Statistical Physics (A1) — A. Schekochihin
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PART I. KINETIC THEORY

§1. From certainties to probabilities

SUMMARY

Many-body systems:
- Sensitivity to initial conditions
- Too much info
- We want to calculate bulk properties

} ⇒ Statistical description

Distribution functions:

- $f(x_0)dx$ = probability that $x \in [x_0, x_0+dx]$
- Joint pdfs $f(x,y)$ probability density for x AND y
- x, y independent ⇒ $f(x,y) = f(x)f(y)$
- Averages $\langle x \rangle = \int dx\, x\, f(x)$ etc.

  Linear operation: $\langle ax + by + c \rangle = a\langle x \rangle + b\langle y \rangle + c$

  x, y independent ⇒ $\langle xy \rangle = \langle x \rangle \langle y \rangle$

  Variance $\sigma_x^2 = \langle x^2 \rangle - \langle x \rangle^2$ "width of distribution"

- Transformation of variables $(x,y) \to (u,v)$

$$\underbrace{\tilde{f}(u,v)}_{\text{pdf of }(u,v)} = \underbrace{f(x,y)}_{\text{pdf or }(x,y)} \underbrace{\left|\frac{\partial(x,y)}{\partial(u,v)}\right|}_{\text{Jacobian of transformation}}$$

Particle distributions

$$F(\vec{r},\vec{v})\, d^3\vec{r}\, d^3\vec{v} = \#\text{ of particles in cube } d^3\vec{r}$$
with velocities in cube $d^3\vec{v}$

$\int d^3\vec{v}\, F(\vec{r},\vec{v}) = n(\vec{r})$ particle # density

$\int d^3\vec{r} \int d^3\vec{v}\, F(\vec{r},\vec{v}) = N$ total # of particles

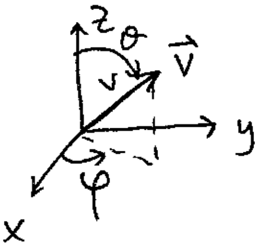
- Homogeneous system: $F(\vec{r},\vec{v}) = F(\vec{v})$ independent of $\vec{r}$
  $n = \text{const}$

$f(\vec{v}) = \frac{1}{n} F(\vec{v})$ velocity pdf $\quad \int d^3\vec{v}\, f(\vec{v}) = 1$

$f(\vec{v})\, d^3\vec{v}$ = fraction of particles in velocity space cube $d^3\vec{v} = [v_x + dv_x] \times [v_y + dv_y] \times [v_z + dv_z]$

- Isotropic system:
  $f(\vec{v}) = f(v)$
  $v = |\vec{v}|$ speed

$= v^2 dv \sin\theta\, d\theta\, d\varphi$



(v, θ, φ) polar coordinates in velocity space

Speed distribution:

$$\tilde{f}(v)\, dv = \int_0^{2\pi} d\varphi \int_0^{\pi} d\theta \sin\theta\, dv\, v^2 f(v) = 4\pi v^2 f(v)\, dv$$

Pressure

p = momentum per unit time per unit area

$$= \int d^3\vec{v}\, m v_x^2 F(\vec{r},\vec{v})$$

homogeneous

$$= \int d^3\vec{v}\, m n v_x^2 f(\vec{v}) = m n \langle v_x^2 \rangle$$

$$= \frac{1}{3} m n \langle v^2 \rangle$$

isotropic