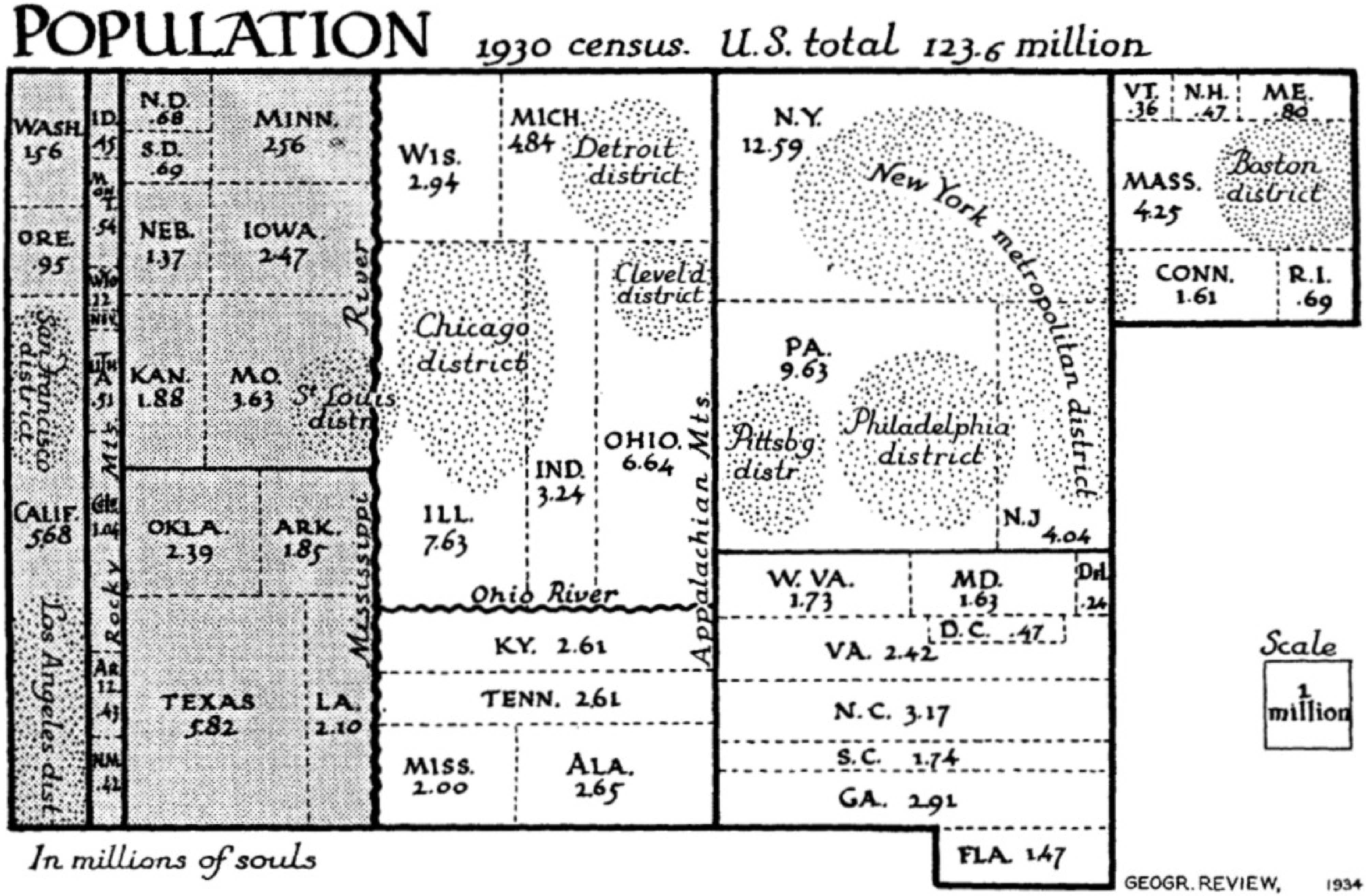
Physics in Unexpected Places

Mark Newman
University of Michigan
and
Santa Fe Institute



E. Raisz, *The Geographical Review* (1934)

The diffusion cartogram

We need a process that moves population away from high-density areas into low-density ones until everything is uniform.

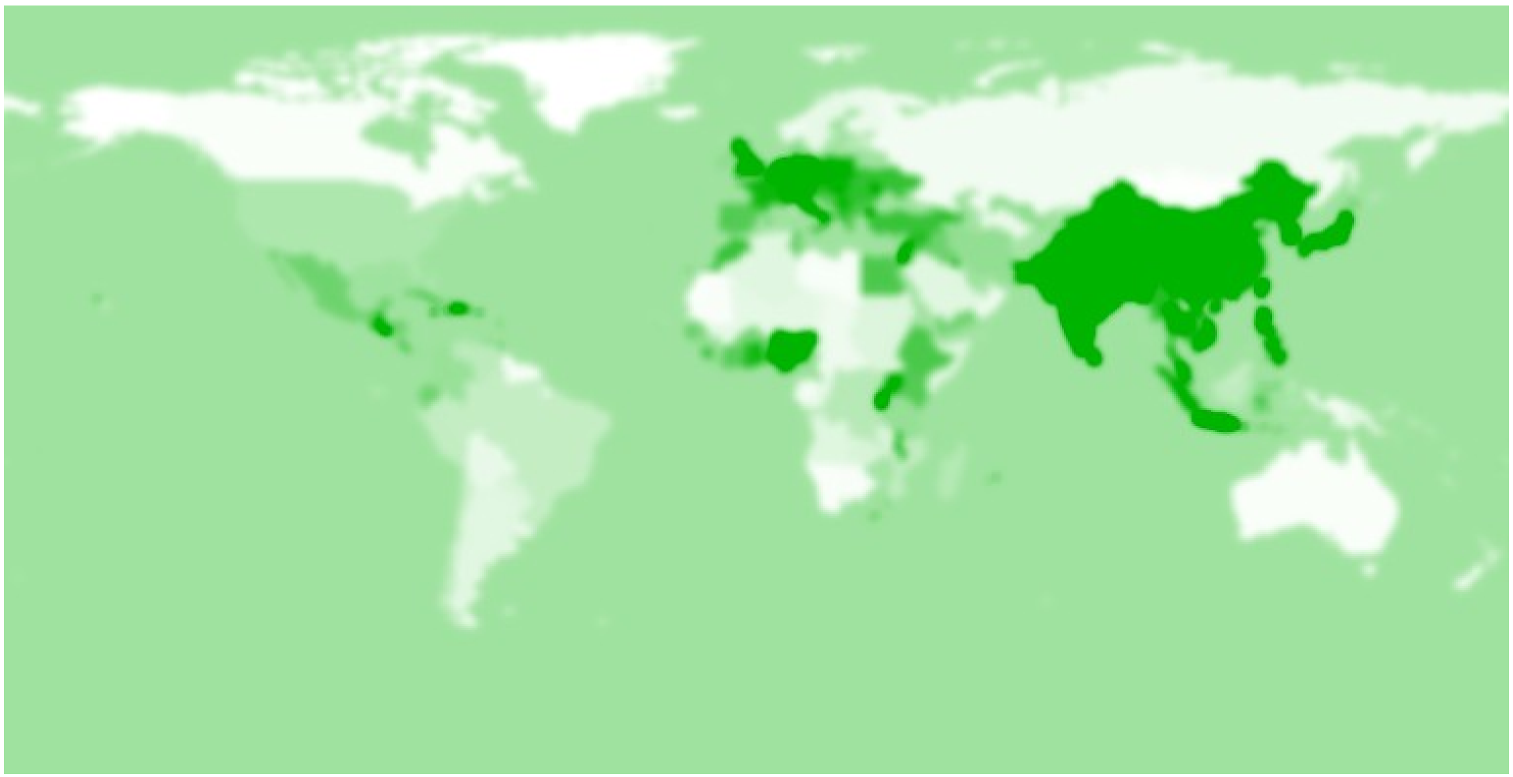
$$\mathbf{J} = \mathbf{v}(\mathbf{r},t)\,\rho(\mathbf{r},t) \qquad \text{and} \qquad \mathbf{J} = -\nabla\rho,$$

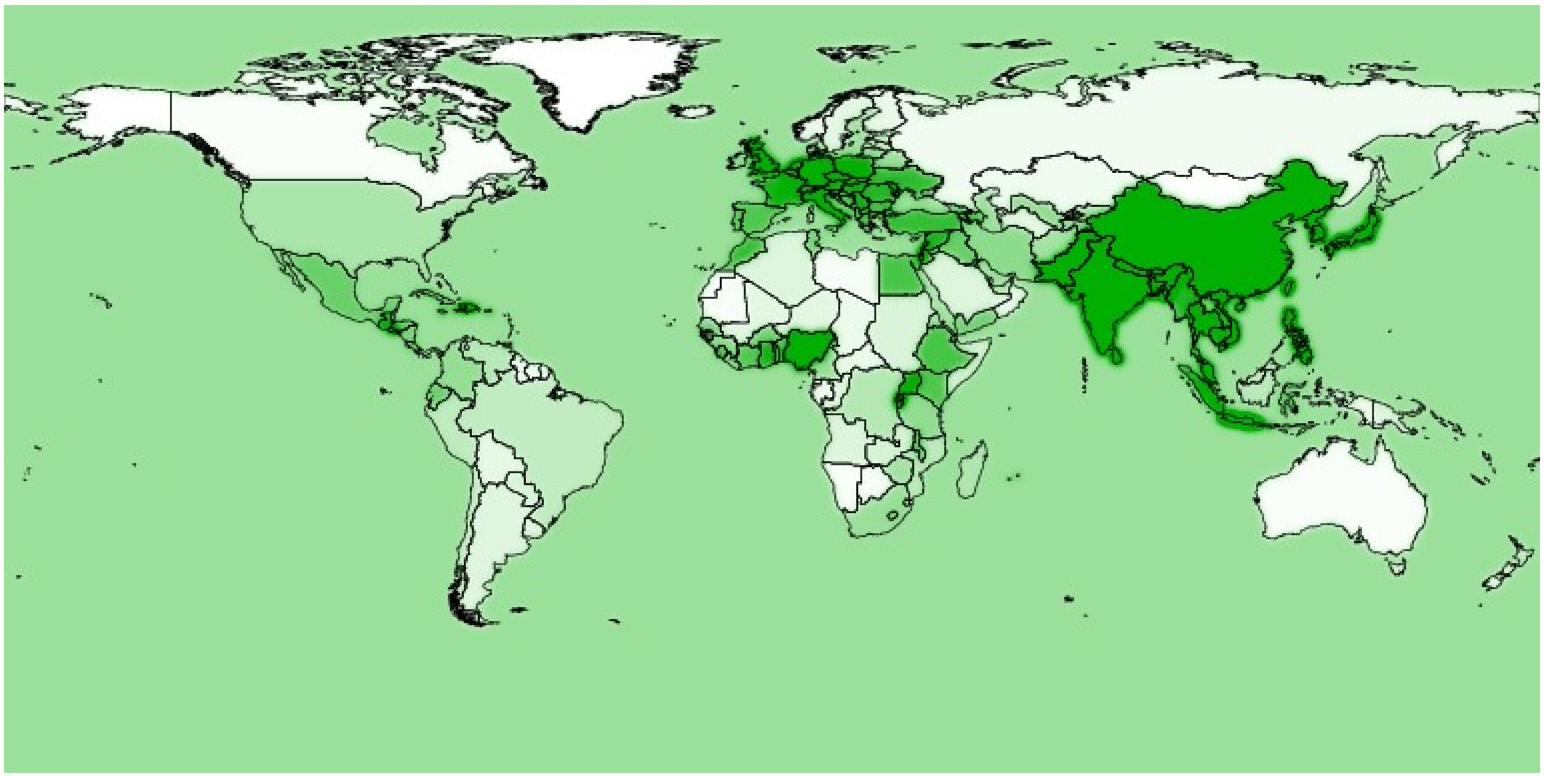
The diffusing population is conserved locally:

