

# *Black Holes and Fundamental Physics*

*Frans Pretorius*  
*Princeton University*

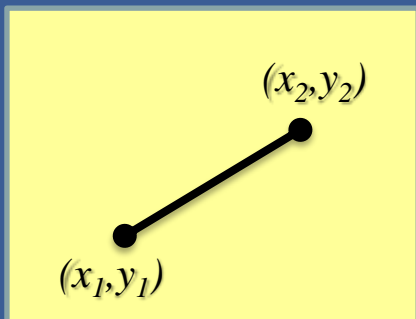
Occam Lecture  
Oxford, Dec 2, 2013

# Outline

- Spacetime geometry, gravity and black holes
- Black holes and thermodynamics
- The AdS/CFT revolution
  - connections between general relativity and quantum field theories
  - the hydrodynamic limit
- Conclusions

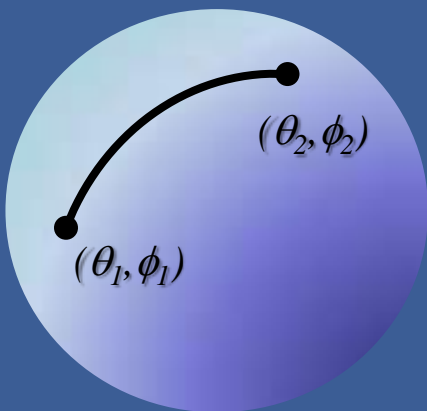
# Space as geometry

- That a space can have a geometric structure is a familiar concept
  - e.g. the shortest distance between two points on a plane is encoded by Pythagoras' theorem :



$$L = \sqrt{\Delta x^2 + \Delta y^2}$$

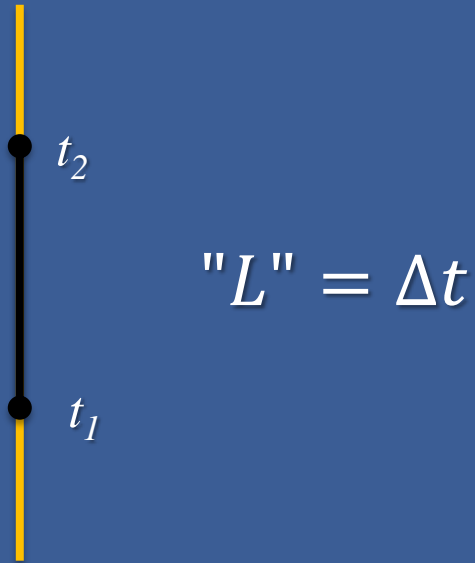
- similar though more complicated results hold in spaces that have curvature, e.g. the surface of a sphere :



$$L = R \cos^{-1}(\sin \varphi_1 \sin \varphi_2 + \cos \varphi_1 \cos \varphi_2 \cos \Delta\theta)$$

# Time as geometry

- In Newtonian mechanics time can also be viewed as being a measure in a 1-dimensional geometry, though this is trivial as a line has no *intrinsic* curvature

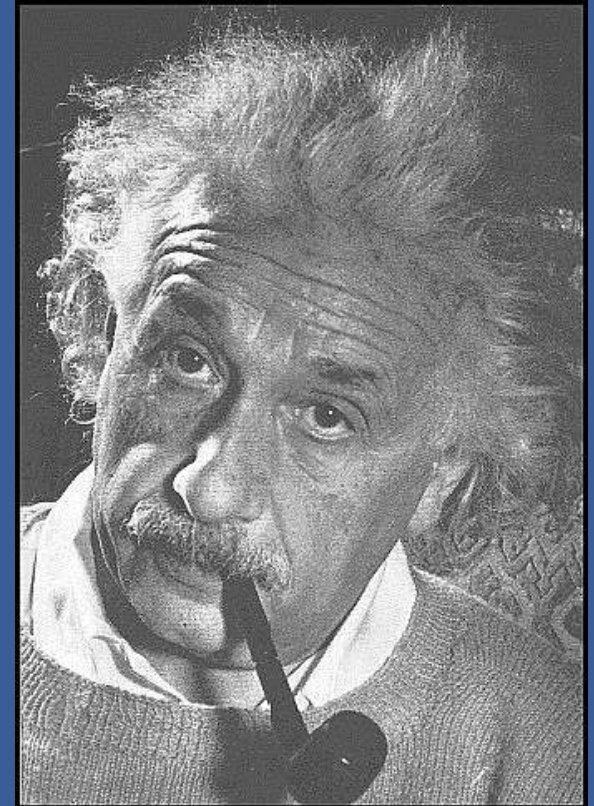


# Special Relativity

- In 1905 Einstein introduced the theory of special relativity, which can most succinctly be summarized as claiming space and time, or *spacetime*, are together manifestations of a 4D geometric structure called Minkowski spacetime
- Minkowski spacetime is flat (no curvature), but the unusual property is that time adds *negative* distance squared in the 4D version of Pythagoras' theorem :

$$L^2 = \Delta x^2 + \Delta y^2 + \Delta z^2 - c^2 \Delta t^2$$

- Some interesting (not obvious!) consequences are
  - the speed of light  $c$  is a universal constant of nature in any reference frame
  - “nothing” can travel faster than light
  - energy and mass are intimately related concepts, quantified by the famous formula  $E=mc^2$



