Handout 9

Thermal Radiation

Basic quantities

Energy density, u — electromagnetic energy per unit volume.

Photon flux, Φ — number of photons incident on unit area per second.

Energy flux, E_e — electromagnetic energy incident on unit area per second.

Radiation pressure, p — pressure exerted on a surface exposed to electromagnetic radiation.

From kinetic theory,

$$\begin{aligned} p &= \frac{1}{3}u \qquad \text{isotropic radiation} \\ &= u \qquad \text{beam of radiation.} \end{aligned}$$

These expressions are valid whether or not any radiation is reflected by the surface (if radiation is reflected then both p and u increase accordingly).

Other relations from kinetic theory:

$$\begin{aligned} \Phi &= \frac{1}{4}nc \\ E_e &= \frac{1}{4}uc. \end{aligned}$$

Thermal radiation spectral function

The **spectral energy density**, describes how electromagnetic energy is distributed with wavelength or (angular) frequency:

$u_\lambda \mathrm{d}\lambda$ = energy density contained in radiation with wavelengths between λ and $\lambda + \mathrm{d}\lambda$

$u_\omega \mathrm{d}\omega$ = energy density contained in radiation with angular frequencies between ω and $\omega + \mathrm{d}\omega$

Hence,

$$u = \int_0^\infty u_\lambda \mathrm{d}\lambda = \int_0^\infty u_\omega \mathrm{d}\omega \tag{1}$$

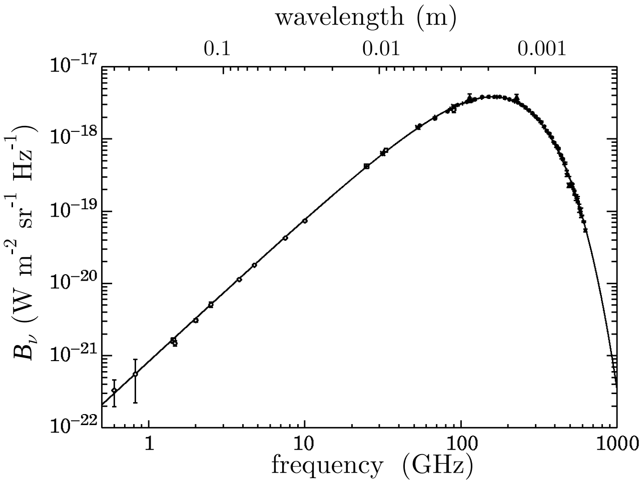
In lectures we derived the spectral energy density by treating thermal radiation in a cavity as a set of standing-wave modes with quantized energy energy levels. This results in

$$\begin{aligned} u_\lambda &= \frac{8\pi hc}{\lambda^5}\frac{1}{\mathrm{e}^{\beta hc/\lambda} - 1} \\ u_\omega &= \frac{\hbar}{\pi^2 c^3}\frac{\omega^3}{\mathrm{e}^{\beta\hbar\omega} - 1} \end{aligned} \tag{2}$$

The spectrum of thermal radiation described by these expressions is known as **Planck's law** (though why a function should be called a law is a mystery to me).

Cosmic microwave background.
The spectrum of radiation from the Big Bang fits Planck's law almost perfectly with a temperature $T = 2.73\,\text{K}$. The quantity plotted on the vertical axis is proportional to u_ω.

[Figure courtesy of S.J. Blundell & K.M. Blundell, *Concepts in Thermal Physics*, (OUP, 2006)]



Kirchhoff's radiation law

The **spectral absorptivity**, α_λ, is the fraction of incident radiation absorbed at wavelength λ. A **black body** is a material for which $\alpha_\lambda = 1$ for all λ.

The **spectral emissive power**, $e_\lambda \mathrm{d}\lambda$, is the power emitted per unit area with wavelengths between λ and $\lambda + \mathrm{d}\lambda$.

Kirchhoff's radiation law states that the ratio of emissive power to absorptive power $e_\lambda/\alpha_\lambda = f(\lambda, T)$, a universal function of wavelength and temperature, independent of the nature or shape of the cavity. This law accounts for the fact that for a given wavelength of radiation, good absorbers are also good emitters.

Stefan–Boltzmann law

The **Stefan–Boltzmann law** states that the total power radiated is proportional to T^4. The total power can be obtained by integration — eqs. (1) and (2). For a black body this gives

$$u = AT^4,$$

where $A = \pi^2 k_\mathrm{B}^4/(15c^3\hbar^3)$. Alternatively, the radiated power per unit area is given by

$$E_e = uc/4 = \sigma T^4,$$

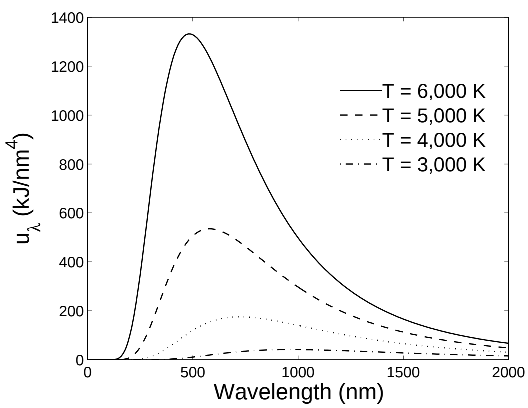
where $\sigma = \pi^2 k_\mathrm{B}^4/(60c^2\hbar^3) = 5.67 \times 10^{-8}\,\mathrm{W}^{-2}\mathrm{K}^{-4}$ is the **Stefan–Boltzmann constant** (also known as **Stefan's constant**).

Peak radiance

The maximum of the spectral energy density u_λ occurs at a wavelength λ_max which satisfies

$$\lambda_\mathrm{max} T = constant.$$

This is known as **Wien's law**. The constant is $hc/(4.97k_\mathrm{B})$.
The temperature dependence of u_λ is illustrated in the figure on the right.



ATB
Hilary 2012