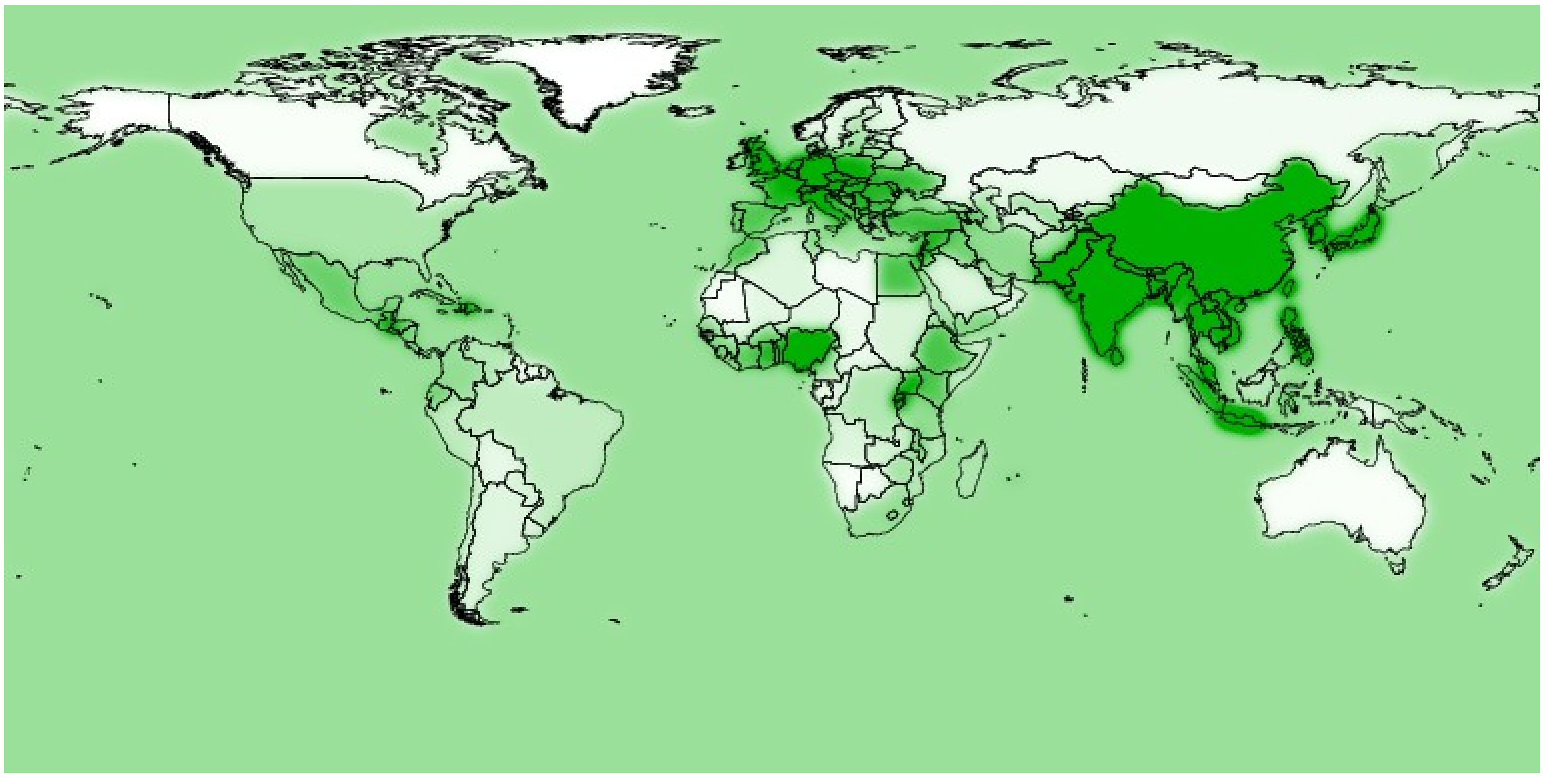
$$\nabla \cdot \mathbf{J} + \frac{\partial \rho}{\partial t} = 0.$$

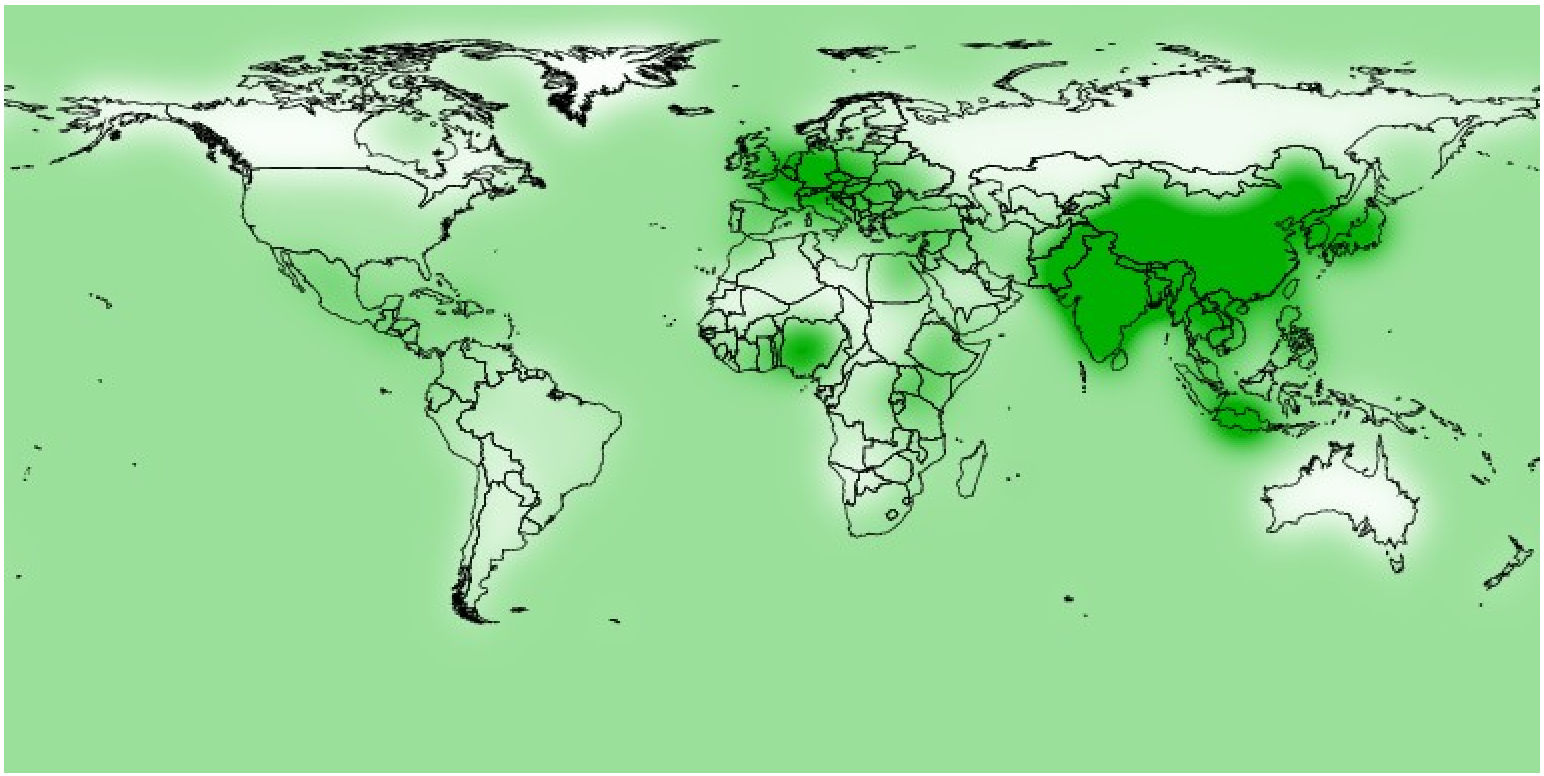
Hence

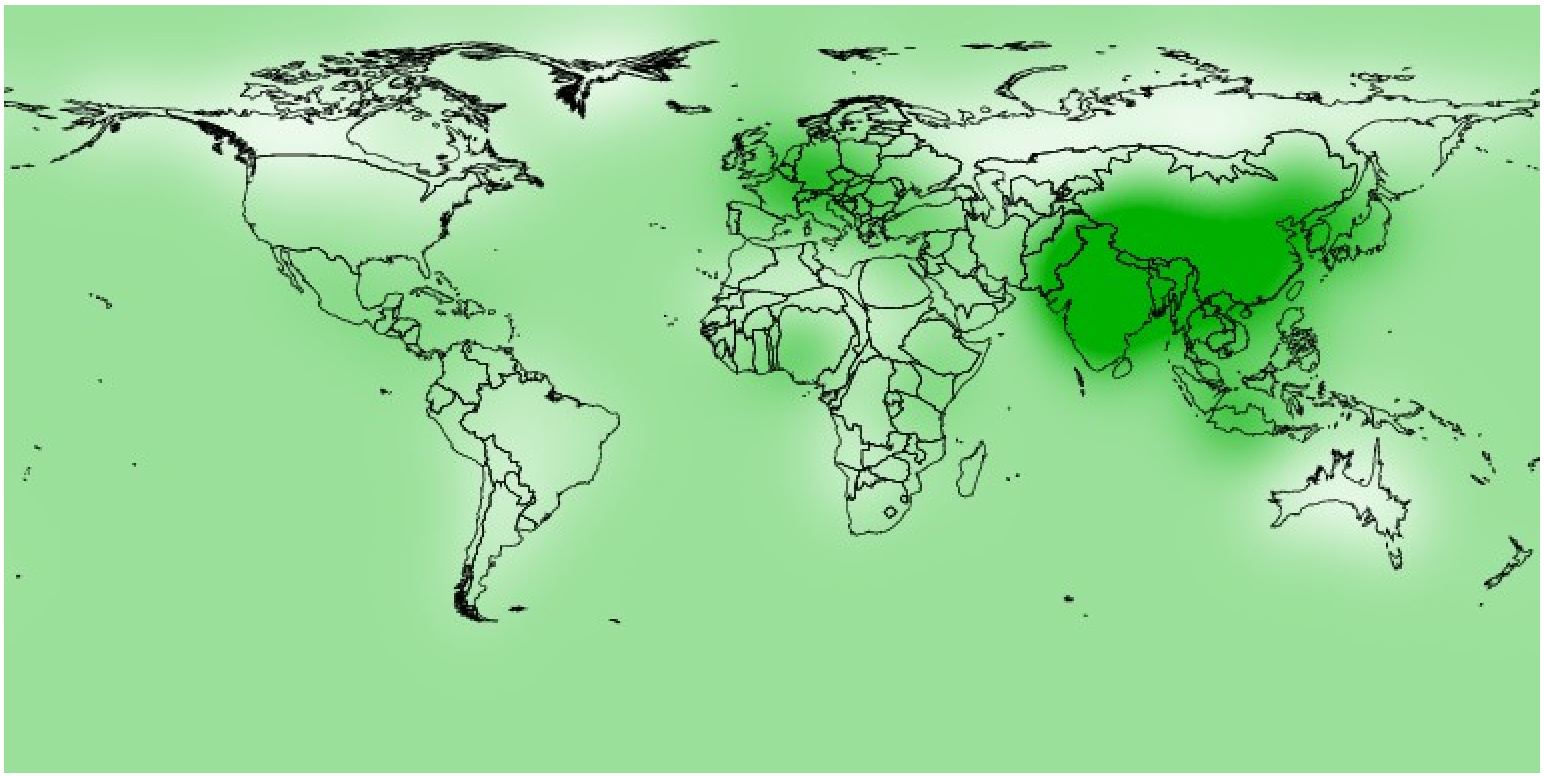
$$\nabla^2\rho - \frac{\partial \rho}{\partial t} = 0 \qquad \text{and} \qquad \mathbf{v}(\mathbf{r},t) = -\frac{\nabla\rho}{\rho}.$$