# General relativity

- Einstein immediately realized Newton's theory of gravity was inconsistent with SR, as it allowed instantaneous propagation of gravitational fields → lead him (after arduous work) to general relativity (GR), published in 1915
- In SR spacetime is *passive*, i.e. it is the arena in which events happen, but none of the events can influence it
- In GR spacetime becomes a dynamical structure, not only influencing the objects that exist in it, but matter/energy can now shape the geometric structure of spacetime
  - exactly how this happens is described by the *Einstein field equations*
  - a small test mass with no forces acting on it follows a *geodesic* of the spacetime geometry, i.e. a curve extremizing the distance between two points
  - curved geometry with the geodesic hypothesis is enough to completely describe what we think of as the “force of gravity”

# Point particle force in Newtonian gravity

- Consider a simple question in gravitational physics: *what is the gravitational field produced by a point particle of mass  $M$  ?*
- In Newton's theory, the answer is as simple: *the field is an attractive force, experienced by all other matter in the universe, mathematically described by Newton's universal law of gravitation  $F = - G M m / r^2$* 
  - $F$  is the force experienced by a test particle of mass  $m$ ,  $G$  is Newton's constant, and  $r$  is the separation between  $m$  &  $M$

## The analogous question in GR

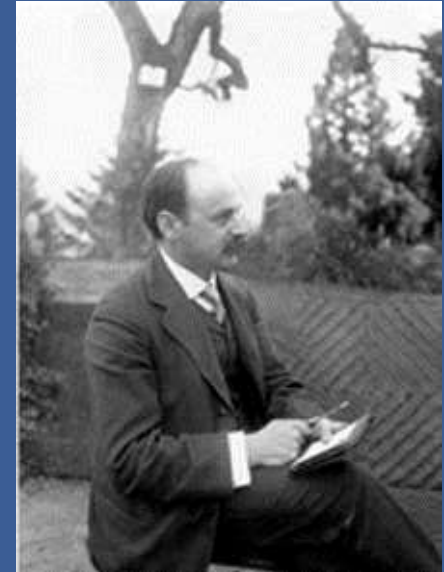
- *What geometric structure is produced by a point particle of mass  $M$*

*Answer: a black hole!*



# The first black hole solution

- In **1916**, just one year after Einstein published the theory of general relativity, Karl Schwarzschild found the first exact solution to the field equations
  - though not fully appreciated until several decades later, Schwarzschild's solution describes the spacetime of a *non-rotating black hole*
- It is characterized by a single free parameter, the *Schwarzschild Radius*  $r_s$ , which can be related to the mass  $M$  of the black hole via  $r_s = 2GM/c^2$
- $r_s$  is the location of the event horizon of the black hole. Nothing inside the event horizon, not even light, can escape
- $r_s$  also tells us much you have to compress an object before it will collapse into a black hole
  - For the sun,  $r_s \sim 3\text{km}$ ; for the earth,  $r_s \sim 1\text{cm}$

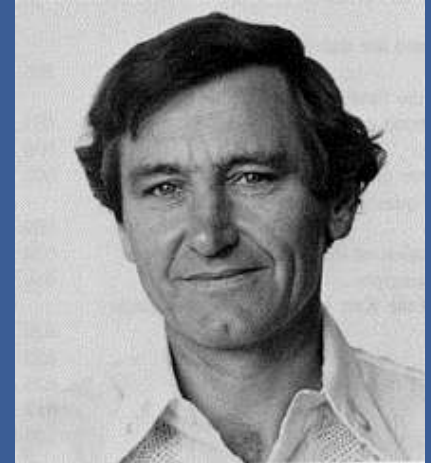


# Golden Age of Black Hole Physics

- The 1960's and 1970's witnessed an explosion in our theoretical understanding of black holes, and the realization that black holes could be important denizens of our universe
- In 1963, Roy Kerr discovered a black hole solution describing rotating black holes

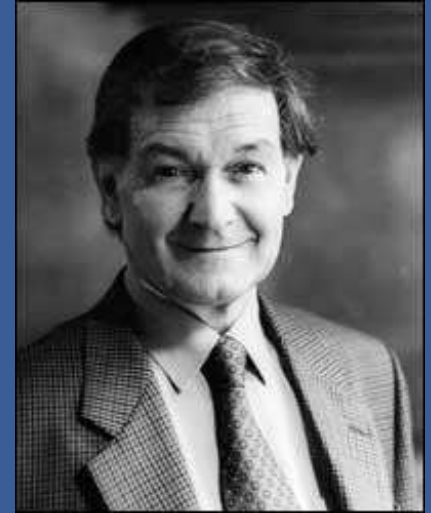


- In 1967, Werner Israel proved the “no-hair” theorem for non-rotating black holes in vacuum. Later extended to rotating black holes by Carter and Robinson; it said that only *two* numbers, mass  $M$  and angular momentum  $J$ , uniquely describes *any* black hole geometry
  - The astonishing consequence of this is that the Kerr solution describes the *exact* geometry of *all* isolated, macroscopic black holes in the universe



# Golden Age of Black Hole Physics

- In 1964 Roger Penrose proved that *singularities* in the spacetime geometry generically arise inside black holes; this implies that there is a time inside the black hole when general relativity fails to provide a correct description of nature



- As an aside, all this work was done before the nomenclature “black hole” existed! John Wheeler is attributed to coining the name in 1968

