
Basic Thermodynamics

Handout 5

Thermodynamic coefficients and moduli

Heat capacities:

$$C_V = T \left(\frac{\partial S}{\partial T} \right)_V, \quad C_p = T \left(\frac{\partial S}{\partial T} \right)_p$$

Compressibilities:

$$\kappa_T = -\frac{1}{V} \left(\frac{\partial V}{\partial p} \right)_T, \quad \kappa_S = -\frac{1}{V} \left(\frac{\partial V}{\partial p} \right)_S$$

Bulk moduli:

$$B_T = \frac{1}{\kappa_T} = -V \left(\frac{\partial p}{\partial V} \right)_T, \quad B_S = \frac{1}{\kappa_S} = -V \left(\frac{\partial p}{\partial V} \right)_S$$

Thermal expansivities:

$$\beta_p = \frac{1}{V} \left(\frac{\partial V}{\partial T} \right)_p, \quad \beta_S = \frac{1}{V} \left(\frac{\partial V}{\partial T} \right)_S$$

Relations between coefficients and moduli

- $C_p - C_v = VT\beta_p^2/\kappa_T$
- $\kappa_T/\kappa_S = C_p/C_V = \gamma$

Non p - V systems

- Elastic rod: $dU = TdS + fdL$
- Liquid surface: $dU = TdS + \gamma dA$
- Paramagnetism: $dU = TdS - mdB$ (magnetic material + field)
or $dU = TdS + Bdm$ (magnetic material + field + source of field)

Magnetic refrigeration

Entropy of a paramagnetic salt as a function of temperature for several different magnetic fields. Magnetic cooling from T_i to T_f is achieved in 2 steps:

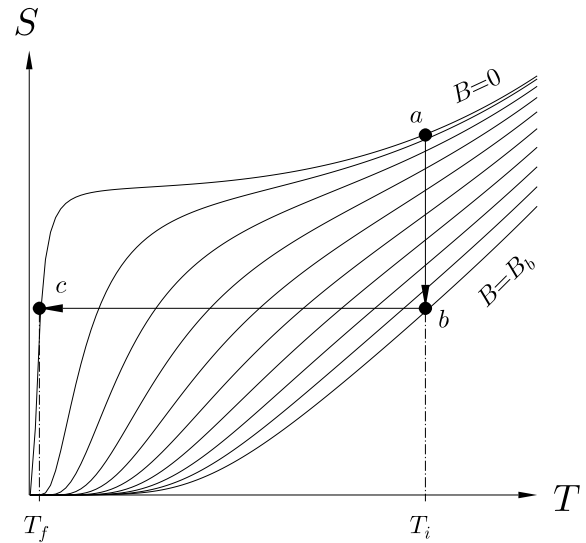
- ($a \rightarrow b$): isothermal magnetization with an applied field that increases from zero to B_b ;
- ($b \rightarrow c$): adiabatic demagnetization, in which the salt is thermally isolated and the field reduced from B_b to zero.

The $S(T)$ curves have been calculated for a spin- $\frac{1}{2}$ paramagnet from

$$S = k_B \ln Z + k_B T \left(\frac{\partial \ln Z}{\partial T} \right)_V,$$

with $Z = 2 \cosh(\mu_B B / k_B T)$. A small term proportional to T^3 has been added to simulate the entropy of the lattice vibrations. The $B = 0$ curve is actually calculated with a small non-zero B to allow for residual fields in the magnet.

Diagram of an **adiabatic demagnetization refrigerator**. The paramagnetic salt is suspended inside an evacuated volume. To magnetize the salt isothermally, the volume is filled with helium exchange gas which thermally couples the salt to the reservoir of liquid helium at a temperature of 4.2 K. The sample volume is then evacuated to thermally isolate it from the reservoir, and the magnetic field reduced to zero adiabatically. The liquid nitrogen jacket serves to reduce the heat conduction rate from the outside.



[Figure courtesy of S.J. Blundell and K.M. Blundell, *Concepts in Thermal Physics*, (OUP, 2006)]