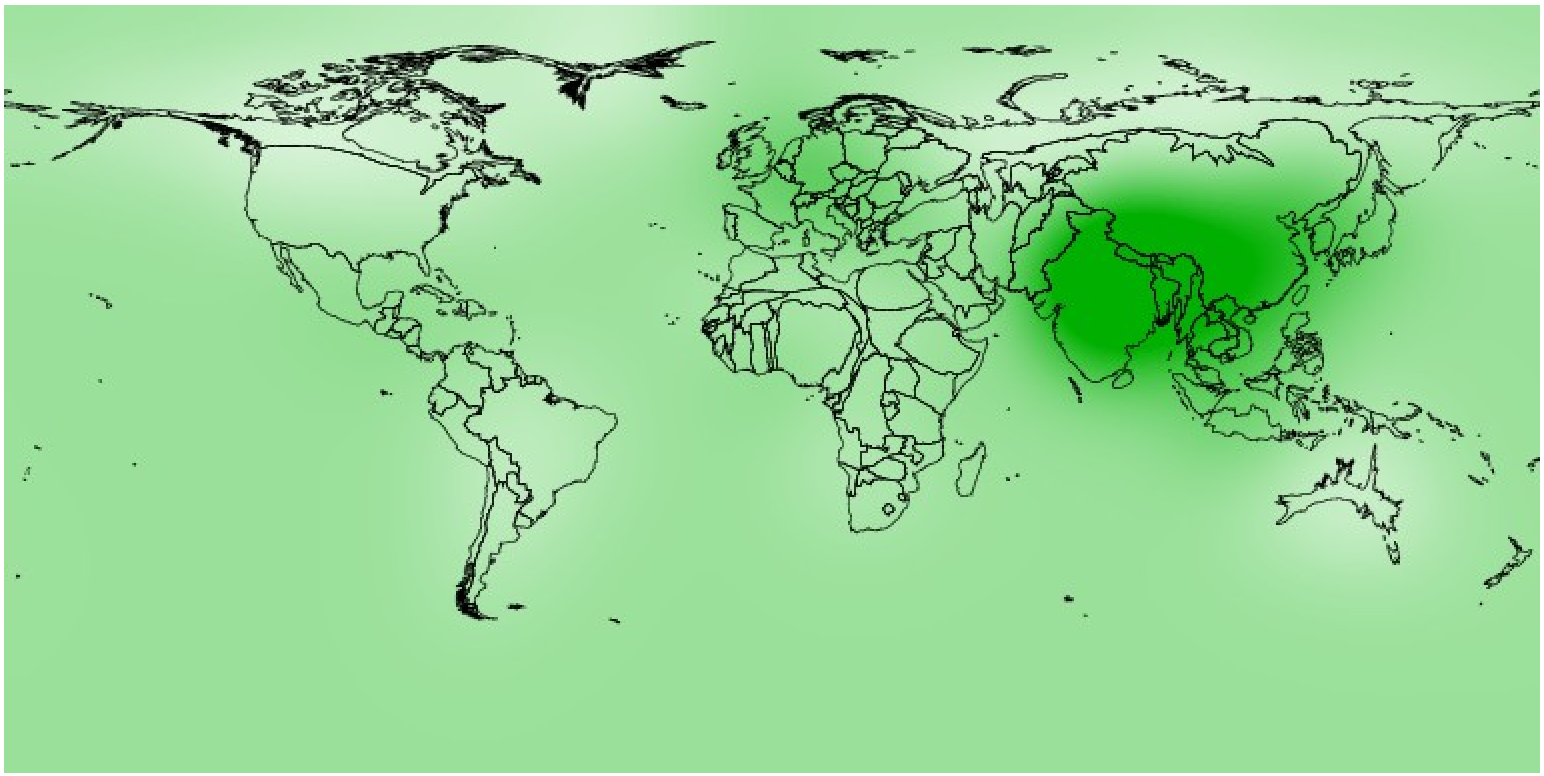


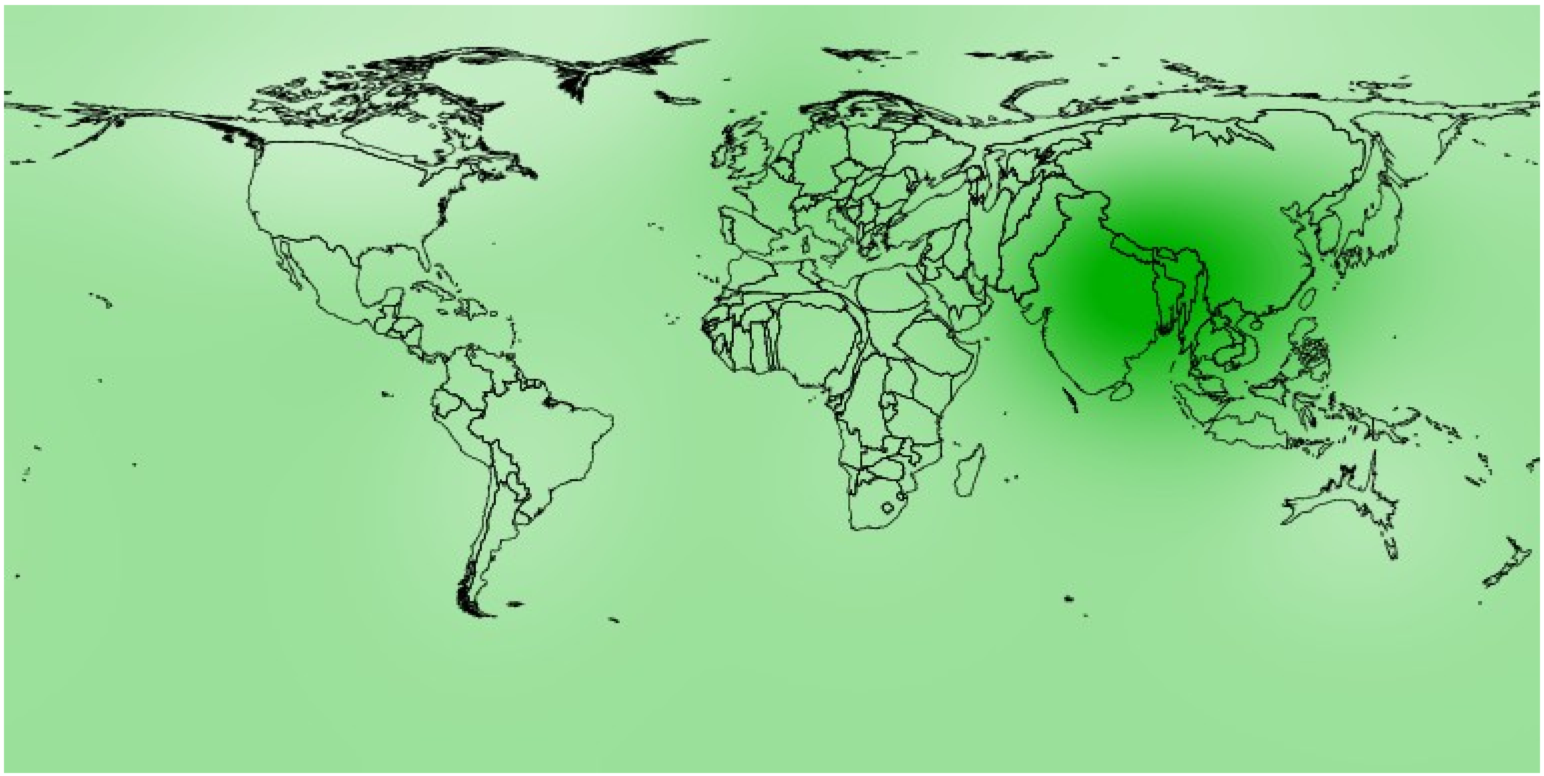


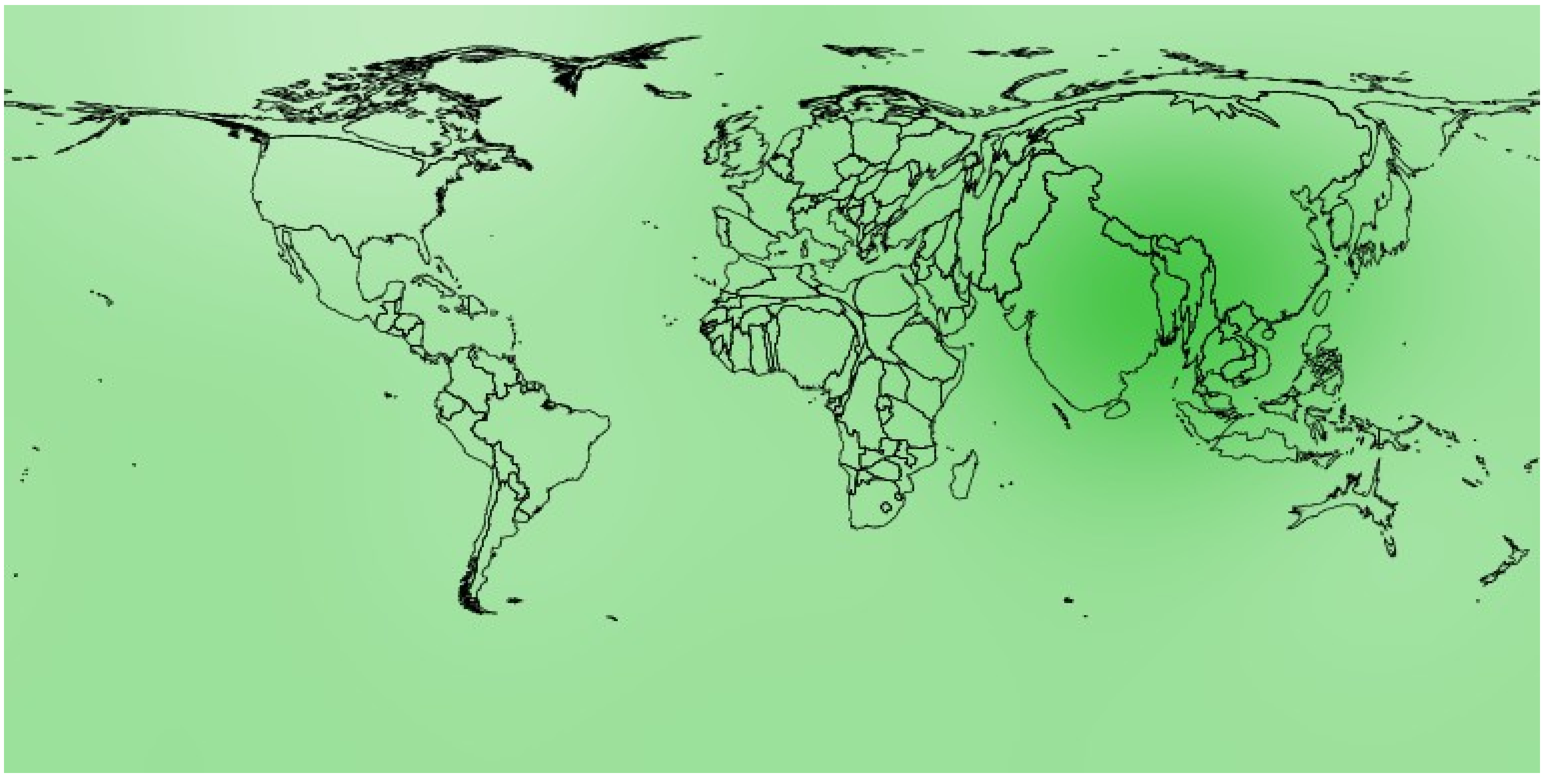








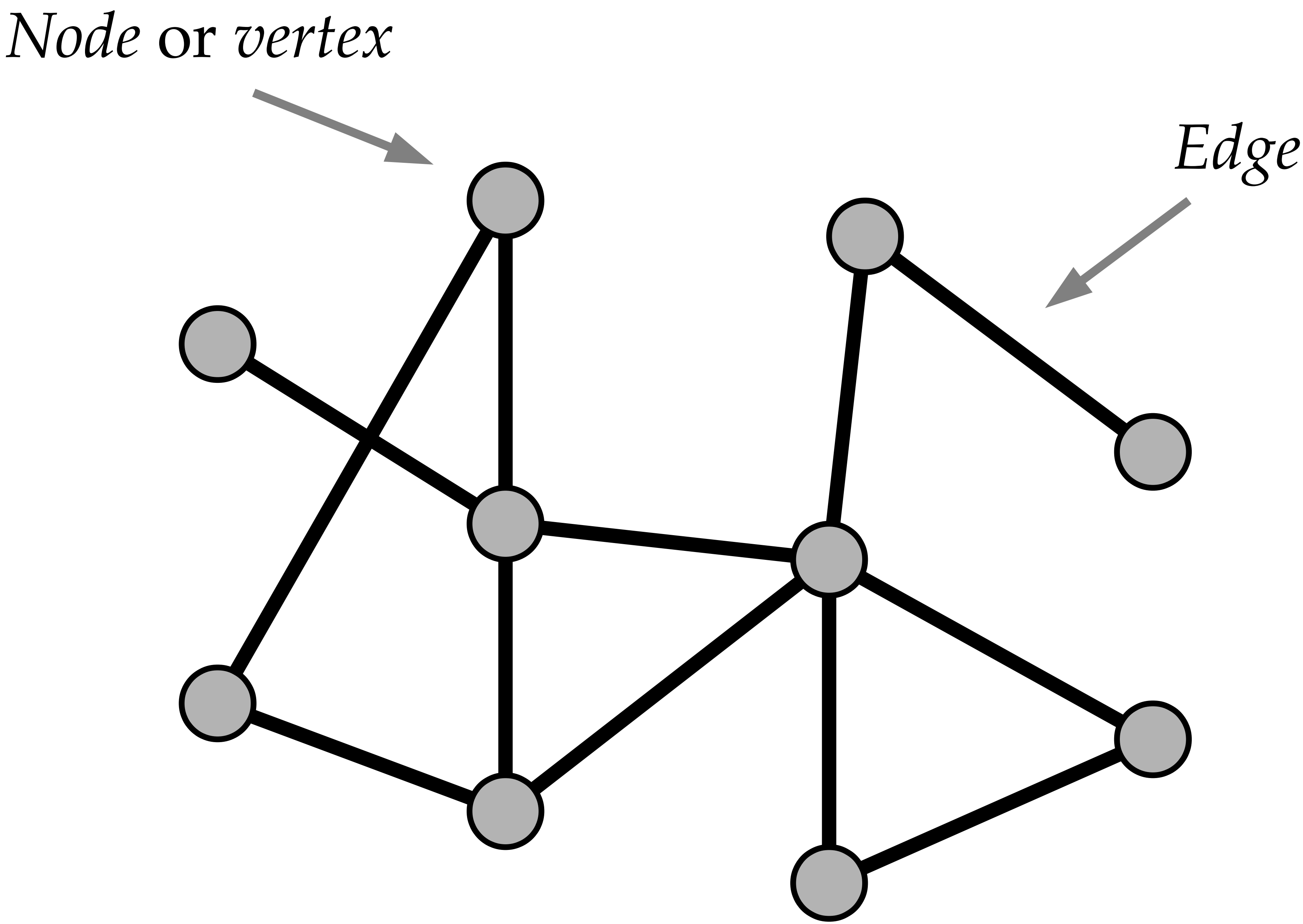