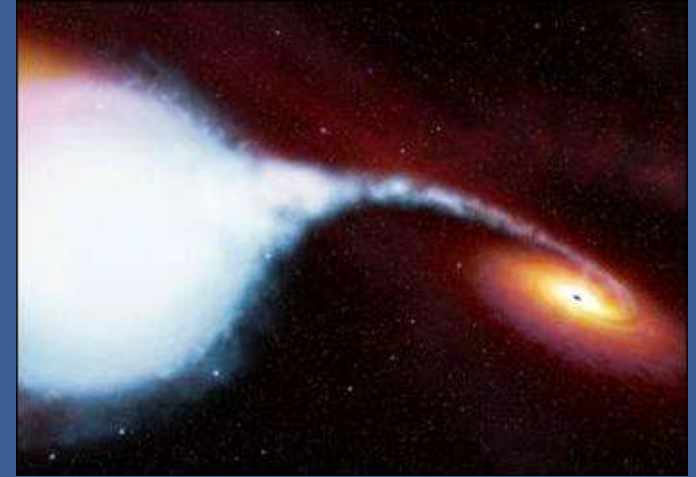
# Golden Age of Black Hole Physics



- Contemporaneously astronomers were finding evidence that black holes might actually exist in our universe
- In 1971, Louise Webster and Paul Murdin, and Thomas Bolton independently discovered a 15-20 solar mass “invisible” companion to a star associated with the x-ray source Cygnus X-1; can only be a black hole within conventional theory



- In 1969, Donald Lynden-Bell proposed that newly discovered quasars, almost point-like sources of light with extreme luminosity, could be explained if the gravitational potential energy of a black hole was the source of the power





# Black Holes and Thermodynamics

- The hints that black holes have “non-gravitational” properties began in the early 70’s (Bardeen, Carter, Hawking, Bekenstein) when it was realized that throwing matter into a black hole changed its properties in a manner exactly analogous to the laws of thermodynamics:

$$dE = TdS + \Omega dJ$$

$$dS \geq 0$$

if the identifications (Planck units)

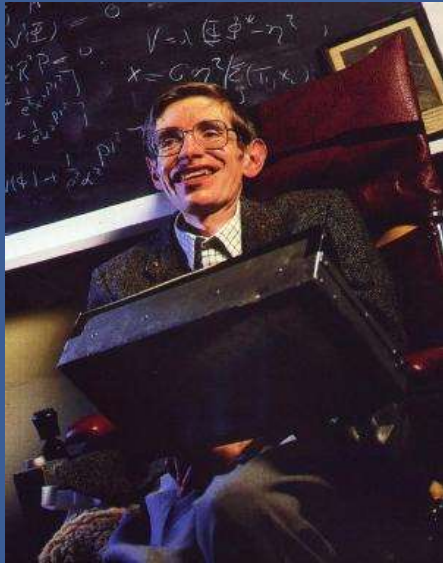
$$E \Leftrightarrow M$$

$$T \Leftrightarrow \kappa/2\pi$$

$$S \Leftrightarrow A/4$$

are made, where  $\kappa=M/r_s^2$  is the “surface gravity” of the black hole of mass  $M$ , area  $A$ , horizon angular velocity  $\Omega$  and angular momentum  $J$

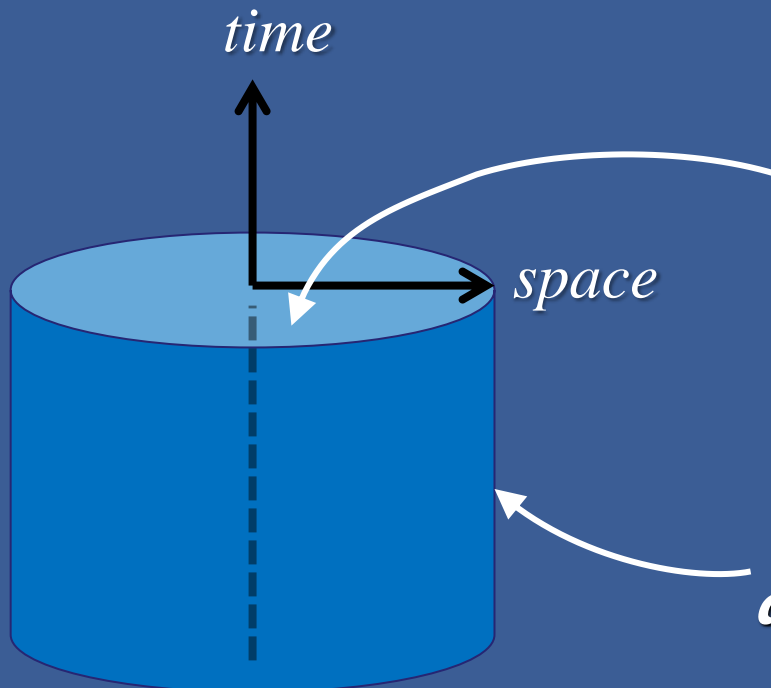
# Black Holes and Thermodynamics



- These laws were thought to be a mere curiosity until Hawking's remarkable discovery in 1974 that when quantum effects are included black holes radiate like thermal black bodies with a temperature  $T$ 
  - $T$  scales inversely with the mass of the black hole, and is extremely small for black holes in the universe; e.g.  $T_{\odot} \sim 60$  nano K
- More important than just the discovery that black holes can evaporate, was what this implied about GR : there is a more fundamental, likely quantum theory, underlying the geometry of spacetime
  - this has served as a Rosetta stone guiding the search for a quantum theory of gravity , in that any candidate theory (barring any better suggestions from experiment!) must be able to account for the entropy of a black hole via “counting microstates”
  - still a source of inspiration and confusion with the *information loss problem*

# To the 21<sup>st</sup> Century : AdS/CFT

- In 1998, Juan Maldacena solidified ideas that were floating around about the likely “holographic” nature of certain quantum theories in what has become known as the *Anti de Sitter/Conformal Field Theory* correspondence conjecture



