of $\vec{E} \times \vec{B}$ tilting as particle streams along \vec{b}

From (58), we calculate

$$\begin{aligned} p'_{\parallel} &= \sum_{\alpha} m_{\alpha} \int d^3 \vec{w} w_{\parallel}^{1/2} F_{\alpha} \quad \leftarrow \int d^3 \vec{w} = \int dw_{\perp} w_{\perp} \int dw_{\parallel} \cdot 2\pi \\ &\qquad\qquad\qquad = 2\pi B \int dw_{\perp} dw_{\parallel} \\ p_{\perp} &= \sum_{\alpha} m_{\alpha} \int d^3 \vec{w} \mu B F_{\alpha} \\ p &= \sum_{\alpha} m_{\alpha} \int d^3 \vec{w} F_{\alpha} \end{aligned} \quad (59)$$

These go into the momentum equation (35), which also needs \vec{B} — it gets it from induction equation (34).

But what about E_{\parallel} — where do we get that from?

Note that (31) was only \vec{E}_{\perp} ($\gg E_{\parallel}$ evidently).

Well, the only equation we have not used is Gauss Law:

$$\nabla \cdot \vec{E} = 4\pi \sum_{\alpha} q_{\alpha} n_{\alpha} \quad (60)$$

this term is small

Estimate: $\frac{|\nabla \cdot \vec{E}|}{4\pi n e} \sim \frac{k u B}{4\pi n e c} \sim \frac{k m_e u e B}{4\pi e^2 n_e m_e c} \sim$

Physically, $\left. \begin{array}{c} \sim \frac{k u \Omega_e}{\omega_{pe}^2} \sim \frac{k^2 v_{the}^2}{\omega_{pe}^2} \frac{u \Omega_e}{k v_{the}^2} \\ \sim \frac{k^2 \lambda_{De}^2}{k \rho_e} \frac{u}{v_{the}} \sim \underbrace{k \lambda_{De}}_1 \frac{\lambda_{De}}{\rho_e} \frac{u}{v_{the}} \frac{M_{ai} \sqrt{\frac{m_e}{m_i}}}{1} \end{array} \right\}$
 Quasineutrality only violated at $\omega \approx \omega_{pe}$, but we have usually safely small

$$\omega \ll \lambda_{De} \ll \omega_{pe}$$

Thus, E_{\parallel} is determined implicitly from

$$\sum_{\alpha} q_{\alpha} n_{\alpha} = \sum_{\alpha} q_{\alpha} \int d^3 \vec{w} \vec{f}_{\alpha} = 0 \quad (61)$$

This completes our "hybrid" fluid-kinetic system, known as "Kinetic MHD".

We are ready to start studying its implications but this time, I will go for delayed, rather than instant, gratification and spend some time tertiating back to basics and discussing how kinetic equations are derived directly from particle motion. This will serve two purposes:

- solidify the physical interpretation of kMHD in terms of particle motion and μ conservation
- offer a general scheme for systematic improvement of approximations
- set us up for the derivation of low-flow drift kinetics.

In this discussion, I will be strongly influenced by Felix Parra's notes for this course.