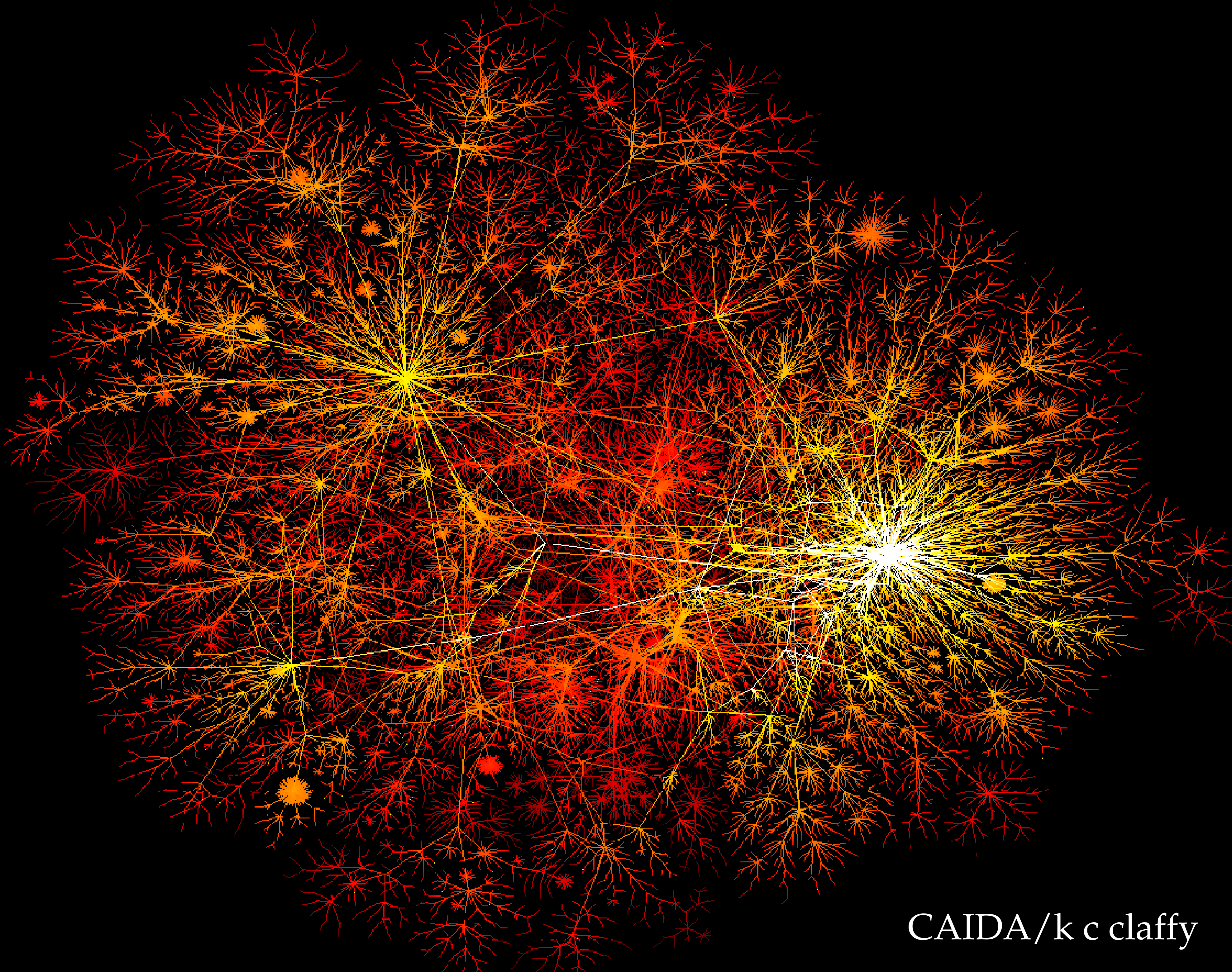


Examples

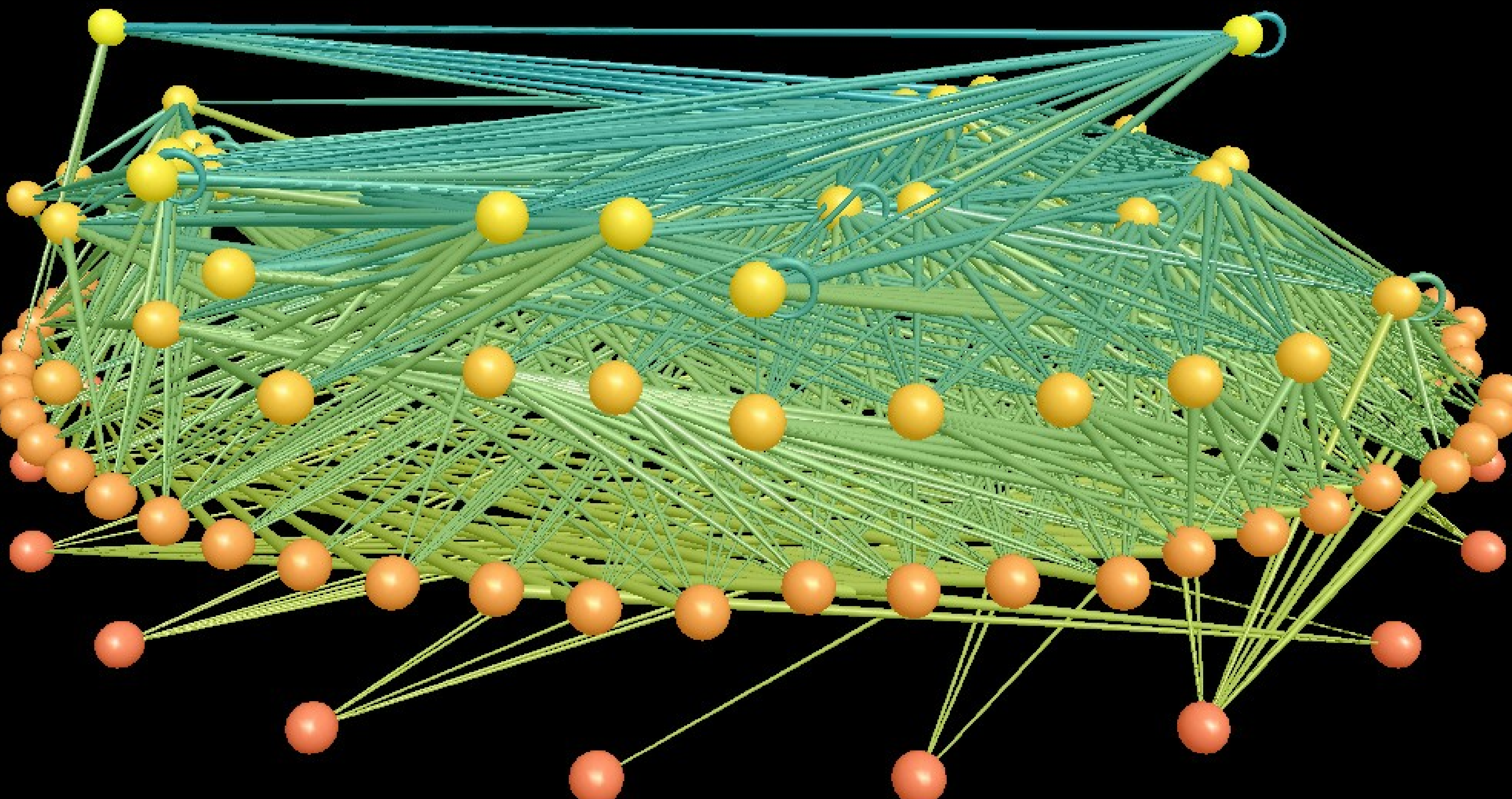
- A large selection of example maps made using this technique can be found at:

`http://www.worldmapper.org`

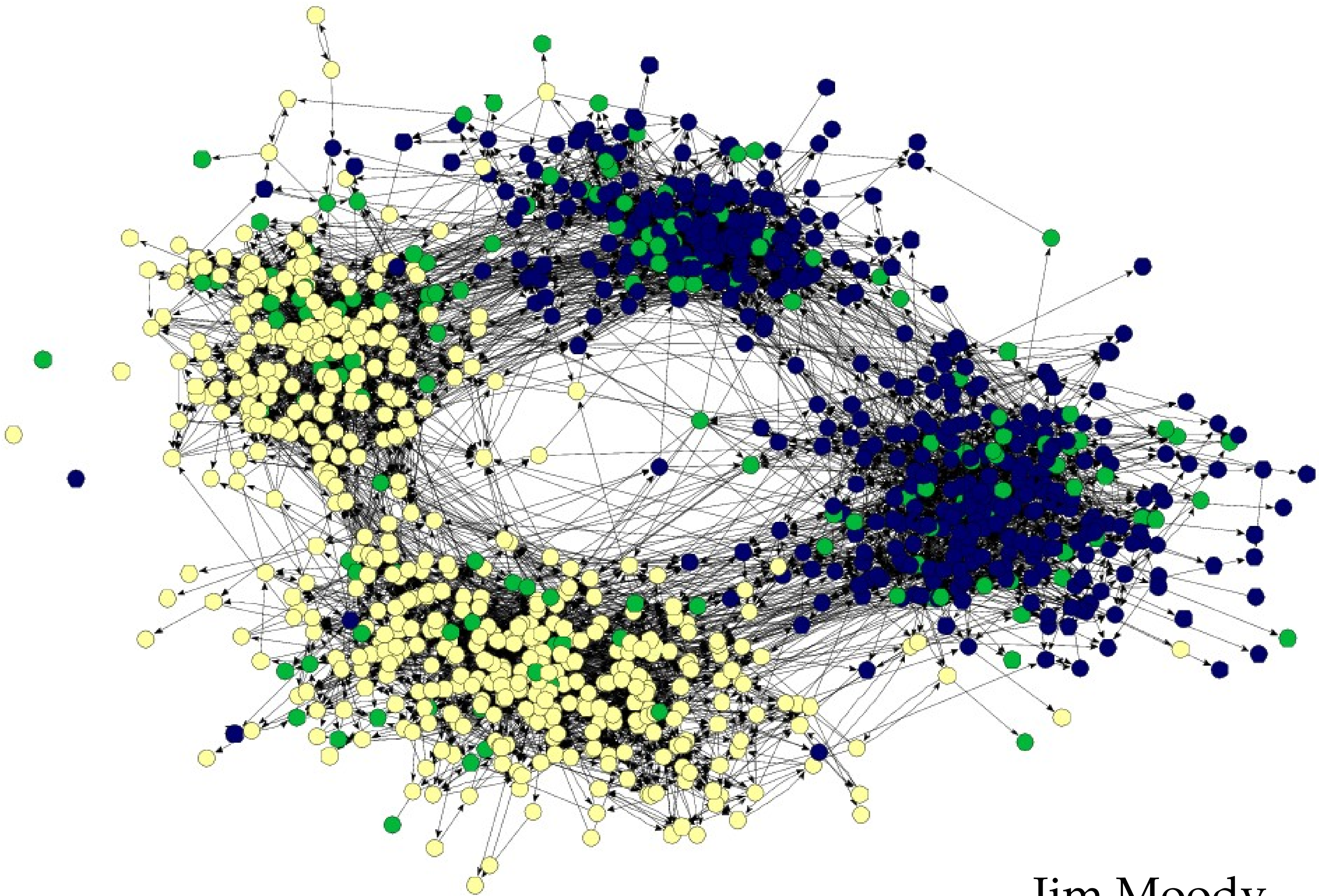

Node or *vertex*
Edge



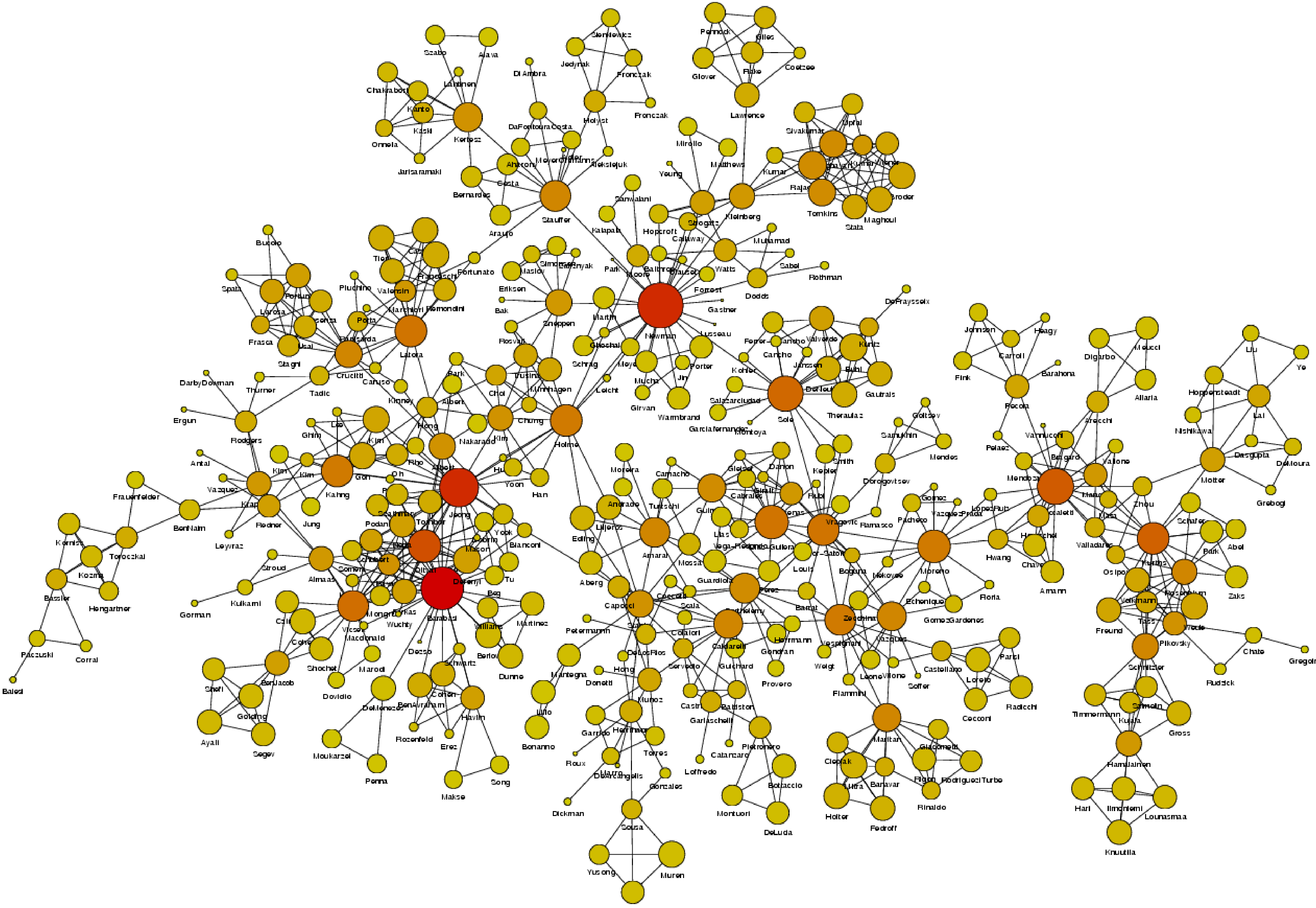
CAIDA/k c claffy



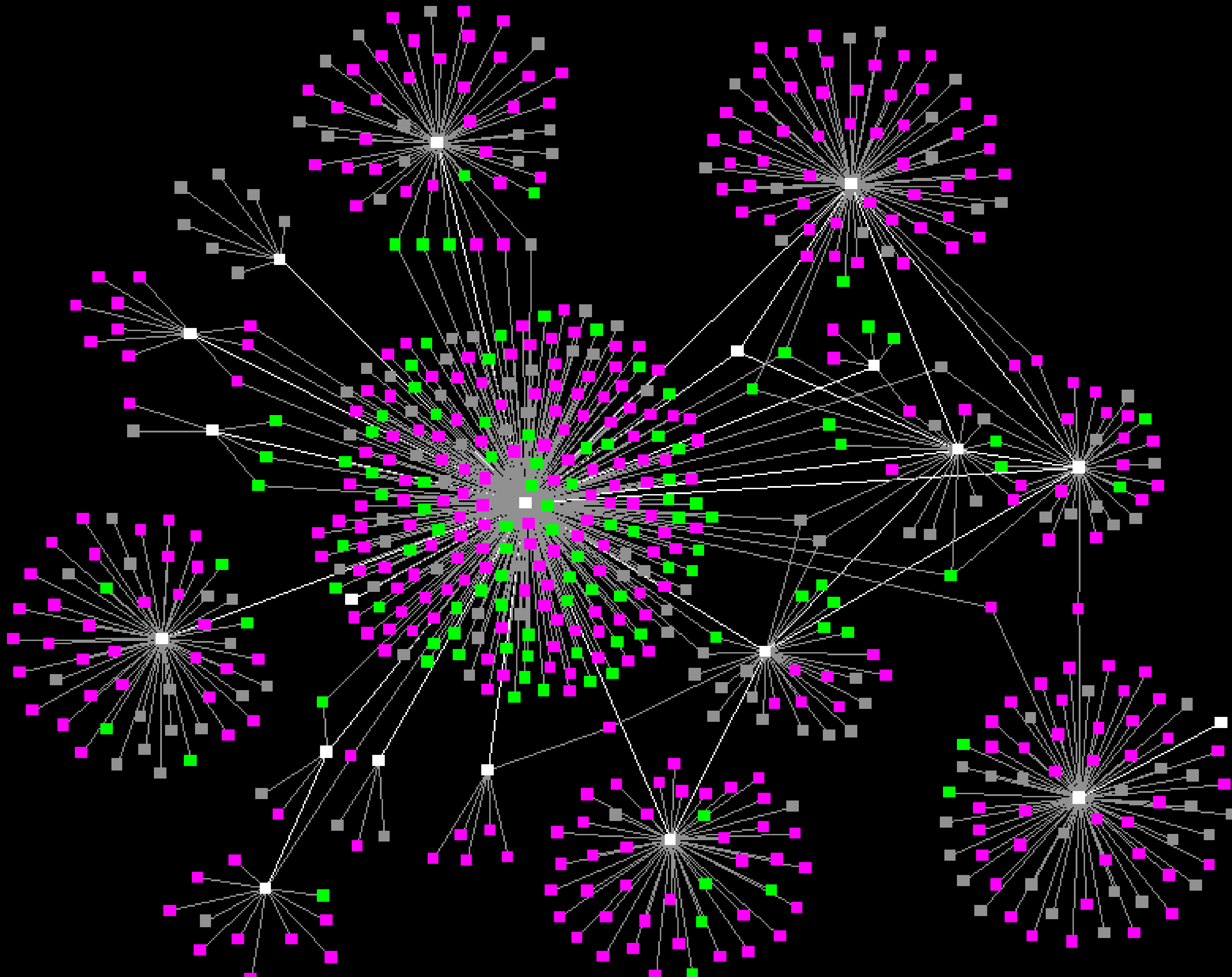
Neo Martinez and Rich Williams



Jim Moody



Szabo
Alava
Lahtinen
Chakraborti
Kanto
Kaski
Onnela
Kertesz
Jarisaramaki
DiAmbra
Sienkiewicz
Jedynak
Fronczak
Holyst
DaFontouraCosta
Aharony
Bernardes
Costa
Araujo
Stauffer
Pennock
Giles
Glover
Flake
Coetzee
Lawrence
Sivakumar
Upfal
Kumar
Tomkins
Stata
Maghoul
Broder
Mirollo
Matthews
Young
Hopcroft
Strogatz
Callaway
Kleinberg
Muhamad
Watts
Sabel
Rothman
Dodds
Forrest
Gastner
Newman
Moore
Park
Erikson
Maslov
Bak
Sneppen
Rosvall
Trusina
Minnhagen
Martin
Schrag
Leicht
Girvan
Warmbrand
Porter
Jin
Spata
Frasca
Stagni
Fortunato
Pluchino
Marchiori
Remondini
Porta
Latora
Crucitti
Caruso
Tadic
DarbyDowman
Thurner
Ergun
Rodgers
Antal
Vazquez
Frauenfelder
BenNaim
Leyvraz
Redner
Jung
Kahng
Goh
Kim
Oh
Lee
Ghim
Choi
Chung
Holme
Albert
Nakarado
Hong
Kinney
Rho
Jeong
Yoon
Han
Yook
Bianconi
Tu
Barabasi
Oltvai
Dezso
Schwartz
Cohen
Havlin
BenAvraham
Rozenfeld
Erez
Makse
Song
Williams
Martinez
Berlow
Dunne
Wuchty
Macdonald
Dovidio
DeMenezes
Moukarzel
Penna
Almaas
Kulkarni
Gorman
Stroud
Shochet
BenJacob
Shefi
Golding
Ayali
Segev
Toroczkai
Kozma
Hengartner
Bassler
Paczuski
Corral
Moreira
Andrade
Camacho
Turtschi
Amaral
Mossa
Edling
Liljeros
Aberg
Capocci
Colaiori
DeLosRios
Servedio
Munoz
Caldarelli
Vespignani
Barrat
Boguna
Petermann
Mantegna
Lillo
Bonanno
Donetti
Garrido
Marro
Roux
Dickman
Sousa
Yusong
Muren
Lingjiang
Torres
Gonzales
Castri
Garlaschelli
Battiston
Catanzaro
Loffredo
Pietronero
Montuori
Bottaccio
DeLuca
Provero
Hermann
Weigt
Flammini
Leone
Vilone
Soffer
Maritan
Giacometti
Banavar
Rinaldo
Cieplak
Holter
Fedroff
Castellano
Loreto
Cecconi
Parisi
Radicchi
Moreno
Echenique
GomezGardenes
Floria
Gomez
Pacheco
Hwang
Amann
Dorogovtsev
Mendes
Samukhin
Goltsev
Ramasco
Sole
Valverde
Kohler
Janssen
Gautrais
Theraulaz
GarciaFernandez
Montoya
DeFraysseix
Smith
Kepler
Fink
Johnson
Carroll
Barahona
Pecora
Mendoza
Valladares
Arecchi
Vallone
Zhou
Motter
Dasgupta
DeMoura
Grebogi
Hoppensteadt
Nishikawa
Lai
Liu
Ye
Digarbo
Meucci
Allaria
Abel
Zaks
Pikovsky
Freund
Chate
Gregoire
Rudzick
Schnitzler
Timmermann
Gross
Hamalainen
Hari
Ilmoniemi
Lounasmaa
Knuutila

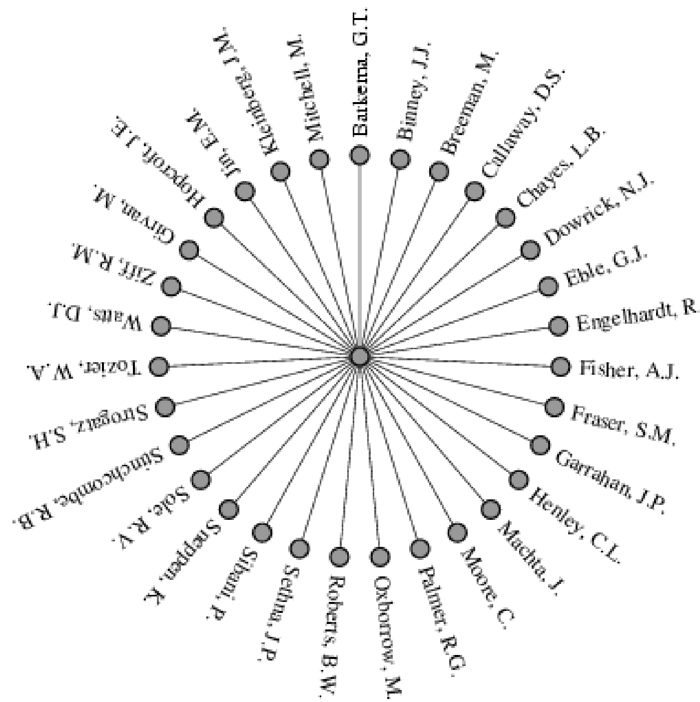


Valdis Krebs

The small-world effect

- Stanley Milgram's 1967 experiment:
  - 296 people were asked to get a letter to a target person in Boston (196 from Nebraska and 100 from Boston)
  - letters could only be passed along a chain of first-name acquaintances
  - 64 letters arrived (29%)
  - they took an average of 6.2 steps to get there
  - about a third went through one particular acquaintance of the target person (his tailor)


Newman, M.E.J.