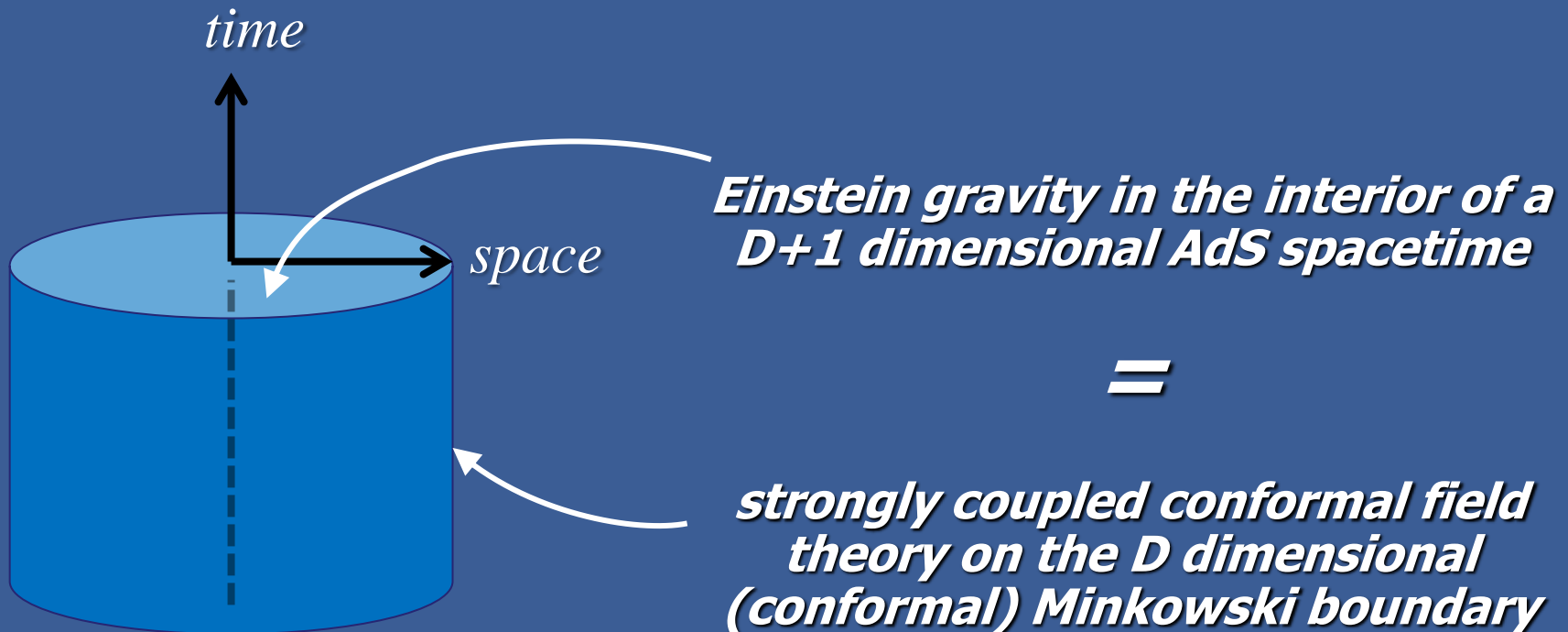
***String theory in the interior of a  $D+1$  dimensional AdS spacetime***

***=***

***conformal field theory on the  $D$  dimensional (conformal) Minkowski boundary***

# Strong coupling limit of AdS/CFT

- This is a quantum  $\leftrightarrow$  quantum holographic duality, but in the limit where the boundary theory becomes strongly interacting, quantum fluctuations in the bulk are suppressed and string theory reduces to classical Einstein gravity : *it becomes a classical  $\leftrightarrow$  quantum duality!*



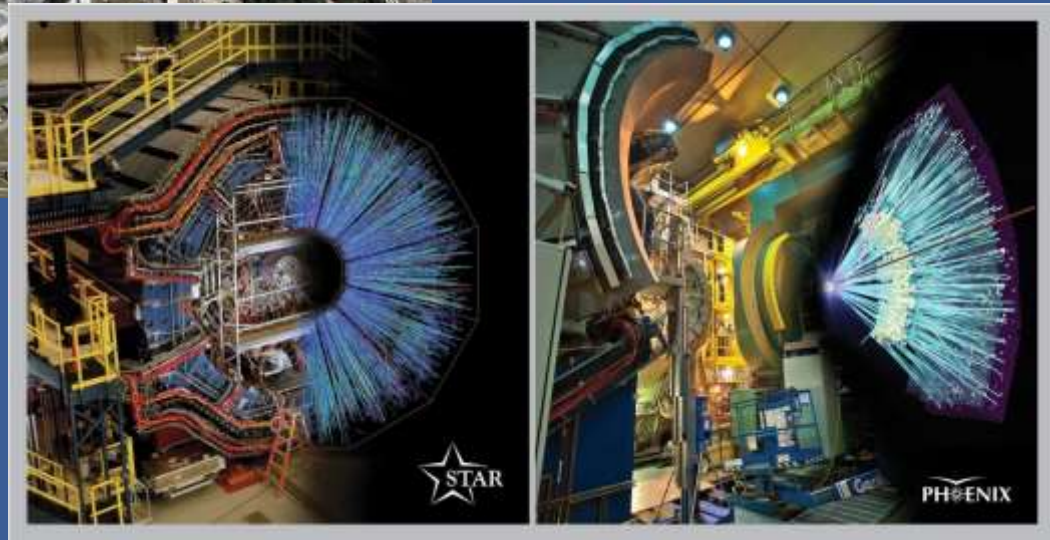


# “Real world” applications of AdS/CFT

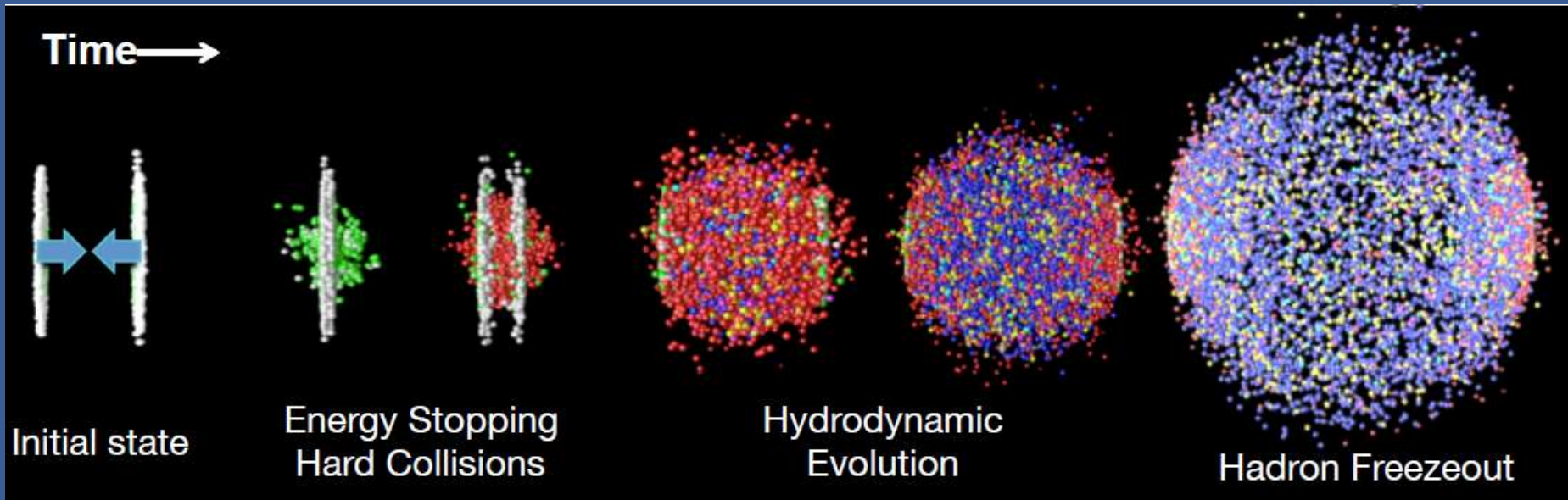
- Quantum Chromodynamics (QCD) is an example of a strongly coupled quantum field theory, as are many effective field theories describing condensed matter physics, including superconductors.
- Though different in many details from the type of theory arising in AdS/CFT, QCD is similar enough that it has been suggested AdS/CFT can be used to study some aspects of QCD, in particular quark-gluon plasma formation in heavy ion collisions
  - Why bother? Strongly coupled QFTs very difficult to solve. Classical gravity is easier : solve the “dual” problem in the former, then map the answer back to the QFT problem using the dictionary of AdS/CFT
  - Note : this can be a perfectly valid tool even if string theory is not the correct theory of quantum gravity!

# Quark-Gluon Plasma formation in Relativistic Heavy Ion Collisions

- The Relativistic Heavy Ion Collider at Brookhaven, and now the Large Hadron Collider at CERN, study the properties of the QGP formed in heavy ion collisions