Barkema, G.T.
Binney, J.J.
Breeman, M.
Callaway, D.S.
Chayes, L.B.
Dowrick, N.J.
Eble, G.J.
Engelhardt, R.
Fisher, A.J.
Fraser, S.M.
Garrahan, J.P.
Henley, C.L.
Machta, J.
Moore, C.
Palmer, R.G.
Oxborrow, M.
Roberts, B.W.
Sethna, J.P.
Sibani, P.
Sneppen, K.
Sole, R.V.
Stinchcombe, R.B.
Strogatz, S.H.
Tozier, W.A.
Watts, D.J.
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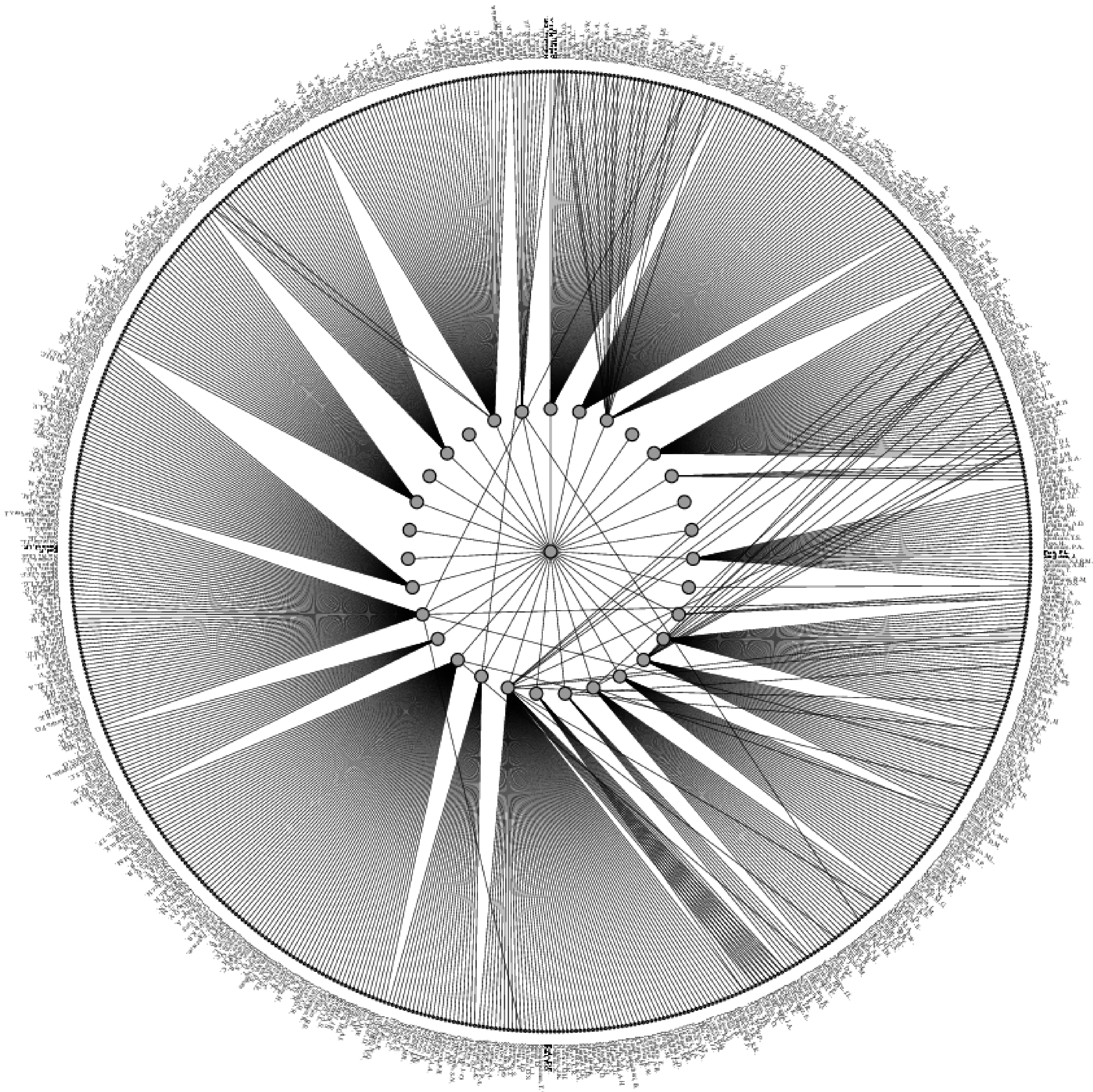
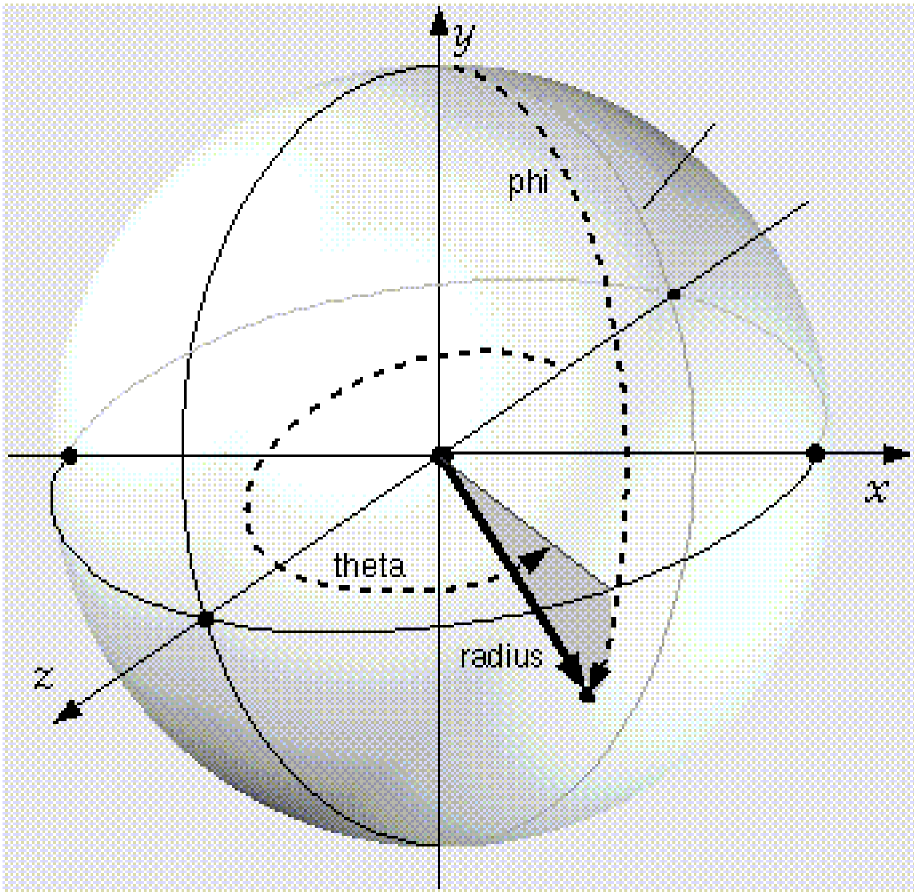
Girvan, M.
Hopcroft, J.E.
Jin, E.M.
Kleinberg, J.M.
Mitchell, M.



The small-world effect

- If each person knows 100 people then:
  - Number of people 1 step away from you is 100
  - People 2 steps away is 100 x 100 = 10,000
  - People 3 steps away is 100 x 100 x 100 = 1,000,000
  - People 4 steps away is 100,000,000
  - People 5 steps away is 10,000,000,000
- But 10 billion is more than the total number of people in the world