# Heavy Ion Collisions



*From T. Nayak, Pramana 79 (2010)*



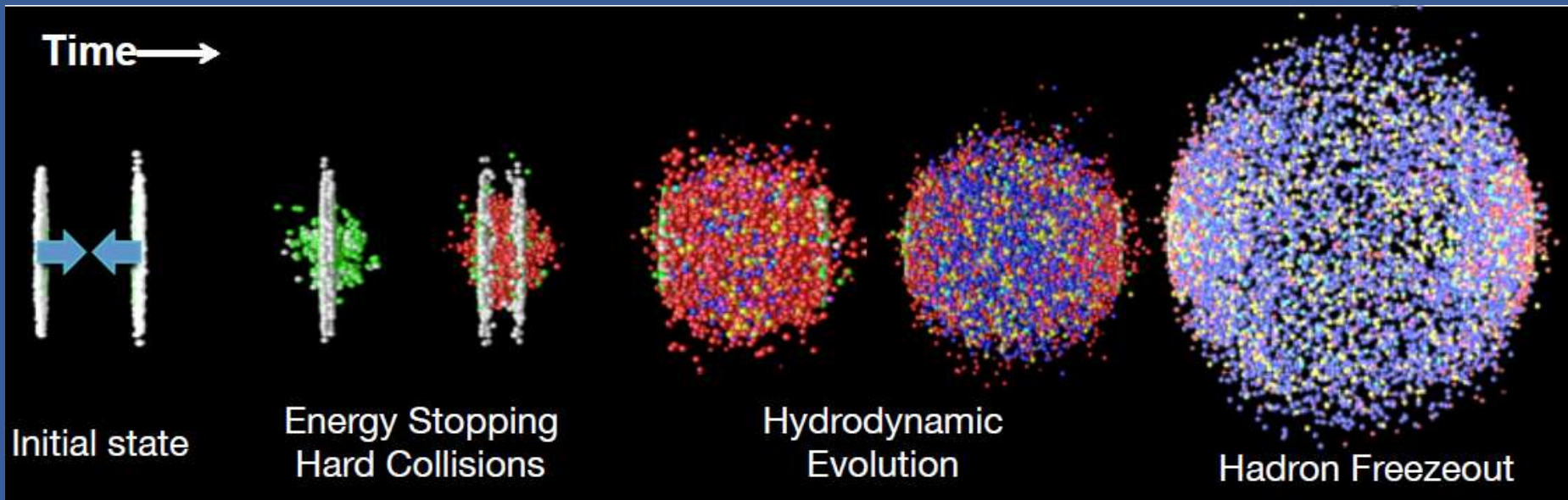
# Gravity Dual

Two  
gravitational  
“shock”  
waves  
collide

A highly  
distorted,  
dynamical  
black hole  
forms

The black  
hole “rings  
down” to a  
static  
solution

No  
confinement  
in a CFT



*From T. Nayak, Pramana 79 (2010)*

# Black Holes and the QGP

- The preceding approach has proven reasonably successful in describing the pre-hadronization physics
  - The inferred thermalization timescales consistent with black hole formation time, as are properties of the black hole that map to the CFT description

- Temperature  $\leftrightarrow$  black hole size
- Hydrodynamic properties (sound speed, viscosity)  $\leftrightarrow$  properties of black hole quasi-normal modes; i.e. how the black hole “loses it’s hair”

(interesting, some of this was already hinted at in the 70’s and 80’s with the “membrane paradigm” of Damour, Thorne, Price & MacDonland)

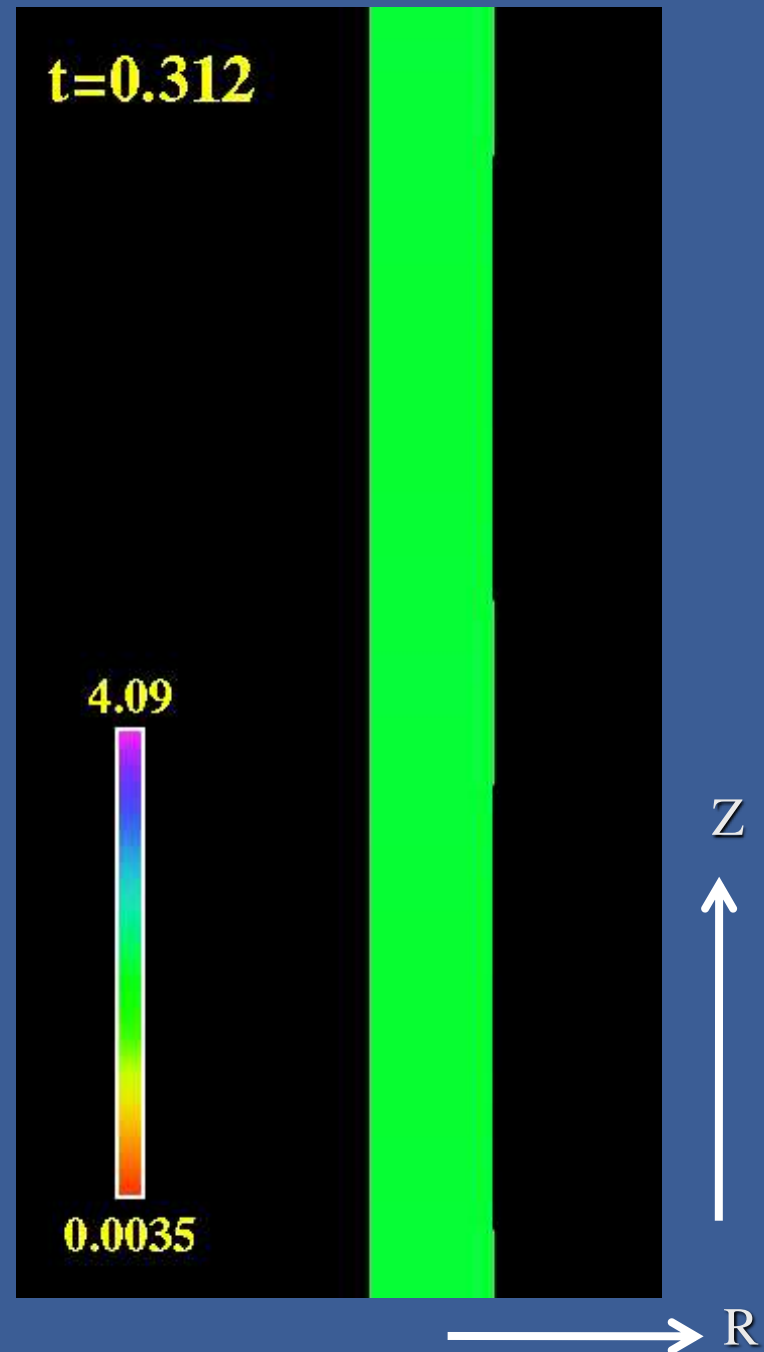
# Black Holes and Hydrodynamics

- Inspired by AdS/CFT and the success of describing the hydrodynamic phase of the QGP with it, people have realized that Einstein gravity admits a hydrodynamic limit, regardless of the correspondence [Bhattacharyya et al, Emparan et al, etc.]
  - The fluctuations of black hole spacetimes governed by the Einstein equations can be mapped to a relativistic Navier-Stokes equation with particular transport properties
  - This was shown perturbatively; relativists argued this is as far as it went, as we “knew” the two theories have vastly different non-linear regimes : black holes and gravity do not exhibit properties common to fluids such as shocks, turbulence, instabilities that can “break” the flow, etc.

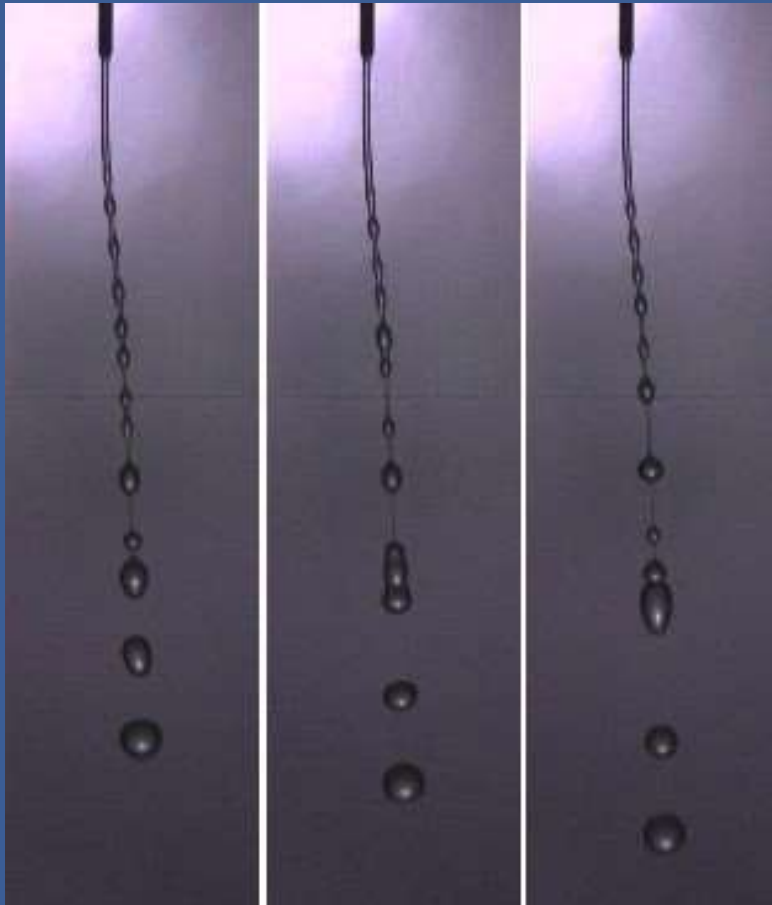
# Evolution of an 5D Black String

- A 5D black string has the geometry of a 4D Schwarzschild black hole with an extra, flat spatial
- Gregory & LaFlamme (1993) showed it was unstable to perturbations in the extra dimension

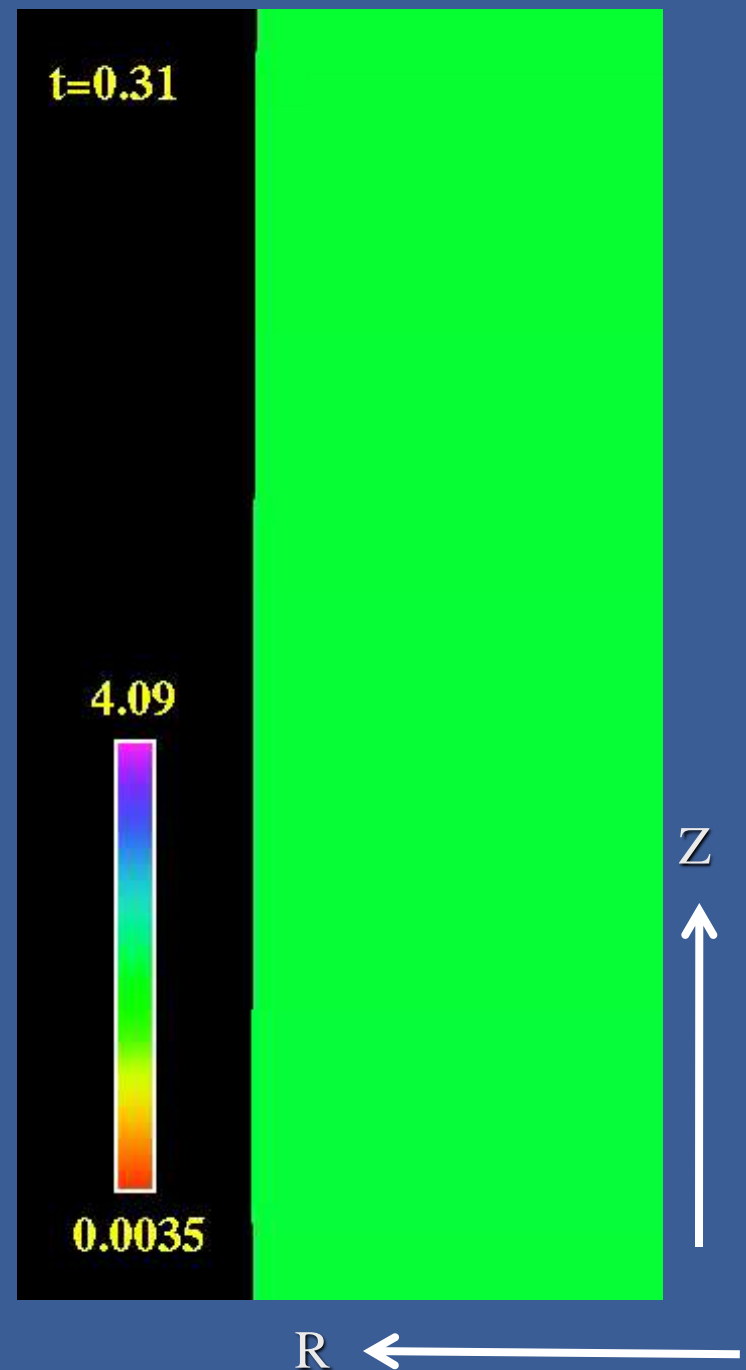
*Lehner and FP, PRL 105 (2010)*



close-up and in "real time"