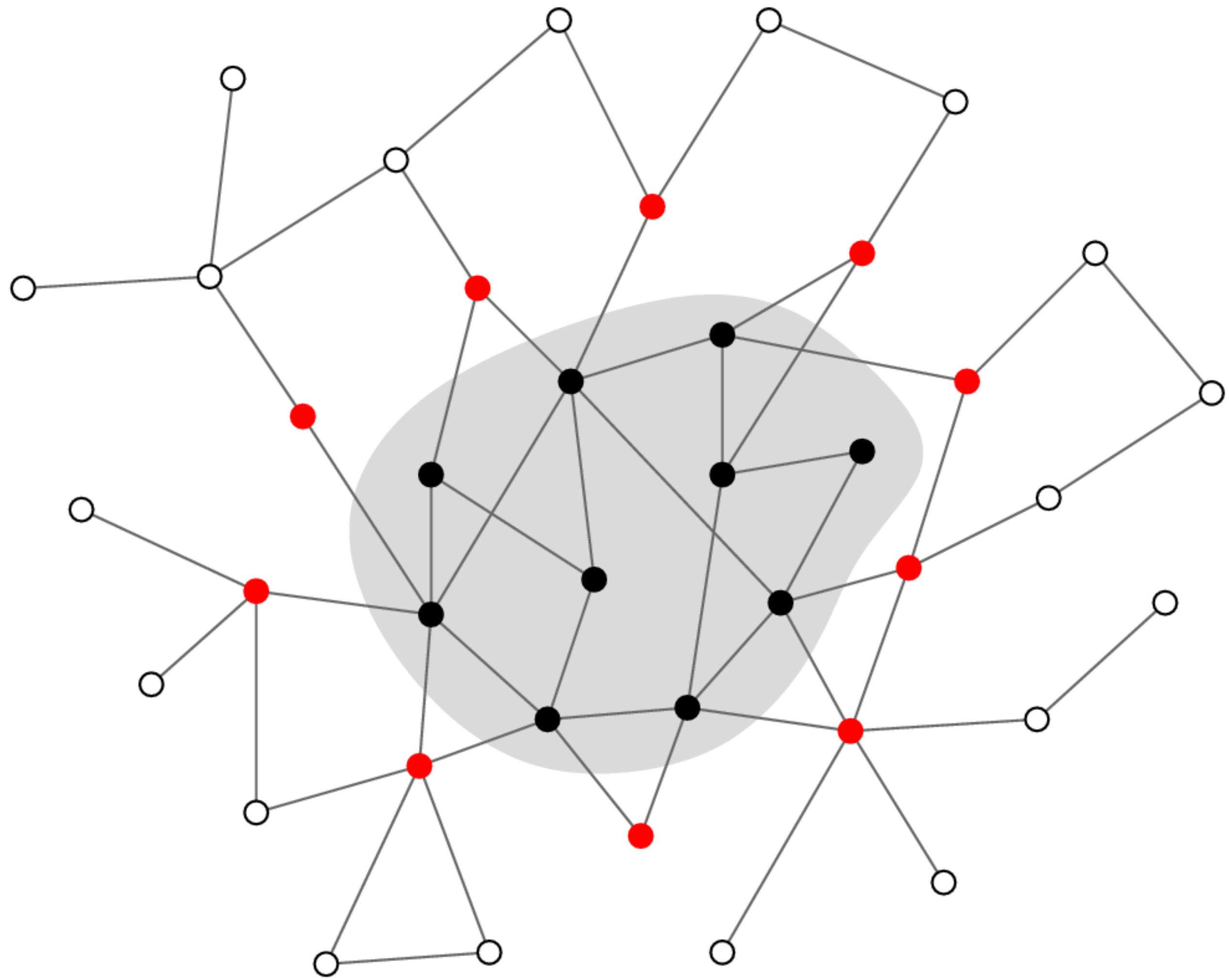
$$A = 4\pi r^2$$

$$V = \frac{4}{3}\pi r^3$$

$$A = \sqrt[3]{36\pi}\, V^{2/3}$$

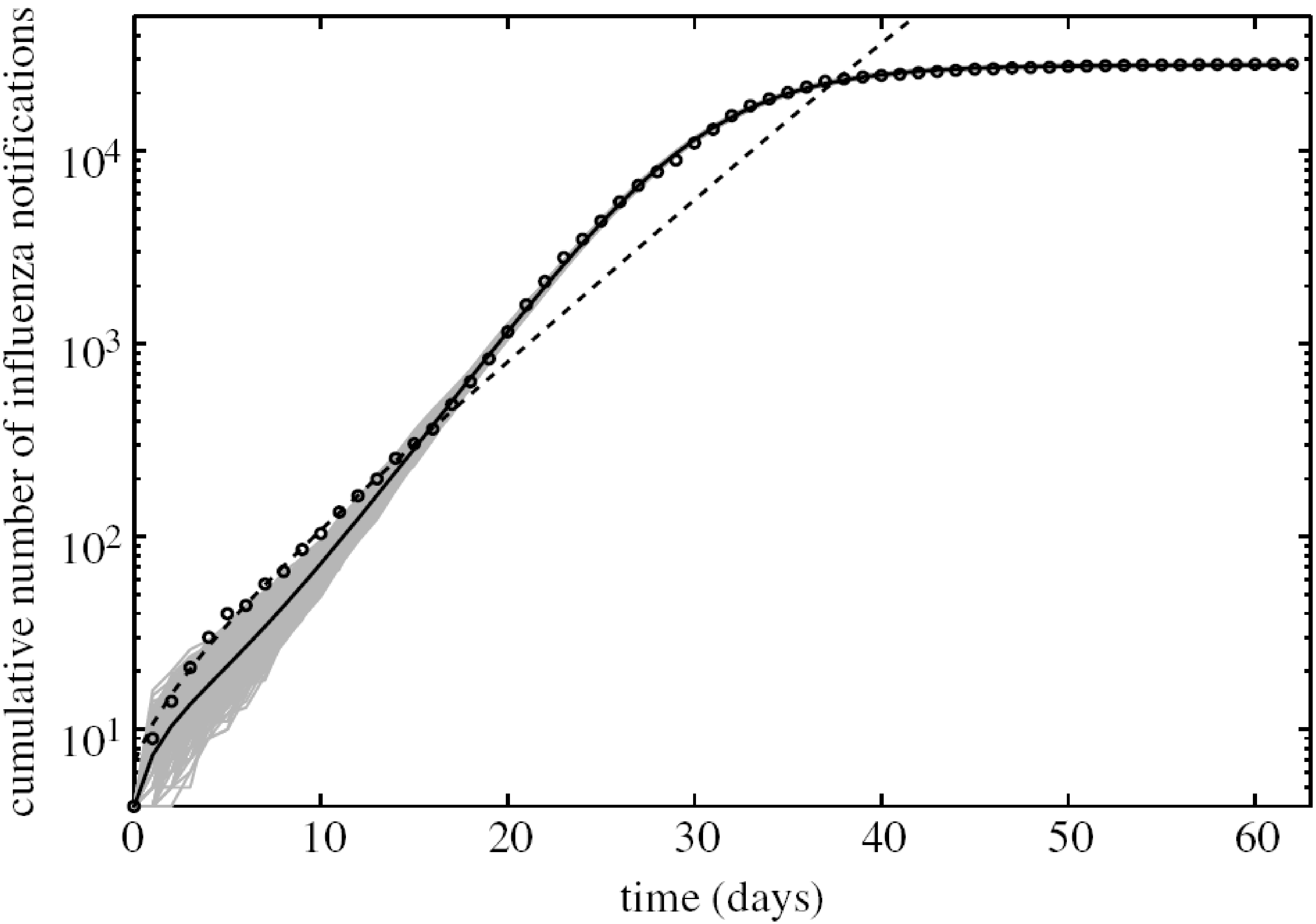
And in general dimension d

$$A \propto V^{(d-1)/d}$$

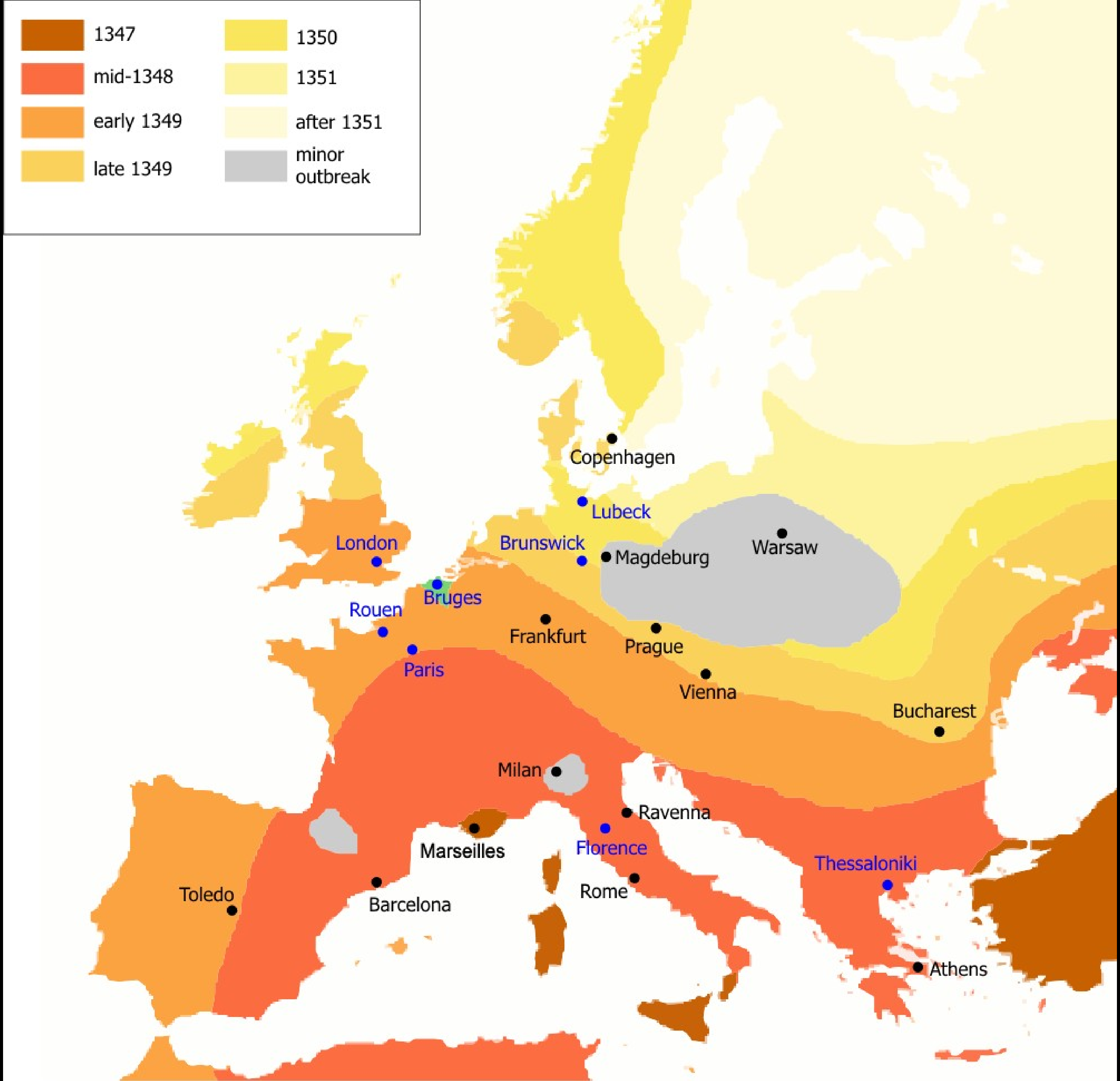


- In a network the surface can be proportional to volume
- Equivalent to choosing d infinite

Spanish flu, San Francisco 1918-1919



Chowell *et al.* 2007



1347
mid-1348
early 1349
late 1349
1350
1351
after 1351
minor outbreak
Copenhagen
Lubeck
London
Brunswick
Magdeburg
Warsaw
Bruges
Rouen
Frankfurt
Prague
Paris
Vienna
Bucharest
Milan
Ravenna
Marseilles
Florence
Thessaloniki
Toledo
Barcelona
Rome
Athens

How many people does a person know?

- Just asking people is a bad way to measure this
  - People are very bad at recalling all the people they know
- Bernard and Killworth suggested a different strategy:
  - Give people a list of surnames
  - Ask them how many people they know with names on the list
  - Scale up to all the surnames in the nation (allowing for the correct [Pareto] distribution of surnames)

Top US Surnames

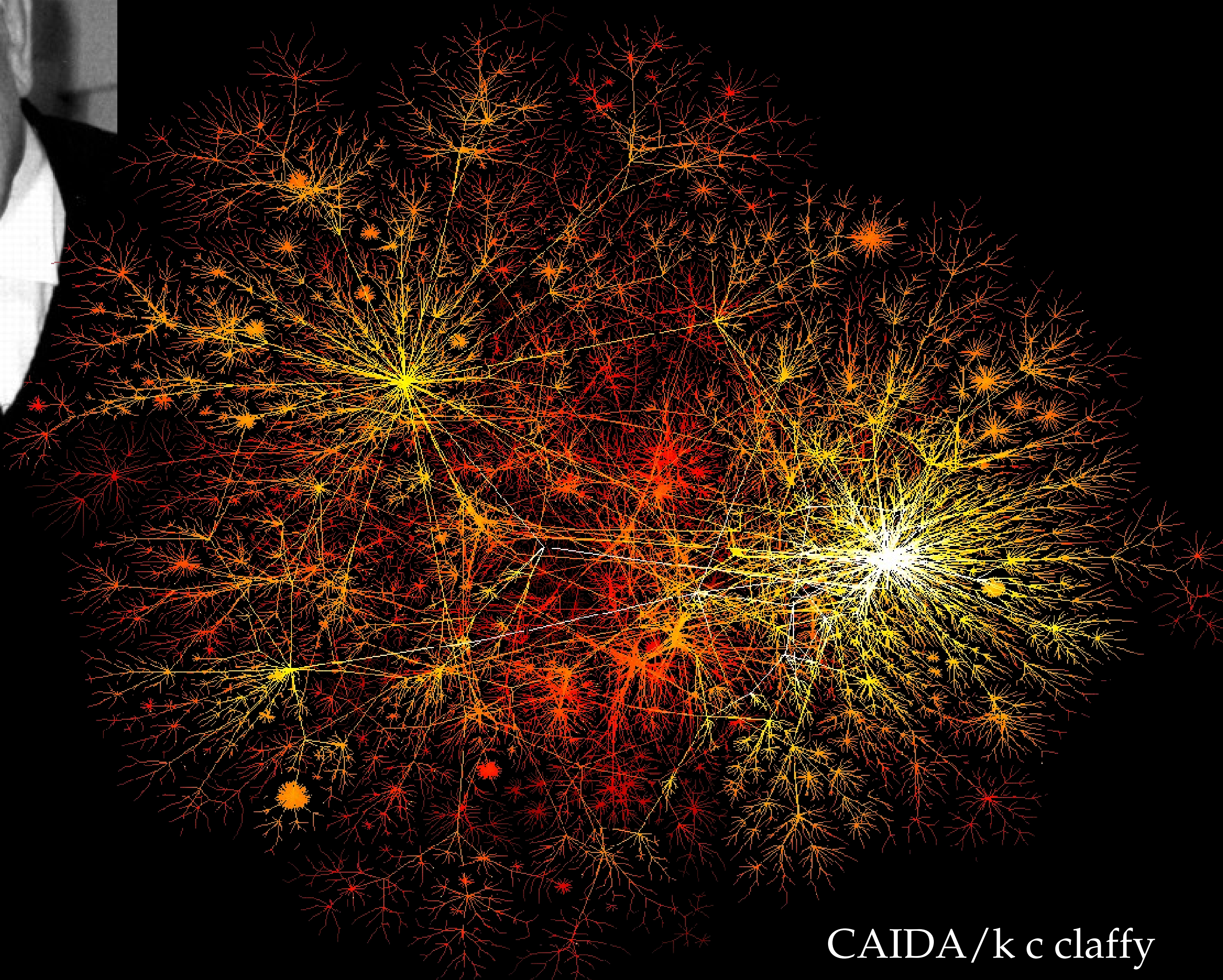
Adams
Allen
Anderson
Baker
Brown
Campbell
Carter
Clark
Collins
Davis
Edwards
Evans

Garcia
Gonzalez
Green
Hall
Harris
Hernandez
Hill
Jackson
Johnson
Jones
King
Lee

Lewis
Lopez
Martin
Martinez
Miller
Mitchell
Moore
Nelson
Parker
Perez
Phillips
Roberts

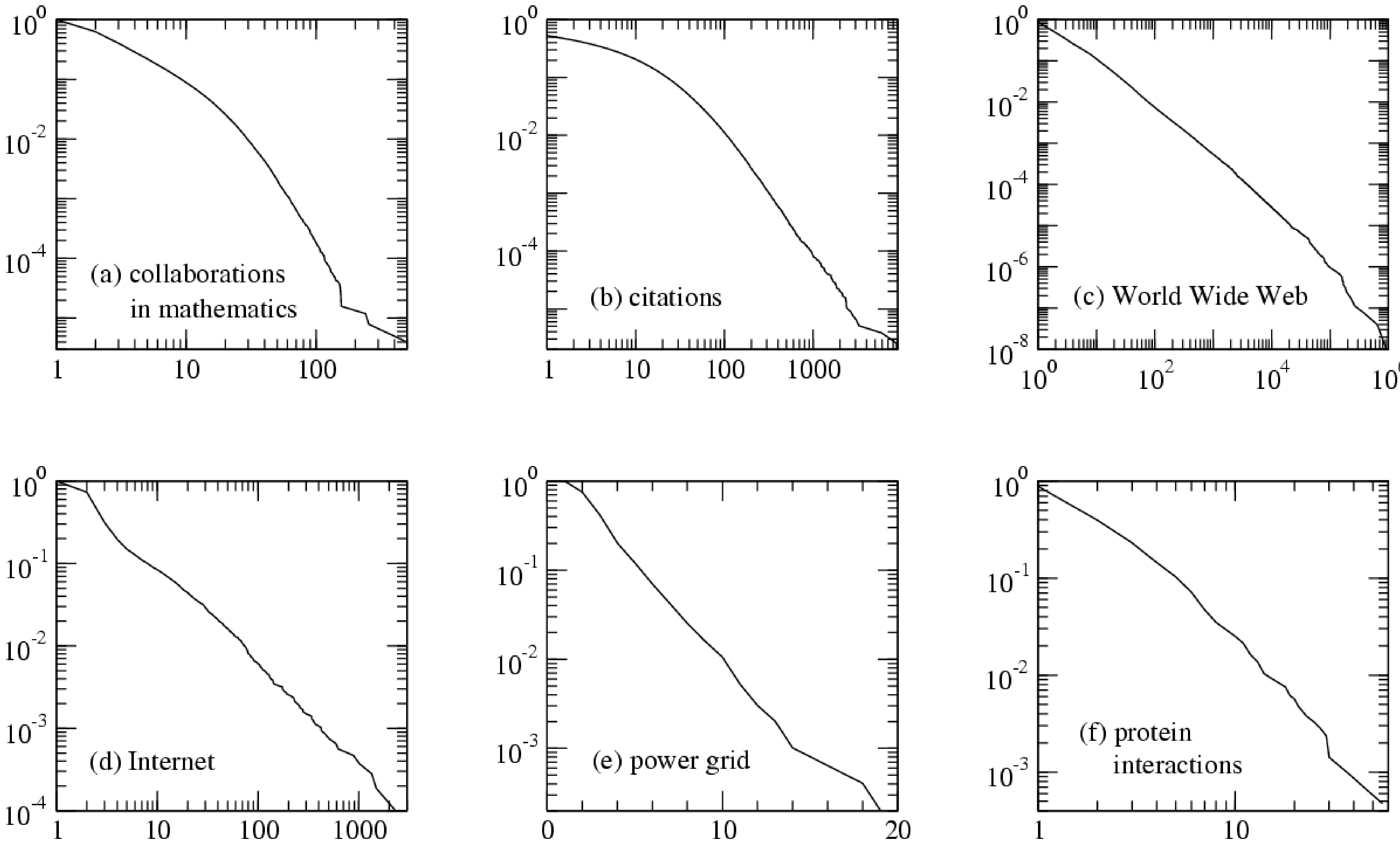
Robinson
Rodriguez
Scott
Smith
Taylor
Thomas
Thompson
Turner
Walker
White
Williams
Wilson