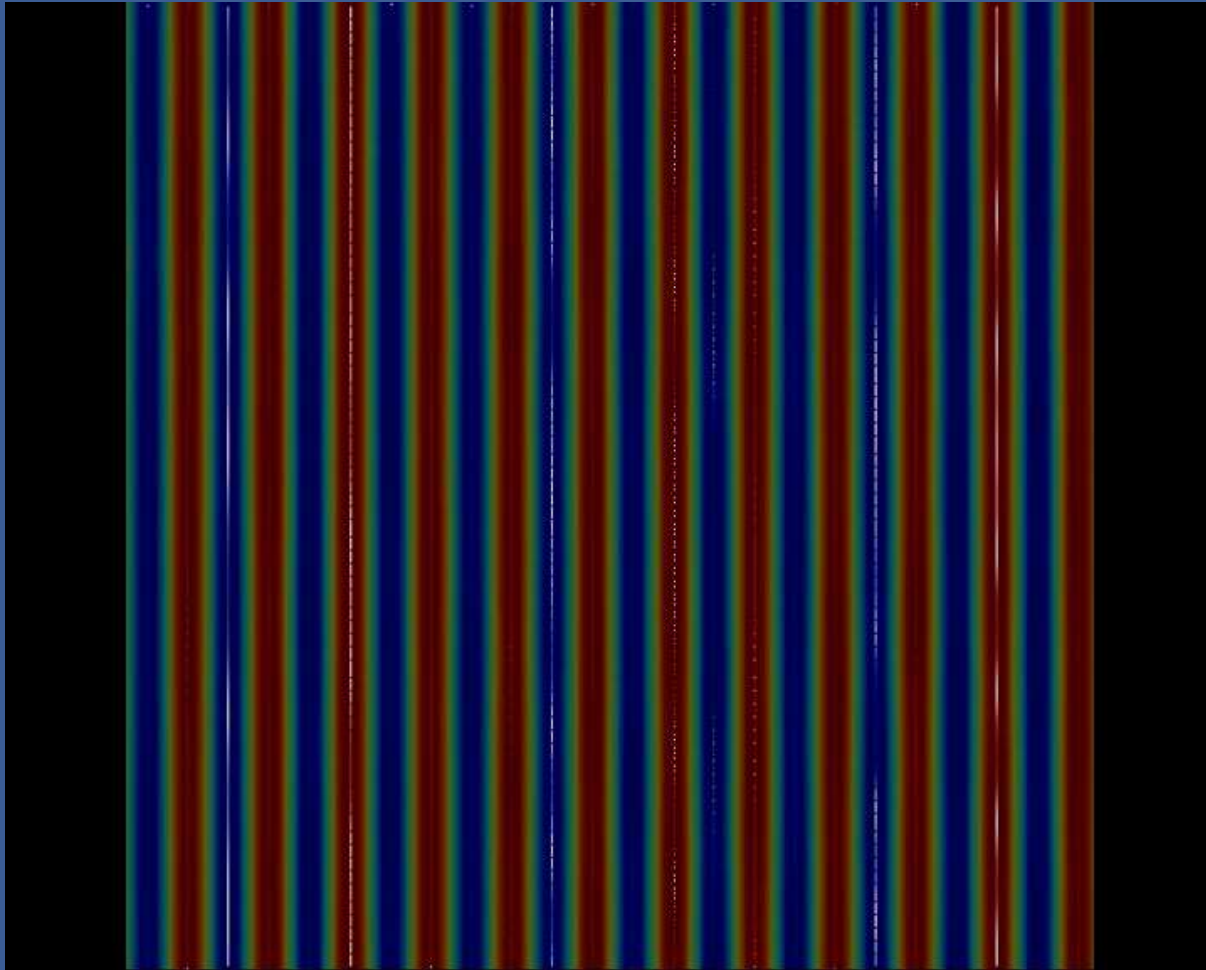


- *Gregory LaFlamme*  $\leftrightarrow$  *Raleigh-Plateau!*





# Turbulence in a perturbed 4D black brane



*Adams, Chesler and Liu (2013)*

# Why was this missed before?

- Most of the knowledge about Einstein gravity gained from looking at 4D, asymptotically flat solutions
  - Uniqueness theorems only allow Kerr solutions, which in the fluid analogue have very low Reynolds number. For a relativistic conformal fluid this number works out to be

$$R_e \sim TL$$

For a Kerr a high temperature ( $T \sim 1/M$ ) means small black holes ( $L \sim M$ ); one can never reach the turbulent regime.

- Cosmic censorship has not been proven in the 4D asymptotically flat case; perhaps sufficiently “violent” perturbations of Kerr *can* break it up?

# Conclusions

- In classical general relativity, the notion of a black hole can succinctly be described by the following (very technical!) definition of an event horizon

*A non-empty boundary of the causal past of future infinity*

- Spacetimes that possess such structures have profound and rich dynamics, whether the “familiar” 4D black holes we think exist in our universe, or ones that live in higher dimensional theoretical constructs
- In many cases the properties of black holes can be mapped, often exactly, to phenomena and laws in entirely different branches of physics – *why?*