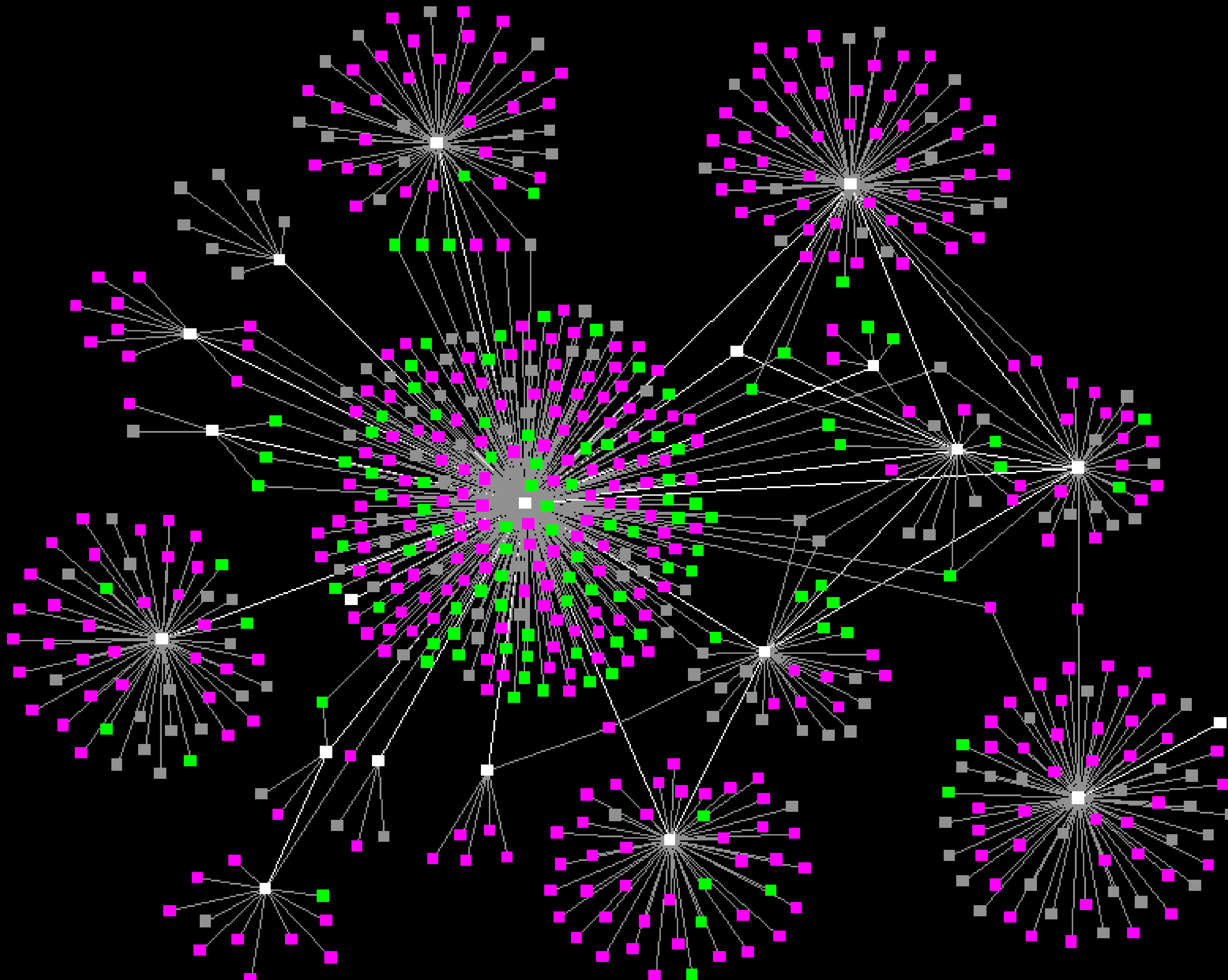
Anatol Rapoport

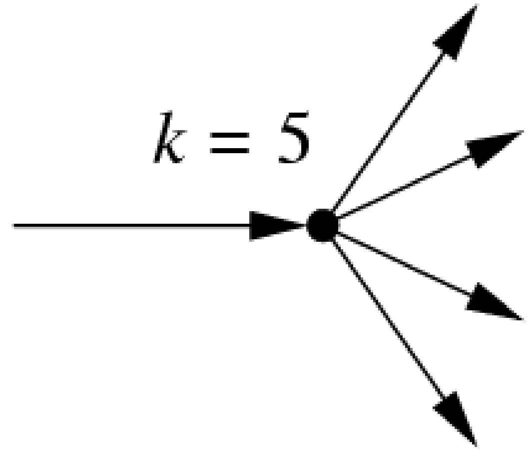


CAIDA/k c claffy

Degree distributions







Number of secondary cases is $T(k-1)$ where T is the transmissibility.

$$R_0 = T\frac{\sum_i k_i(k_i-1)}{\sum_i k_i} = T\frac{\langle k^2\rangle - \langle k\rangle}{\langle k\rangle}.$$

Disease spreads if $R_0 > 1$ or if

$$T > \frac{\langle k\rangle}{\langle k^2\rangle - \langle k\rangle}.$$

Epidemic threshold depends on the structure of the network.

Information content of radio transmissions

Suppose we send a message composed of a stream of symbols or characters, and suppose that the character A occurs with probability p_A.

- If $p_A = \frac{1}{2}$, then we need 1 bit to say whether it's present.
- If $p_A = \frac{1}{4}$, then we need 2 bits.
- If $p_A = \frac{1}{8}$, then we need 3 bits.

In general a character i that occurs with probability p_i carries information $\log_2(1/p_i) = -\log_2 p_i$ bits.

Average information per character

$$S = -\sum_i p_i \log_2 p_i.$$

(Shannon 1948.)

What distribution over characters gives the highest mean rate of transmission of information? We maximize the average information S subject to the constraint

$$\sum_i p_i = 1,$$

giving

$$-\frac{\partial}{\partial p_i}\sum_i p_i \log p_i - \alpha\frac{\partial}{\partial p_i}\left[\sum_i p_i - 1\right] = 0,$$

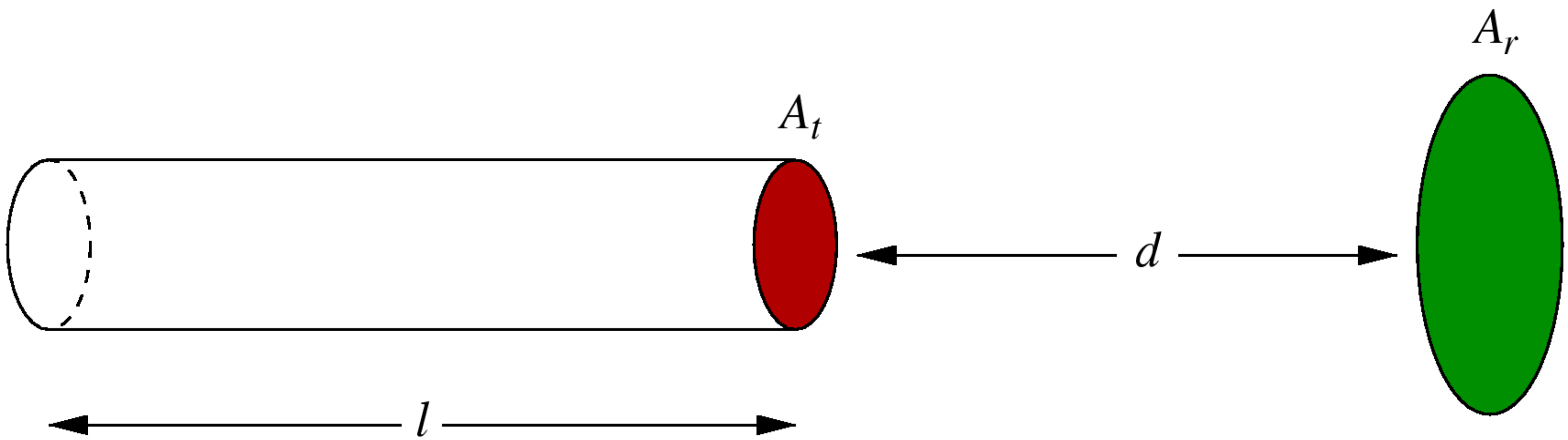
for all i, or

$$p_i = \mathrm{e}^{-\alpha-1} = \text{constant}.$$

So all characters are equally likely.

A message with maximal information rate is indistinguishable from random noise to an observer unfamiliar with the code in which the message is written.

To study the equivalent question for radio waves, consider the following thought experiment:



We set up electromagnetic microstates in the perfectly reflecting interior of the cavity. Each microstate is one “character” of the message.

We have a mean transmission power or "energy budget" of $\langle E \rangle$ per message.

Any number of photons is allowed in the cavity.

Constraints:

$$\sum_i p_i = 1, \qquad \sum_i p_i E_i = \langle E \rangle, \qquad \sum_i p_i N_i = \langle N \rangle.$$

This is a well-known problem:

$$\frac{\partial S}{\partial p_i} - \alpha \frac{\partial}{\partial p_i}\left[\sum_i p_i - 1\right] - \beta \frac{\partial}{\partial p_i}\left[\sum_i p_i E_i - \langle E \rangle\right] - \gamma \frac{\partial}{\partial p_i}\left[\sum_i p_i N_i - \langle N \rangle\right] = 0$$

for all i.

$$p_i = \frac{\mathrm{e}^{-\beta(E_i - \mu N_i)}}{Z},$$

where $\mu = \gamma/\beta$ and $Z = \sum_i \mathrm{e}^{-\beta(E_i - \mu N_i)}$.

The rest of the calculation is well known. We put

$$N = \sum_k n_k, \qquad E = \sum_k n_k \epsilon_k,$$

and perform the sum over all the occupancy numbers n_k to find that

$$Z = \sum_{\{n_k\}} \prod_k \mathrm{e}^{-\beta(\epsilon_k - \mu) n_k} = \prod_k \sum_{n_k=0}^{\infty} \mathrm{e}^{-\beta(\epsilon_k - \mu) n_k} = \prod_k \left[1 - \mathrm{e}^{-\beta(\epsilon_k - \mu)}\right]^{-1}.$$

Then the mean number of photons in the cavity is

$$\begin{aligned} \langle N \rangle &= \beta^{-1} \frac{\partial \ln Z}{\partial \mu} = \sum_k \frac{1}{\mathrm{e}^{\beta(\epsilon_k - \mu)} - 1} \\ &= \sum_k \langle n_k \rangle . \end{aligned}$$

So

$$\langle n_k \rangle = \frac{1}{\mathrm{e}^{\beta \epsilon_k} - 1}.$$

This distribution over modes maximizes the information content.

But now here's the trick. Instead of passing the cavity to the receiver, we just take the lid off and send the message by radio.

The density of single-particle states that have momentum in the right direction to hit the receiver distance d away is

$$\rho(\epsilon) = \frac{2\ell A_t A_r \epsilon^2}{d^2 h^3 c^3}.$$

So the power spectrum of the message is

$$I(\epsilon) = \frac{2\ell A_t A_r}{d^2 h^3 c^3} \frac{\epsilon^3}{e^{\beta\epsilon} - 1}.$$

A message with maximal information rate transmitted using radio waves ***has the energy spectrum of blackbody radiation****.*

Apparent temperature:

$$T^4 = \frac{15h^3c^2}{2\pi^4} \frac{d^2}{A_t A_r} P.$$

Information rate:

$$\frac{\mathrm{d}S}{\mathrm{d}t} = \frac{8\pi^4}{45h^3c^2} \frac{A_t A_r}{d^2} T^3 = \left[\frac{512\pi^4}{1215h^3c^2} \frac{A_t A_r}{d^2} P^3 \right]^{\frac{1}{4}}.$$

For a transmitter and receiver of one square meter each, a meter apart, with a power of $P = 1\,\mathrm{W}$, the information rate is 1.61×10^{21} bits per second.

Note that the information rate increases with the area of the transmitter and receiver, so that the best information rate for a given energy budget is achieved for large antennas and low apparent temperature.

Any sufficiently advanced technology is indistinguishable from noise